QCD and Collider Physics III: Drell – Yan, W and Z production

- Drell – Yan
  - history of experimental significance: J/psi, Z/W ?
- Drell – Yan:
  - Factorisation theorem
  - NLO calculations
  - Q_t resummations (next lecture)
- Drell – Yan:
  - Tevatron / LHC
- Literature:

Ellis, Stirling, Webber: *QCD and Collider Physics*
Field: *Applications of perturbative QCD*
CTEQ summerschool 2003
References in lecture

http://www-h1.desy.de/~jung/qcd_collider_physics_wise_2006

Massive muon pairs

VOLUME 25, NUMBER 21 PHYSICAL REVIEW LETTERS 23 NOVEMBER 1970

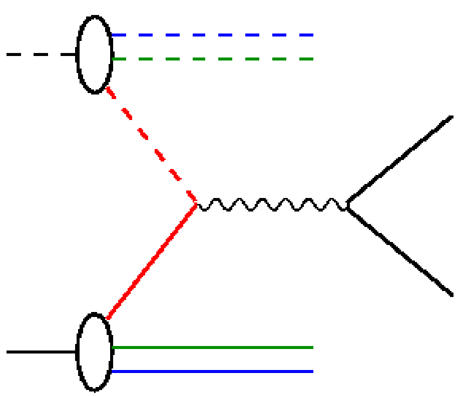
Observation of Massive Muon Pairs in Hadron Collisions*

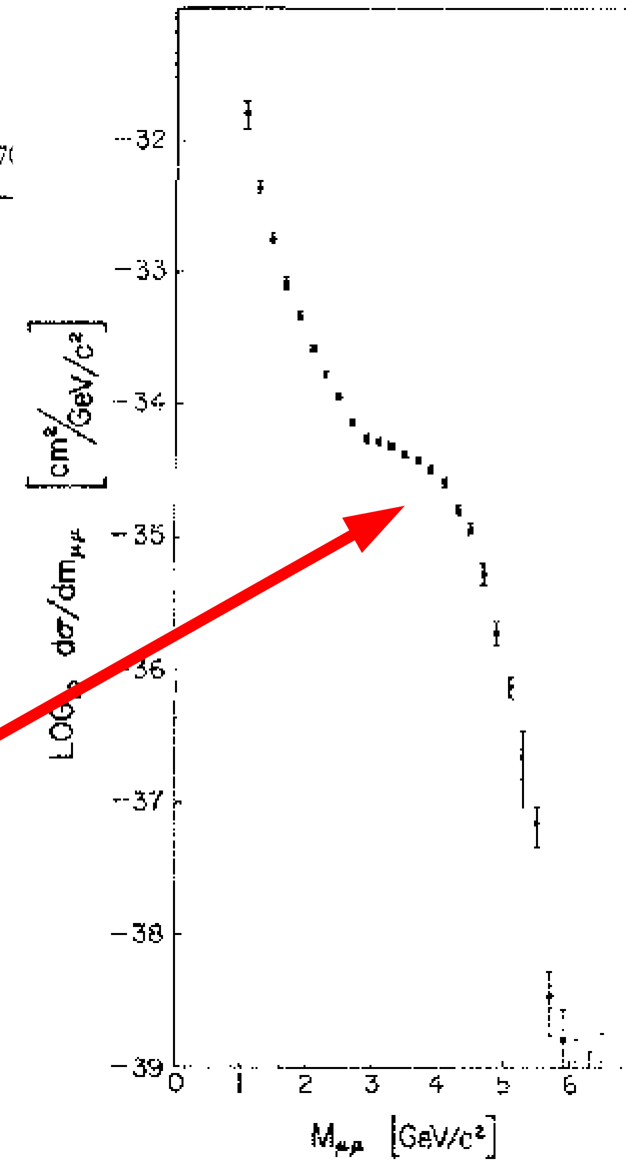
J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, and B. G. Pope
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

E. Zavattini
CERN Laboratory, Geneva, Switzerland

(Received 8 September 1970)





Search for a new weak boson ... not found,

But there were J/psi in the data

Observation of J/psi

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1974

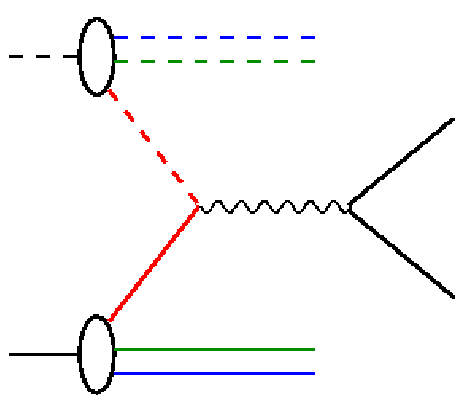
Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \rightarrow e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.



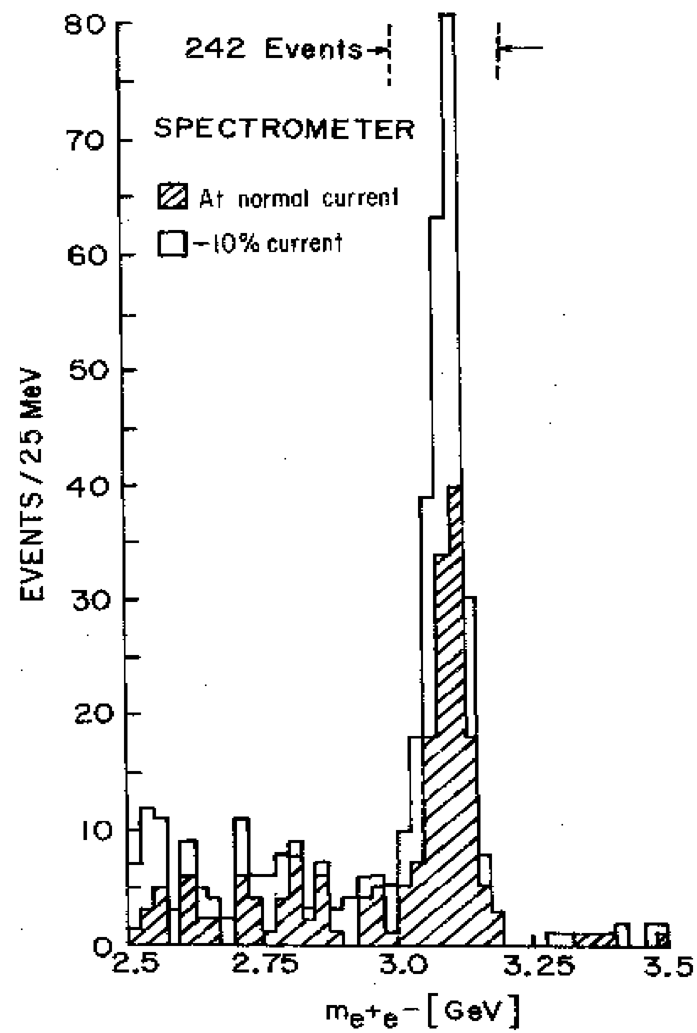


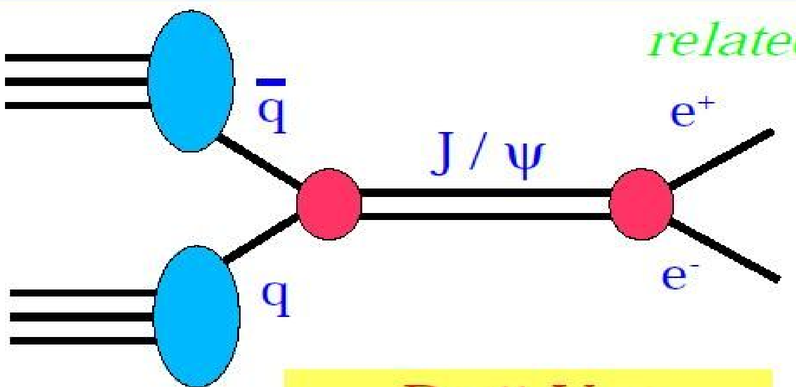
FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

J/psi discoveries

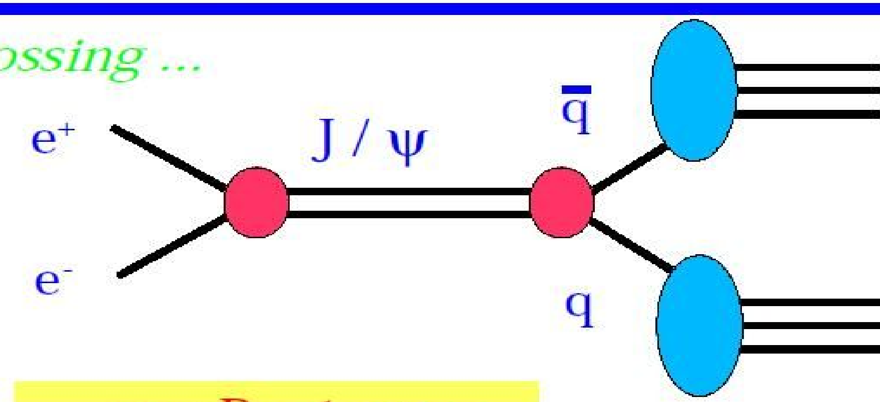
Fred Olness, CTEQ
summerschool 2003

The November Revolution

related by crossing ...



Drell-Yan
Brookhaven AGS



e^+e^- Production
SLAC SPEAR
Frascati ADONE

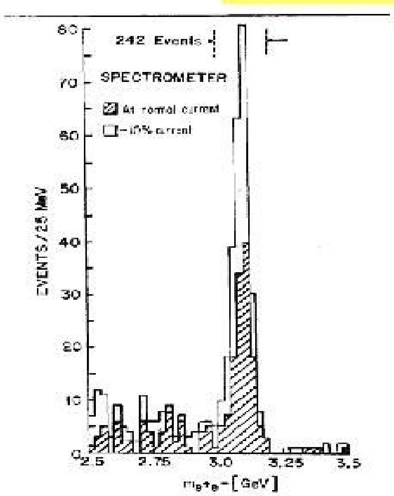
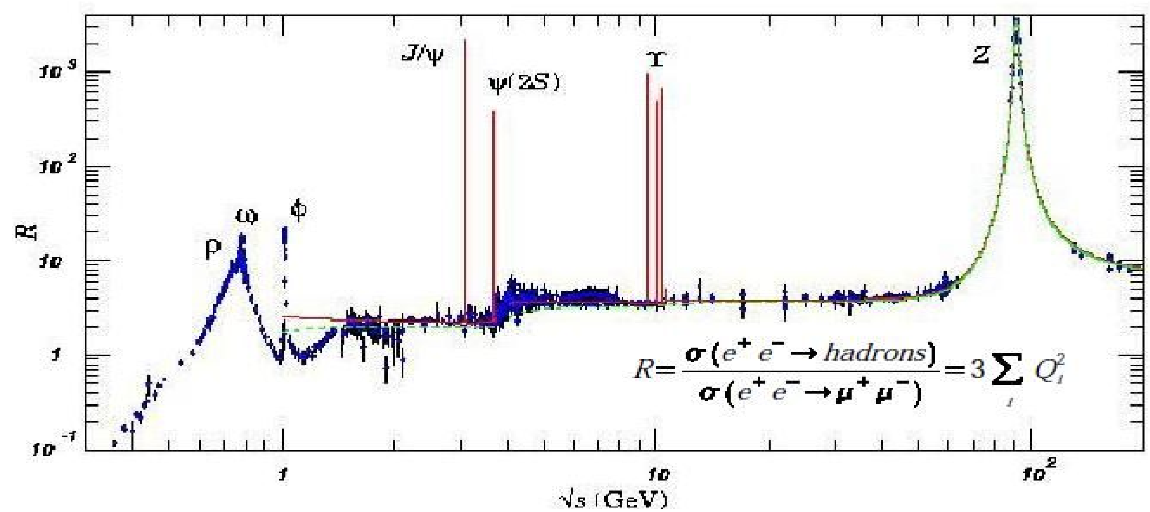


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.



Drell – Yan in lowest order

Study Of Scaling In Hadronic Production Of Dimuons.
J.K.Yoh et al. Phys.Rev.Lett.41:684,1978, Erratum-ibid.41:1083,1978.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|M|^2$$

$$|M_{e^+e^-\to l^+l^-}|^2 = 2\,(4\pi\alpha)^2\,\frac{t^2+u^2}{s^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos\theta)^2$$

$$\sigma(q\bar{q}\to l^+l^-) = \frac{4\pi\alpha^2}{3s}e_q^2$$

$$\frac{d\sigma(q\bar{q}\to l^+l^-)}{dQ^2} = \frac{4\pi\alpha^2}{9Q^4}\sum_q e_q^2\int dx_1\int dx_2 \; f_q(x_1)f_{\bar{q}}(x_2)\delta\left(1-\frac{x_1x_2s}{Q^2}\right)$$

$$\tau = z = \frac{s}{Q^2},\, Q^2 = m^2_{l^+l^-}$$

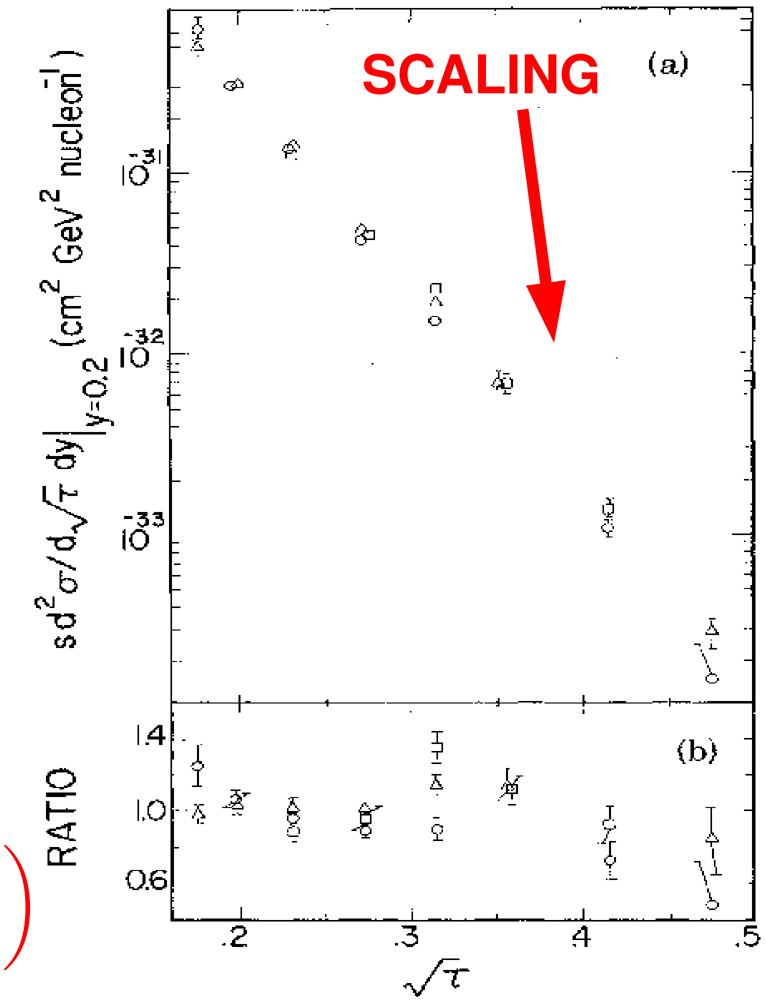


FIG. 3. (a) $s\, d^2\sigma/d\sqrt{\tau}dy|_{y=0.2}$ vs $\sqrt{\tau}$. Circles, triangles, and squares correspond to 400-, 300-, and 200-GeV beam energy, respectively. (b) Above data divided by the overall fit $Ae^{-b\sqrt{\tau}}$.

Drell – Yan at high energies: Z, W

$$\sigma(q\bar{q} \to l^+l^-) = \frac{4\pi\alpha^2}{3s}\frac{1}{N}\left(e_q^2 - 2e_q V_l V_q \chi_1(s) + (A_l^2 + V_l^2)(A_q^2 + V_q^2)\chi_2(s)\right)$$

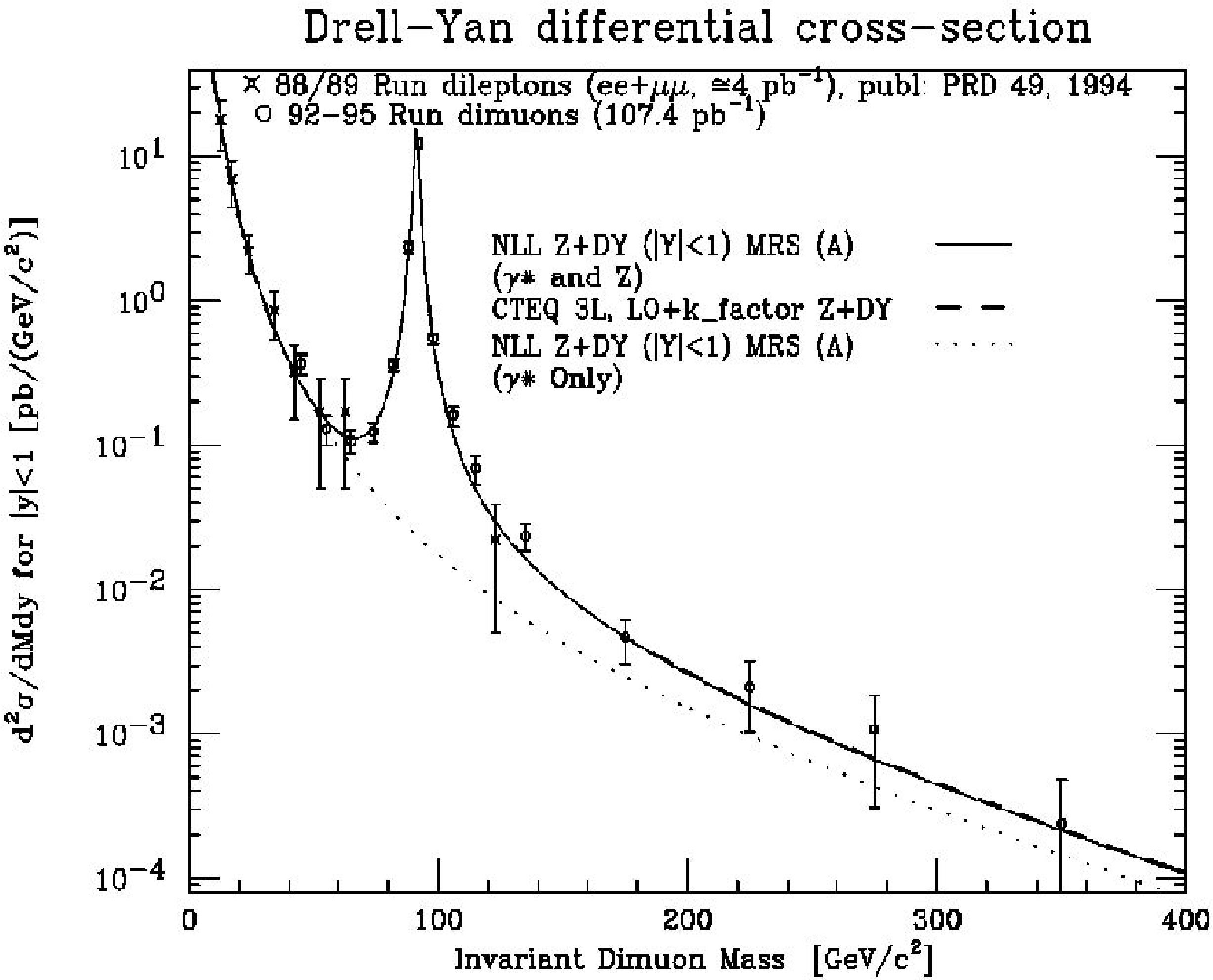
Measurement of Z0 and Drell-Yan production cross-section using dimuons in anti-p p collisions at S**(1/2) = 1.8-TeV. CDF Collaboration F. Abe et al. Phys.Rev.D59:052002,1999.

$$\sigma(q\bar{q} \to \gamma^* \to l^+l^-) = \frac{4\pi\alpha^2}{3s}\frac{1}{N}e_q^2$$

$$\chi_1(s) = \xi \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$$

$$\chi_2(s) = \xi^2 \frac{s^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}$$

$$\xi = \frac{\sqrt{2} G_F M_Z^2}{16\pi\alpha}$$

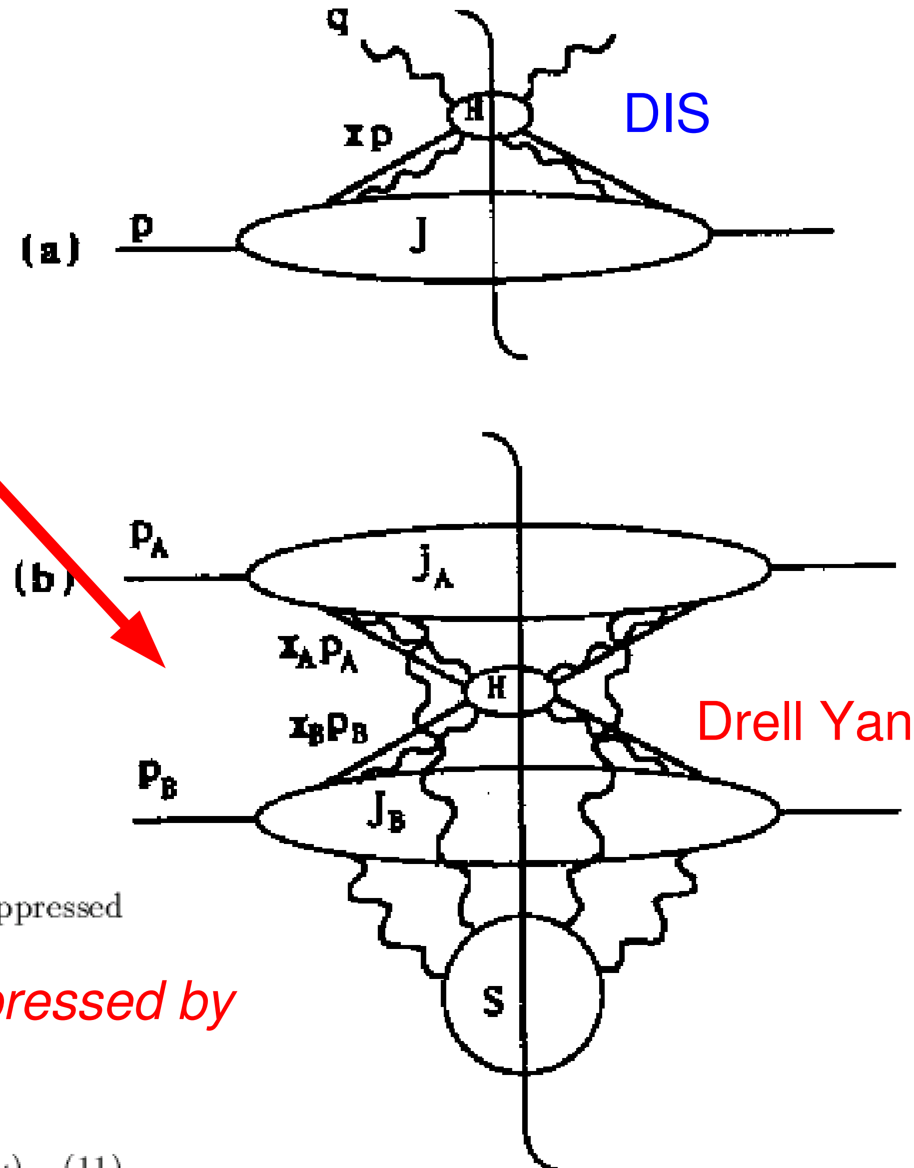


- Z exchange visible !!!
- from now on ignore W/Z exchange, concentrate only on QCD part....

Factorisation in Drell – Yan

Factorization Of Hard Processes in QCD
J C. Collins, D. E. Soper, George Sterman
'Perturbative QCD' (A.H. Mueller, ed.) 1999
Adv.Ser.Direct.High Energy Phys.5:1-91,1988., hep-ph/0409313

- problem are soft gluon fields of 2 incoming hadrons
  - consider A-jet passing through soft color field of B-jet
  - tricky technical proof of factorisation
  - factorisation holds, but not on a graph by graph basis
  - cancellation between differnet graphs connected by soft gluons



From proof of factorisation Collins et al:

The relevant factorization theorem, accurate up to corrections suppressed by a power of Q^2, is

violation suppressed by power of Q^2

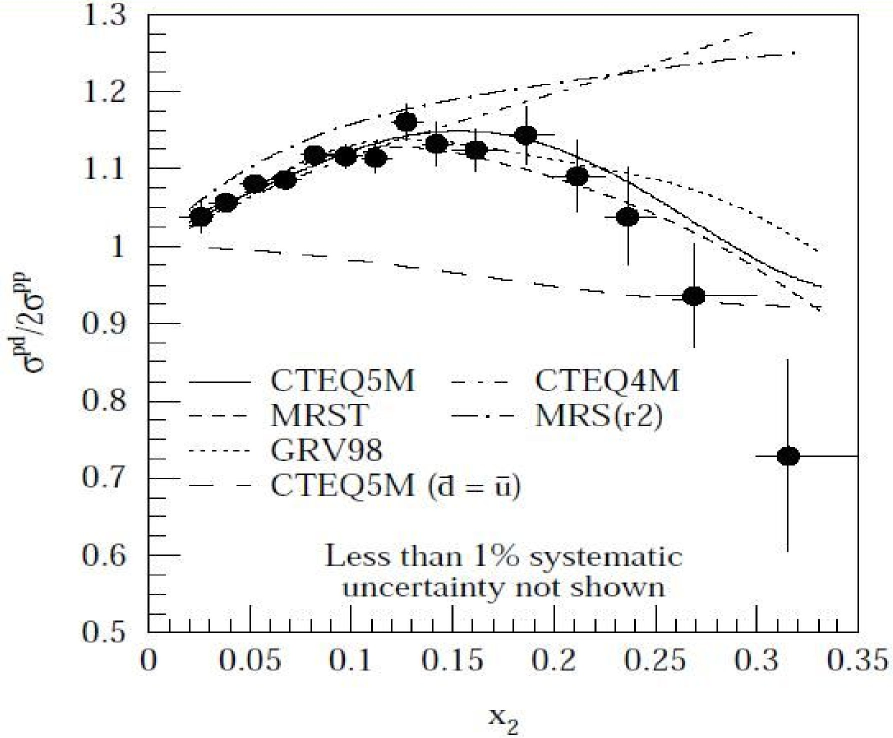
$$\frac{d\sigma}{dQ^2\,dy} \sim \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \times$$

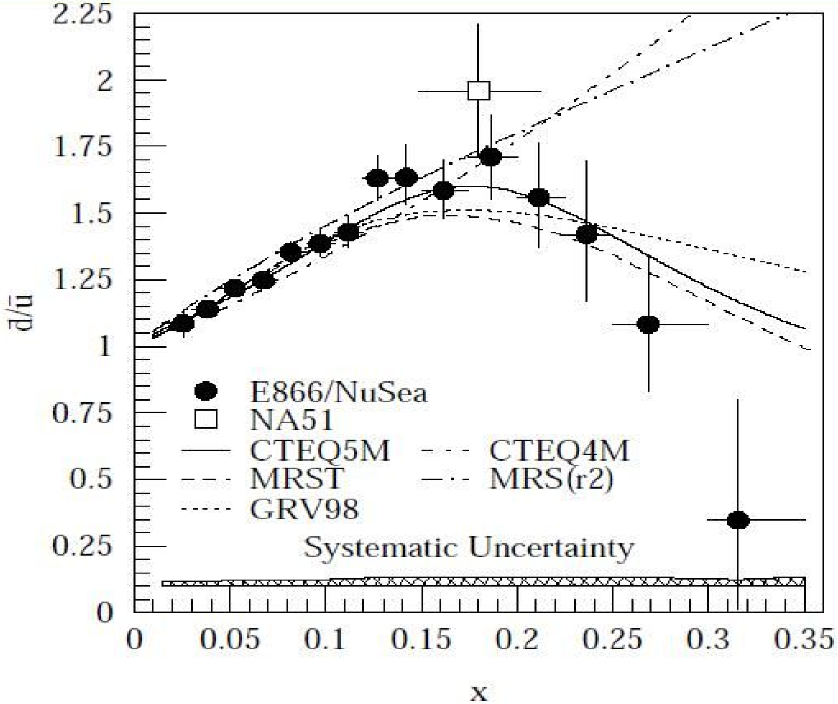
$$\times\, f_{a/A}(\xi_A,\mu)\; H_{ab}\left(\frac{x_A}{\xi_A},\frac{x_B}{\xi_B},Q;\frac{\mu}{Q},\alpha_s(\mu)\right)\; f_{b/B}(\xi_B,\mu). \quad (11)$$

PDFs from Drell – Yan

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \sim \frac{1}{2}\left(1 + \frac{\bar{d}}{\bar{u}}\right)$$

E866 required significant changes in the hi-x sea distributions





With increased flexibility in the parameterization of the sea-quark distributions, good fits are obtained

E.A. Hawker, et al. [FNAL E866/NuSea Collaboration], Measurement of the light antiquark flavor asymmetry in the nucleon sea, PRL 80, 3715 (1998)

H. L. Lai, et al.} [CTEQ Collaboration], Global {QCD} analysis of parton structure of the nucleon: CTEQ5 parton distributions, EPJ C12, 375 (2000)

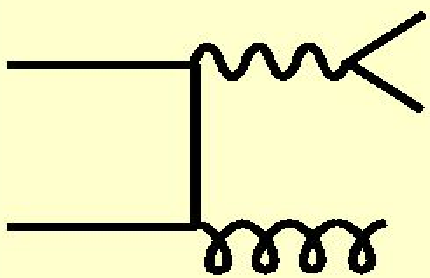
Doing things easier ...

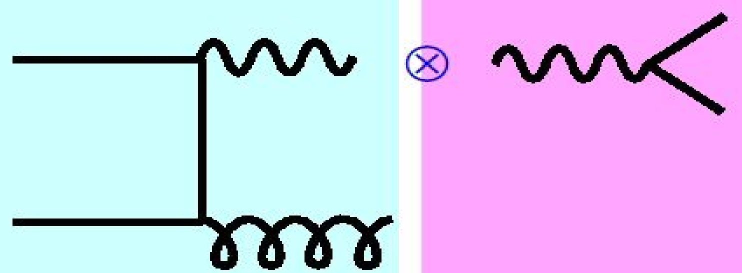
Fred Olness, CTEQ
summerschool 2003

Side Note: From $pp \to \gamma/Z/W$, we can obtain $pp \to \gamma/Z/W \to l^+ l^-$

Schematically:

$$d\sigma(q\bar{q} \to l^+ l^- g) = d\sigma(q\bar{q} \to \gamma^* g) \times d\sigma(\gamma^* \to l^+ l^-)$$



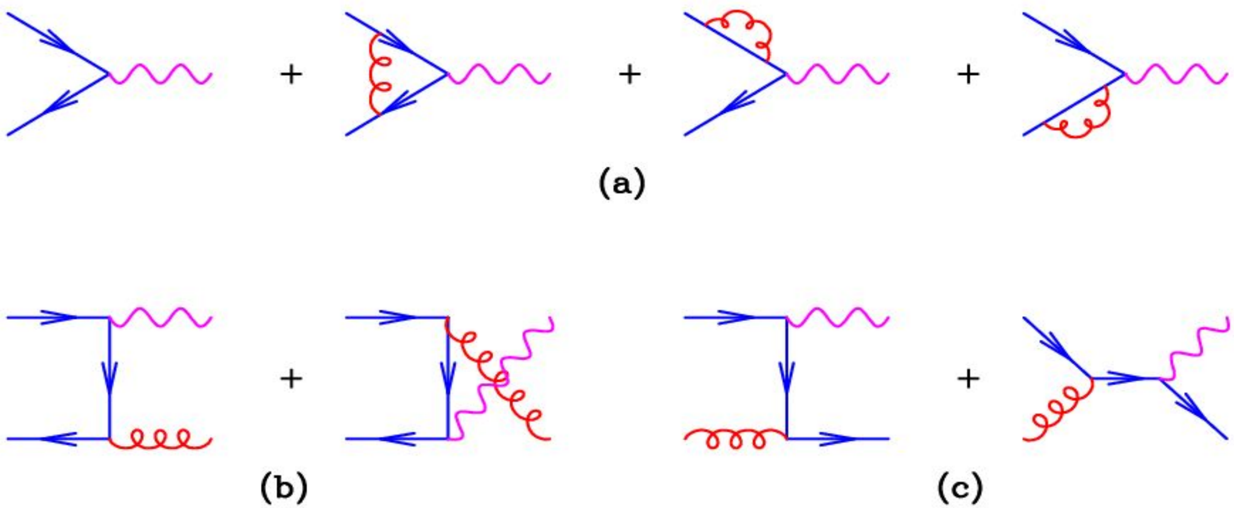


For example:

$$\frac{d\sigma}{dQ^2\, d\hat{t}}(q\bar{q} \to l^+ l^- g) = \frac{d\sigma}{d\hat{t}}(q\bar{q} \to \gamma^* g) \times \frac{\alpha}{3\pi Q^2}$$

QCD corrections for Drell Yan

K. Ellis, LHC lecture,
http://theory.fnal.gov/people/ellis/Talks



- Calculate real correction

$$q + \bar{q} \to \gamma^* + g$$

$$|M|^2 = \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2\,(M^2 s)}{\hat{u}\hat{t}} \right]$$

$$= \left[\left(\frac{1+z^2}{1-z} \right) \left(\frac{-s}{t} + \frac{-s}{u} - 2 \right) \right]$$

- with $z = M^2/s, s+t+u = M^2$
- real diagrams contain collinear divergency $\hat{t} \to 0, \hat{u} \to 0$ and soft divergency $z \to 1$
- coefficient is DGLAP splitting fct:

$$P_{qq}(z) \sim \frac{1+z^2}{1-z}$$

Regularization schemes

R. Field, Appl. of pQCD, p 42

- Massive Gluon (MG) scheme:
  - give gluon fictitious mass, which then is removed
  - ➔ regulate UV divergency by: $\frac{1}{k^2} \to \frac{1}{k^2} \frac{L}{L-k^2}$
  - ➔ regulate IR divergency by: $\frac{1}{k^2} \to \int_{m_g^2}^{L} \frac{dl}{(k^2-l)^2}$

- Dimensional Regularization (DR) scheme:
  - calculate in N rather than in 4 dimensions
  - add real and virtual corrections
  - set N=4

QCD correction in MG scheme

(R. Field, App. pQCD, p179ff)

- using massive gluon, gives for $q\bar{q} \to \gamma^* g$

$$|M|^2 = \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2(M^2 + m_g^2)\hat{s}}{\hat{u}\hat{t}} - M^2 m_g^2 \left(\frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2}\right)\right]$$

$$= \left[\left(\frac{1+z^2}{1-z}\right)\log\frac{t_{max}}{t_{min}} - 4(1-z)\right]$$

$$\sim \left[P_{qq}\log\frac{(1-z)^2 M^2}{z^2 m_g^2} - 2(1-z) + (2\log^2 2 - \frac{\pi^2}{6})\delta(1-z)\right]$$

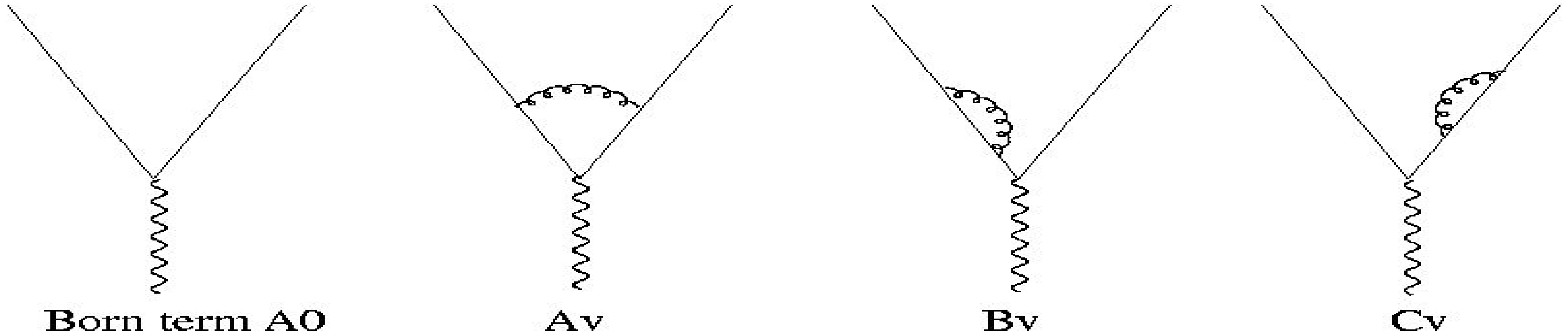
- due to gluon mass, integration over z can be performed, with

$$0 < z < \frac{1}{(1+\sqrt{\beta})^2} \sim 1 - 2\sqrt{\beta} \qquad \beta = \frac{m_g^2}{Q^2}$$

$$\hat{\sigma}_{MG}(real)_{DY} = \frac{2\alpha_s}{3\pi}\hat{\sigma}_0\left[\log^2(\beta) + 3\log(\beta) + \pi^2\right]$$

Virtual corrections

R. Field, App pQCD, p31 ff



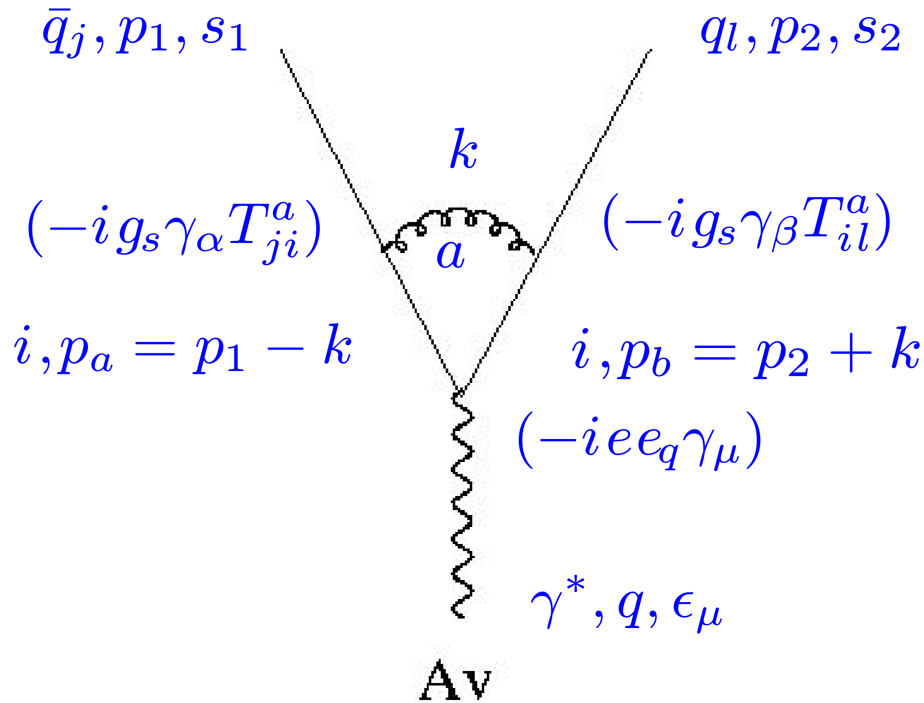
- amplitudes must be added:

$$|A_0 + A_v + B_v + C_v|^2 = |A_0|^2 + 2Re(A_0 A_v^* + A_0 A_v^* + A_0 C_v^*) + |A_v + B_v + C_v|^2$$

- enter again loop integrals which are divergent for $k \to \infty$ and $k \to 0$
- Adding vertex + self-energy diagrams
  - ➔ UV divergencies cancel (similar to that in calc of α_{em})
  - ➔ only IR divergencies stay.... and can cancel real emissions

Virtual Corrections

R. Field, Appl. of pQCD, p 32



divergencies for
$k \to 0$
and
$k \to \infty$

$$A_v = \bar{u}(p_2, s_2)(-i g_s \gamma_\beta T^a_{il}) \left(\frac{i \not{p}_b}{p_b^2}\right) (-i e e_q \gamma_\mu) \left(\frac{i \not{p}_a}{p_a^2}\right) (-i g_s \gamma_\alpha T^a_{ji}) \left[\frac{-i(g_{\beta\alpha} + \eta k_\beta k_\alpha / k^2)}{k^2}\right] v(p_1, s_1)$$

$$\sigma_v(\text{virtual}) = \int \frac{d^4k}{(2\pi)^4} (2 A_0 A_v^*) = \frac{4}{3} \sigma_0 2 g_s^2 (-i) \int \frac{d^4k}{(2\pi)^4} \frac{N(p_1, p_2, k, q)}{(p_1 - k)^2 (p_2 + k)^2 k^2}$$

QCD Corrections to Drell Yan

- Virtual emissions, integrated over z (R. Field, App. pQCD, p179ff): $q\bar{q} \to \gamma^* g$

$$\hat{\sigma}_{MG}(virtual)_{DY} = \frac{2\alpha_s}{3\pi}\hat{\sigma}_0 \left[-\log^2(\beta) - 3\log(\beta) - \frac{7}{2} - \frac{2\pi^2}{3} + \pi^2 \right]$$

$$(\hat{\sigma}_{MG}(real) + \hat{\sigma}_{MG}(virtual))_{DY} = \frac{2\alpha_s}{3\pi}\hat{\sigma}_0 \left[\frac{4\pi^2}{3} - \frac{7}{2} \right]$$

- Define K -factor (1st order): $\hat{\sigma}_{tot}^{DY} = \hat{\sigma}_0 \times (1 + \cdots) = \hat{\sigma}_0 \times K$

$$K^{DY}(\text{1st order}) = 1 + \frac{\alpha_s}{\pi}\left[\frac{8\pi^2}{9} - \frac{7}{3} \right] = 1 + 2.05\alpha_s \sim 2$$

- compare to DIS $$K^{DIS}(\text{1st order}) = 1 - \frac{\alpha_s}{\pi}$$

QCD correction in MG scheme

(R. Field, App. pQCD, p183ff)

- using massive gluon, gives for $qg \to \gamma^* q$

$$
\begin{aligned}
|M|^2 &= \left[\frac{-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\,(M^2 + m_g^2)\hat{u}}{\hat{s}\hat{t}} - M^2 m_g^2 \left(\frac{1}{\hat{s}^2} + \frac{1}{\hat{t}^2} \right) \right] \\
&\sim \left[P_{g\to qq} \log \frac{(1-z)M^2}{z^2 m_g^2} - \frac{1}{2} + z - \frac{3}{2} z^2 \right]
\end{aligned}
$$

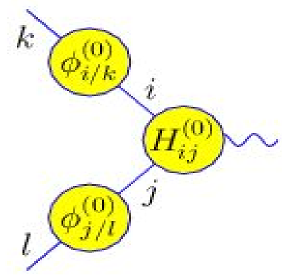
QCD corrections for Drell – Yan III

C.P Yuan,
CTEQ summerschool 2002

- soft divergencies cancelled by real and virtual emissions
- factorise collinear divergency into renormalised parton density

(1)

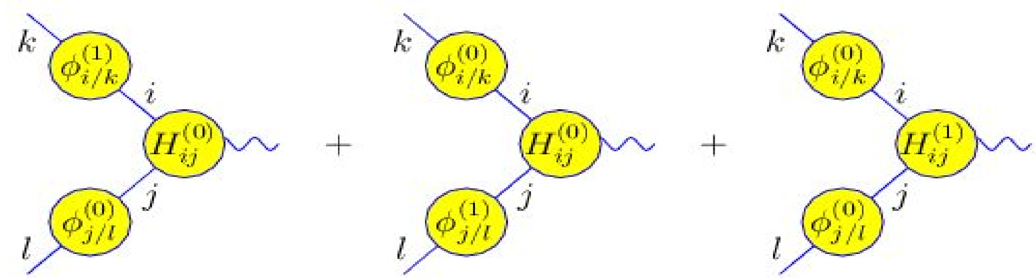
$$\sigma^{(0)}_{kl} =$$



$$\Rightarrow \quad H^{(0)}_{kl} = \sigma^{(0)}_{kl}$$

(2)

$$\sigma^{(1)}_{kl} =$$



$$H^{(0)}_{ij} = \sigma^{(0)}_{ij} = \text{“Born”}$$

suppress "^" from now on

$$H^{(1)}_{ij} = \sigma^{(1)}_{ij} - \left[\ \sigma^{(0)}_{il}\,\phi^{(1)}_{l/j} + \phi^{(1)}_{k/i}\,\sigma^{(0)}_{kj}\ \right]$$

Computed from Feynman diagrams (process dependent)

Computed from the definition of perturbative parton distribution function (process independent, scheme dependent)

Factorization scheme dependent

$$\Rightarrow \quad H^{(1)}_{kl} = \sigma^{(1)}_{kl} - \left[\phi^{(1)}_{i/k} H^{(0)}_{il} + H^{(0)}_{kj}\,\phi^{(1)}_{j/l}\right]$$

Finite

Divergent

$\mathcal{O}(\alpha_s)$ corrections to Drell Yan

Barger,Phillips, p231

- Real and virtual corrections up to $\mathcal{O}(\alpha_s)$ in dim. regularisation:

$$\frac{d\sigma^{DY}}{dM^2}(AB \to l\bar{l}X) = \sum_q e_q^2 \int_0^1 dx_a \int_0^1 dx_b \frac{4\pi\alpha^2}{9s^2} \left(\left[q^A(x_a)\bar{q}^B(x_b) + A \leftrightarrow B \right] \right.$$
$$\left[\delta(1-z) + \theta(1-z)\frac{\alpha_s}{2\pi} 2P_{qq}(z) \left(-\frac{1}{\epsilon} + \ln\frac{M^2}{\mu^2} \right) + \alpha_s f_q^{DY}(z) \right]$$
$$+ \left[(q^A(x_a) + \bar{q}^A(x_a)) g^B(x_b) + A \leftrightarrow B \right]$$
$$\left. \left[\theta(1-z)\frac{\alpha_s}{2\pi} P_{qg}(z) \left(-\frac{1}{\epsilon} + \ln\frac{M^2}{\mu^2} + \alpha_s f_g^{DY}(z) \right) \right] \right)$$

- Splitting $P_{qq}(z), P_{qg}(z)$ functions are the same as in F_2, but non-leading terms f_q^{DY}, f_g^{DY} are different !!!
- absorb $1/\epsilon$ and $\ln(M^2/\mu^2)$ into PDFs

Even higher orders are calculated !

W.L. van Neerven and E.B. Zijistra
NPB 382 (1992) 11

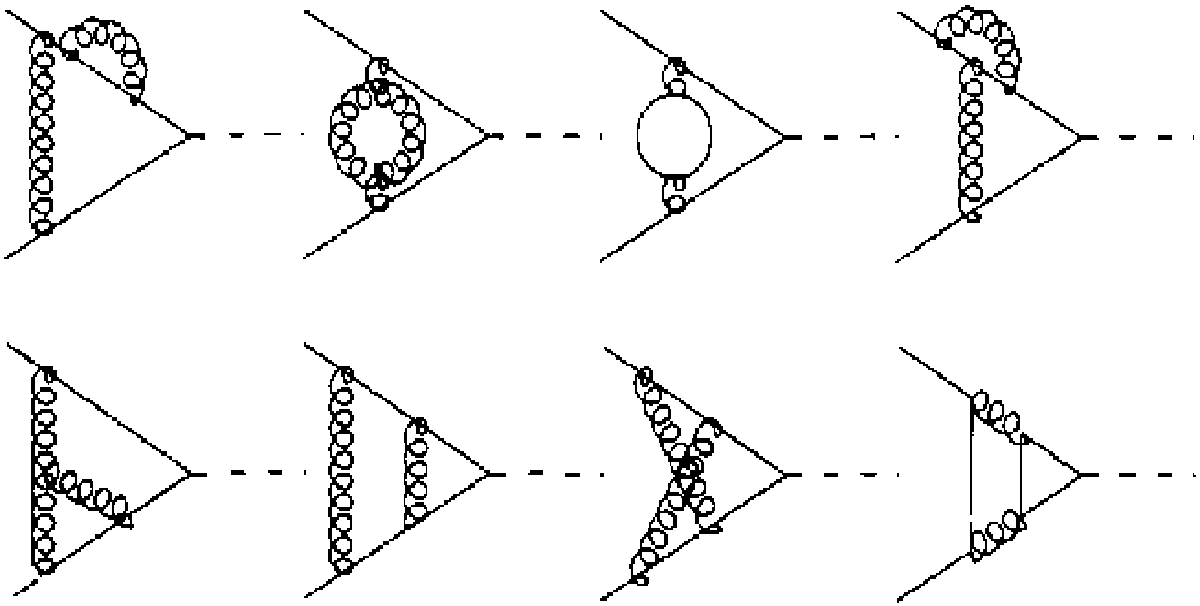


Fig. 4. The two-loop corrections to the process $q + \bar{q} \rightarrow V$.

Even higher orders are calculated !

W.L. van Neerven and E.B. Zijistra
NPB 382 (1992) 11

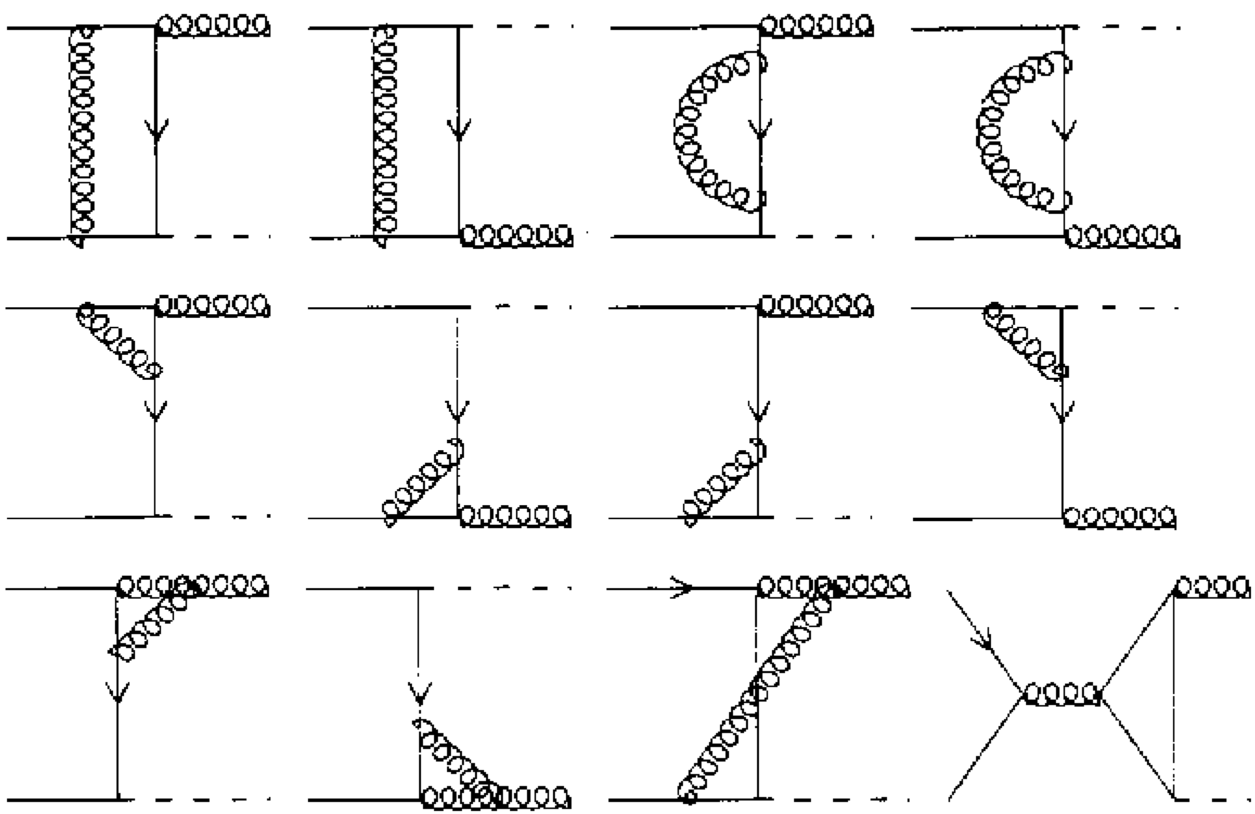


Fig. 5. The one-loop corrections to the process $q + \bar{q} \to V + g$. The diagrams corresponding to the one-loop correction to the subprocess $q(\bar{q}) + g \to V + q(\bar{q})$ can be obtained via crossing.

Even higher orders are calculated !

W.L. van Neerven and E.B. Zijistra
NPB 382 (1992) 11

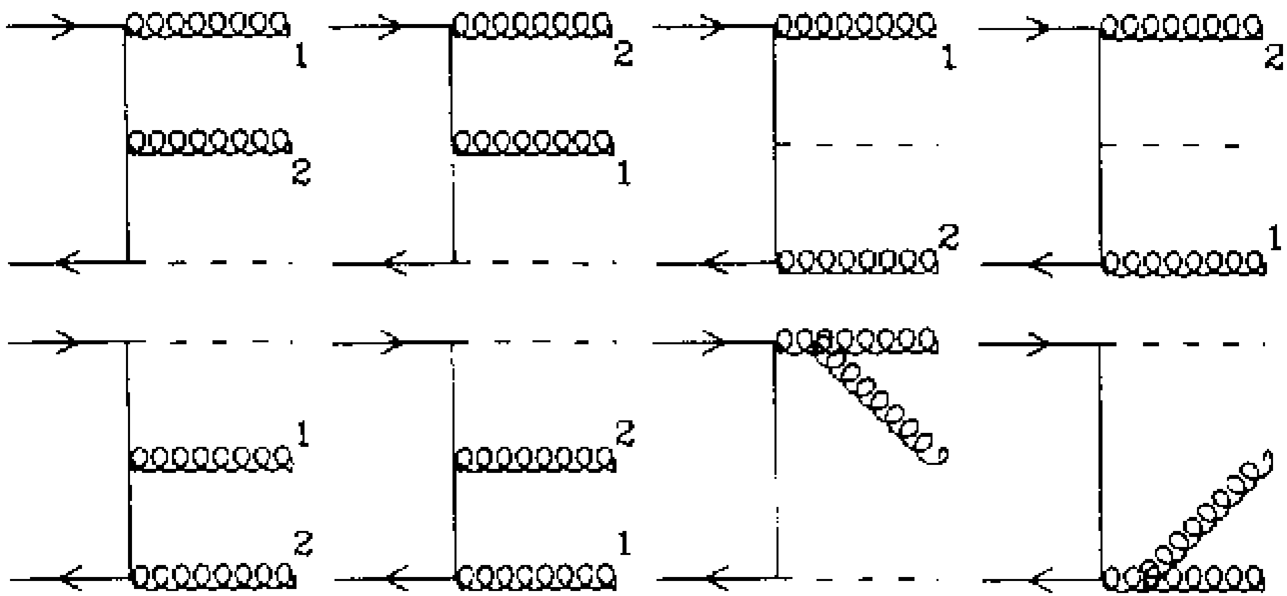


Fig. 6. Diagrams contributing to the subprocess $q + \bar{q} \to V + g + g$. The graphs corresponding to the subprocess $q(\bar{q}) + g \to V + q(\bar{q}) + g$ can be obtained from those presented in this figure via crossing. By crossing two pairs of lines one can obtain the diagrams corresponding to the subprocess $g + g \to V + q + \bar{q}$.

Even higher orders are calculated !

W.L. van Neerven and E.B. Zijistra
NPB 382 (1992) 11

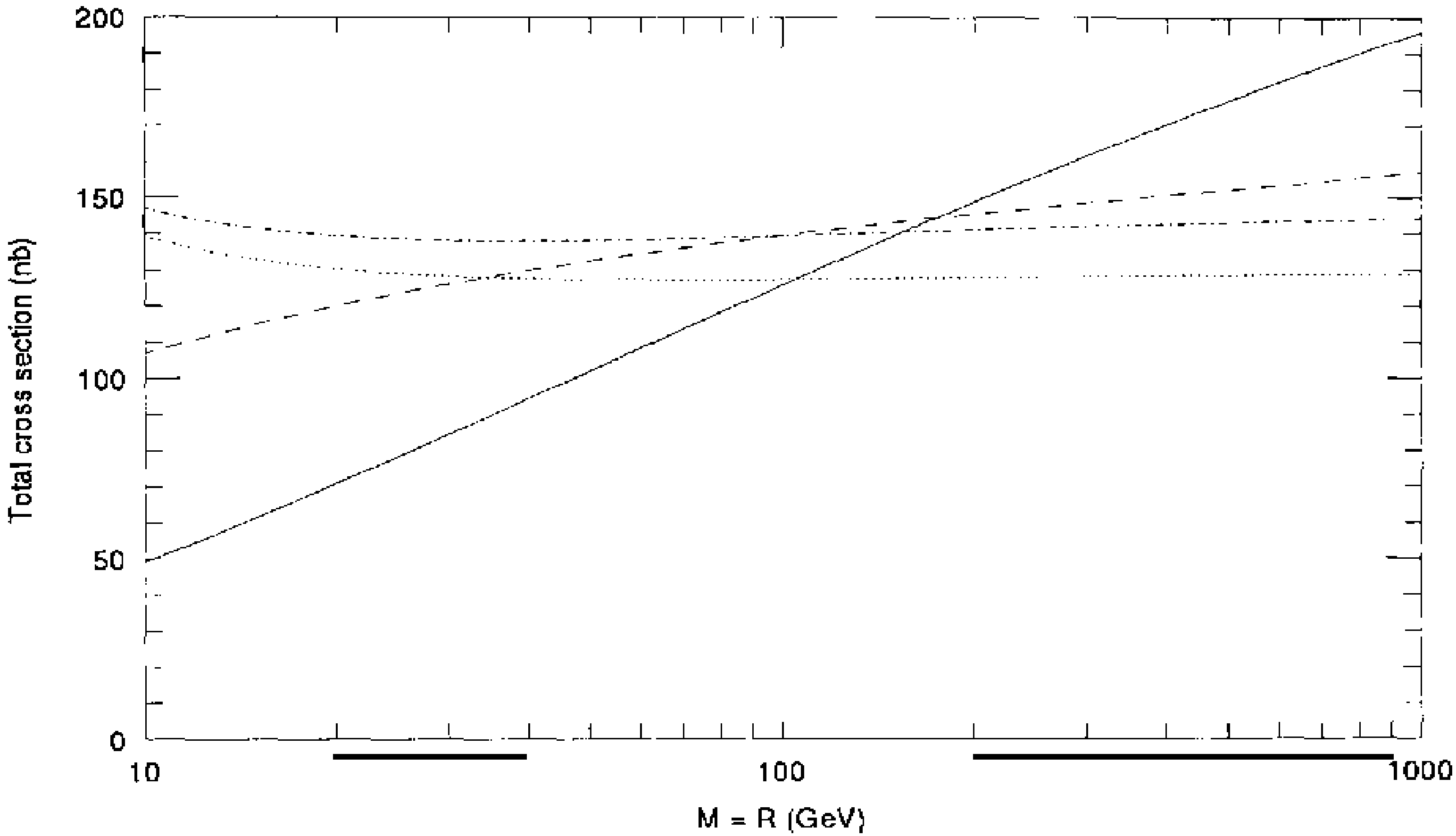
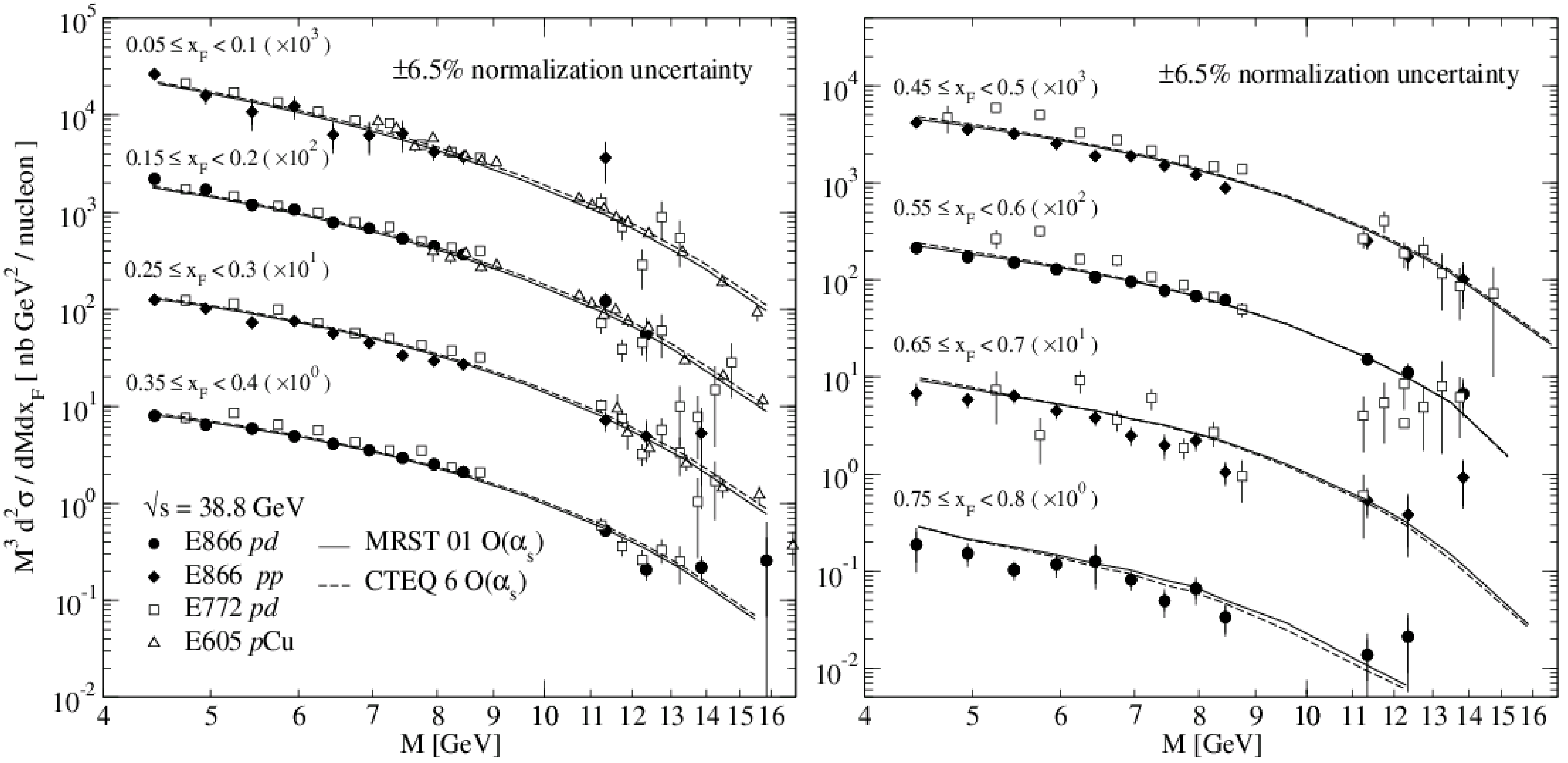


Fig. 16. Mass factorization scale (M) dependence of $\sigma_{W^+W^-}$ for LHC, $\sqrt{S} = 16$ TeV. Solid line: Born, DIS scheme. Long-dashed line: $O(\alpha_s)$, DIS scheme. Dash-dot line: $O(\alpha_s^2)$, DIS scheme. Dotted line: $O(\alpha_s^2)$, $\overline{\rm MS}$ scheme.

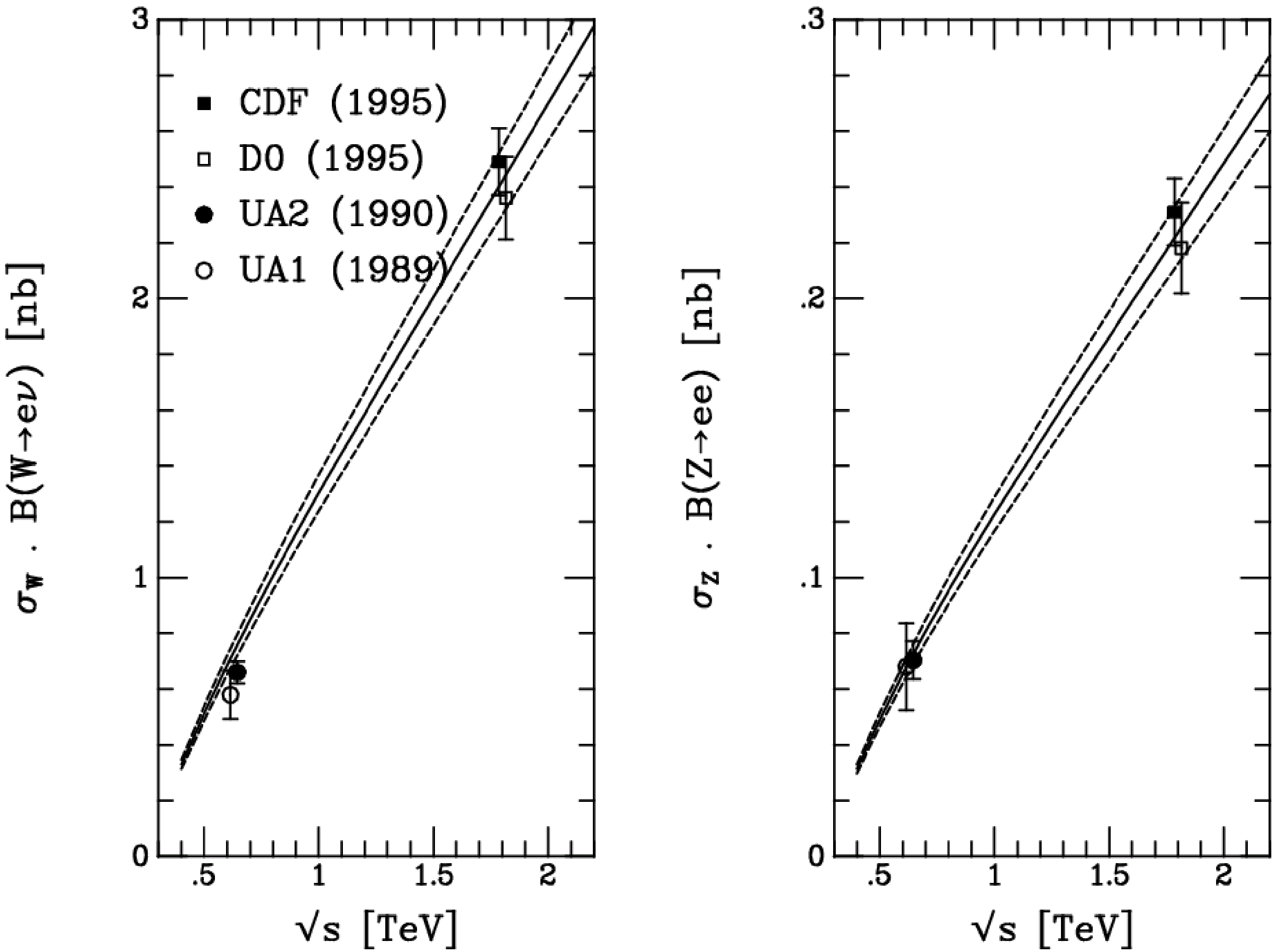
Absolute Drell-Yan Dimuon Cross Sections in 800 GeV/c pp fixed target

J Webb, E866-NuSea hep-ex/0302019



$$x_F = \frac{2}{\sqrt{s}}(p_{l^+} + p_{l^-}) \sim x_1 - x_2$$

W/Z cross sections

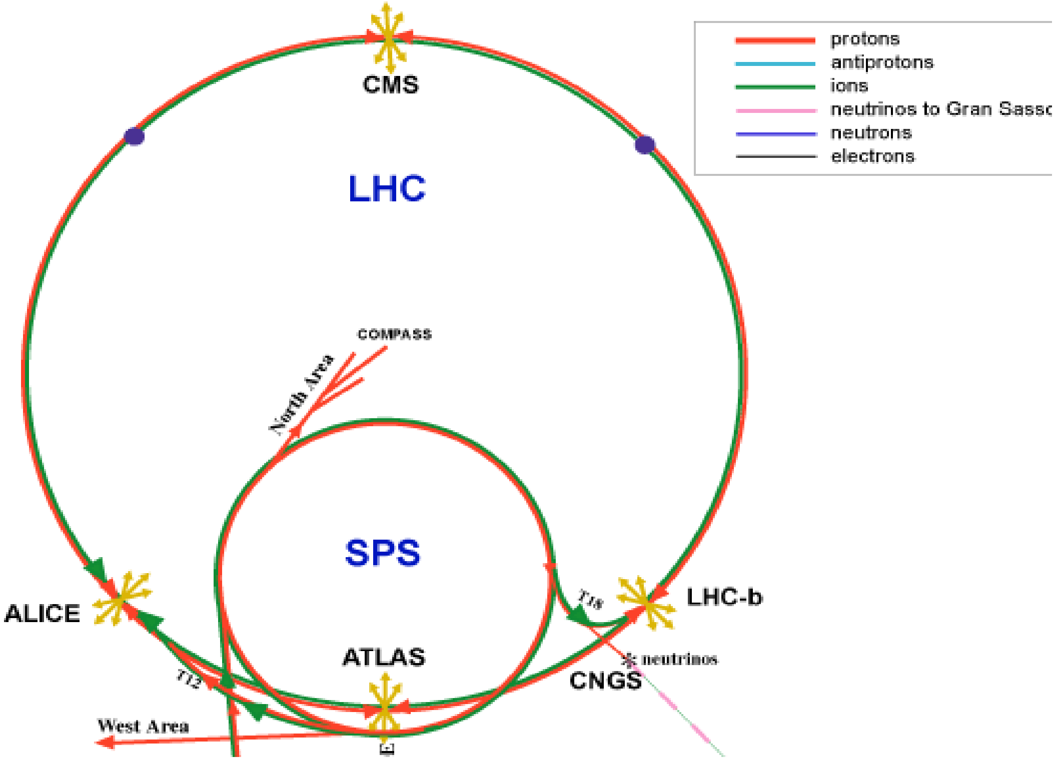


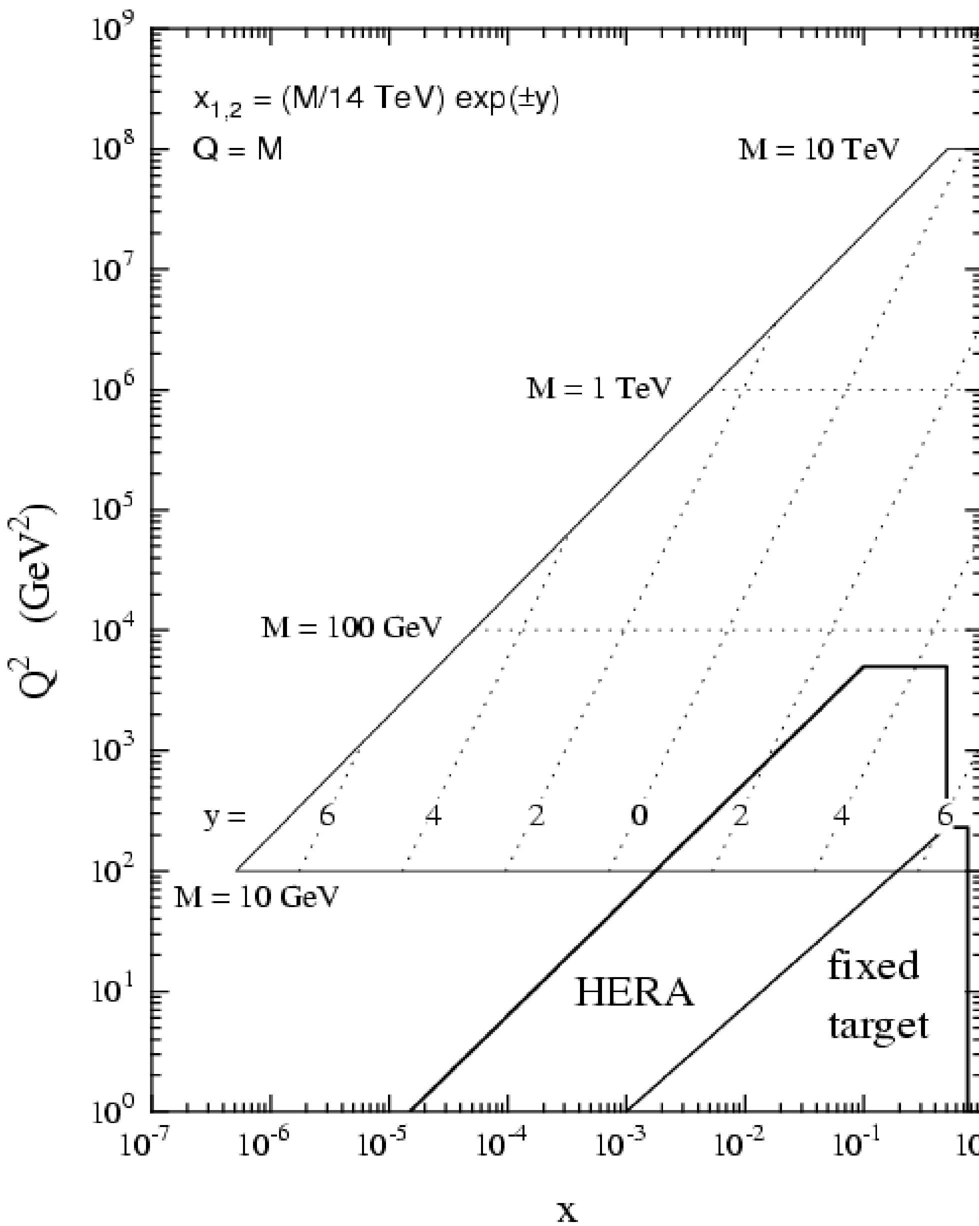
K. Ellis, LHC lecture,
http://theory.fnal.gov/people/ellis/Talks

- Agreement with NLO theory is good (three curves estimate theoretical error).
- LO curves (not shown) lie about 25% too low.

Kinematic reach at LHC ?

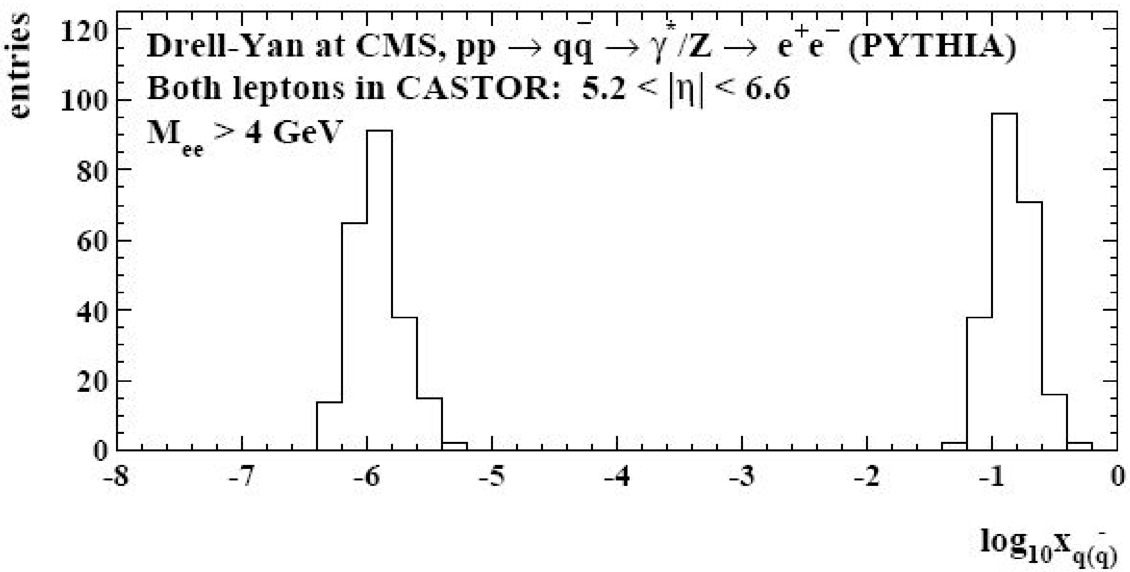
proton proton collider LHC
$\sqrt{s}$ = 14 TeV

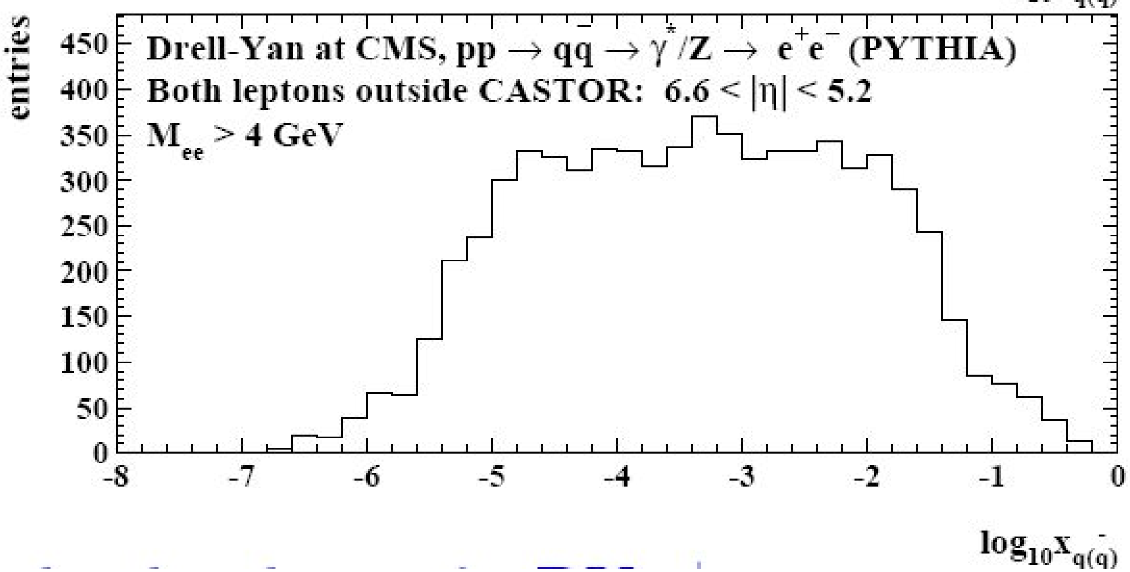




Drell Yan for small x ?

E. Sarkisyan
Van Mechelen





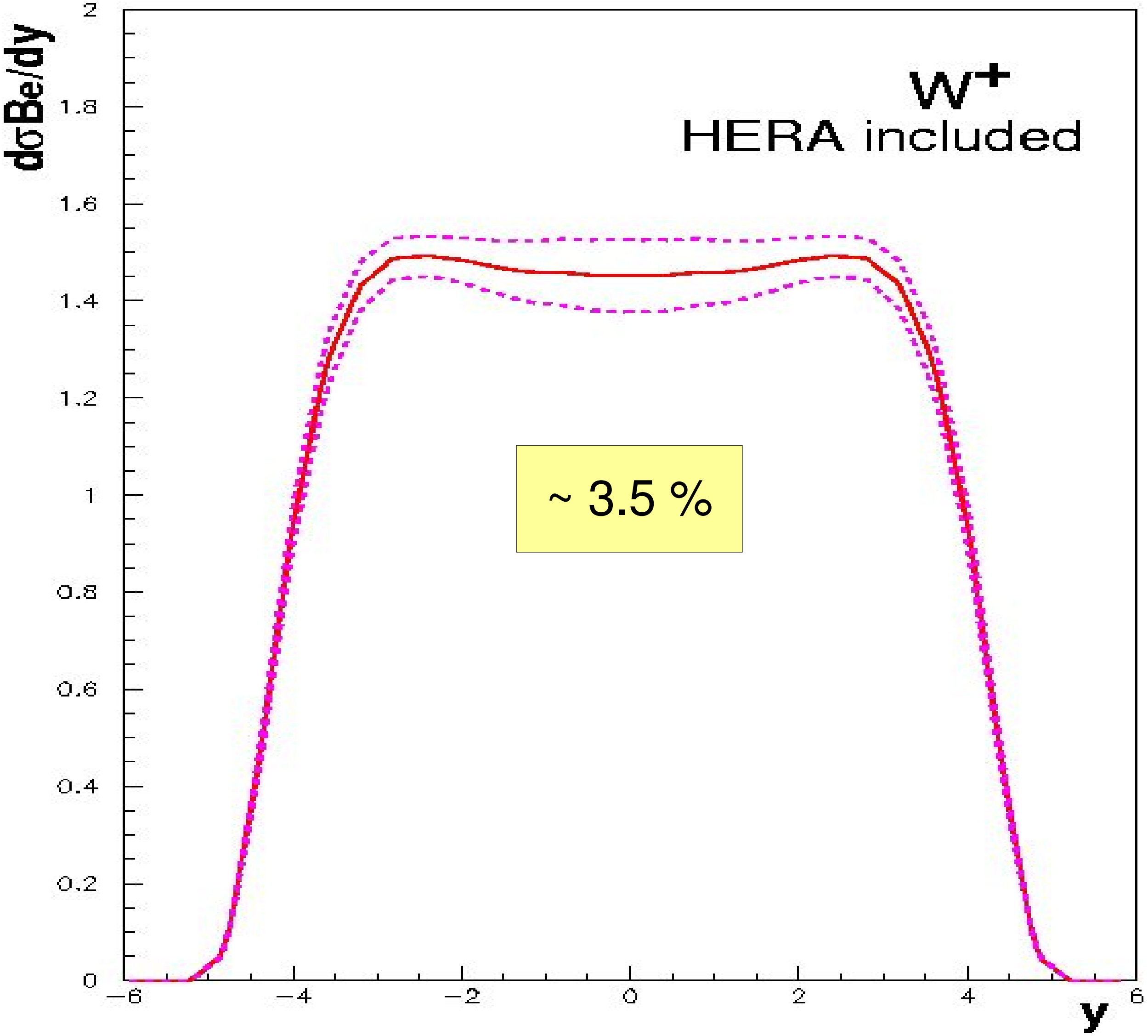
Drell-Yan into electrons

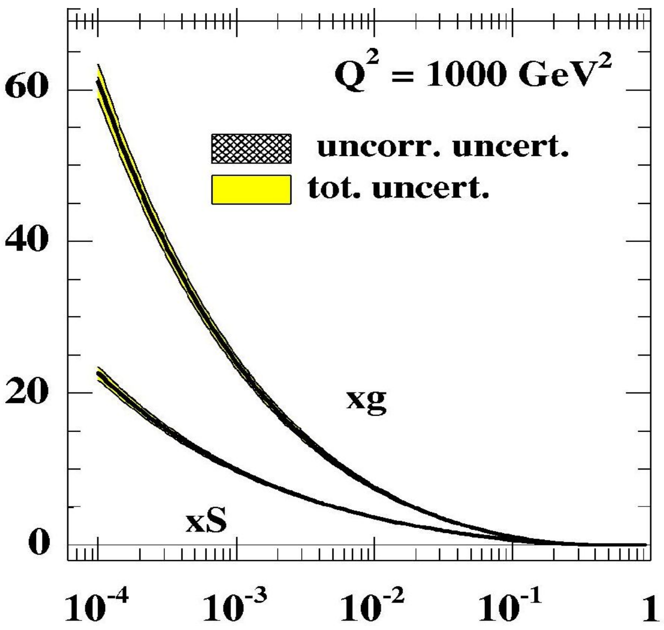
Large rapidity needed
for low-x reach

W/Z for luminosity at LHC

- W production at LHC

HERA – LHC workshop
hep-ph/0601012
hep-ph/0601013

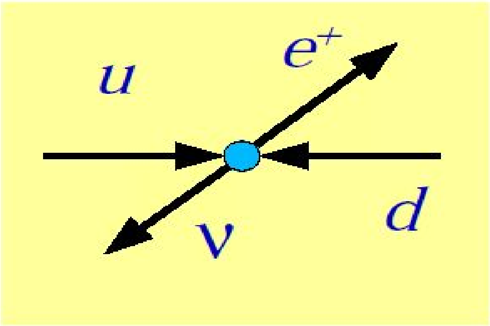




Measurement of W

Fred Olness, CTEQ
summerschool 2003

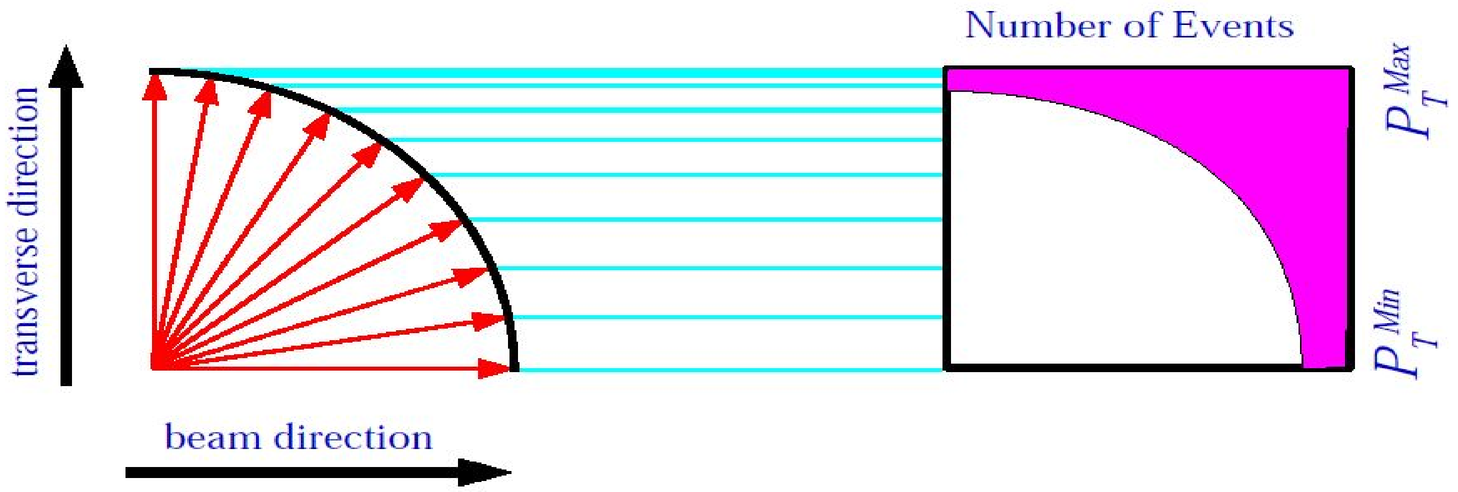
The Jacobian Peak



Suppose lepton distribution is uniform in θ

The dependence is actually $(1+\cos\theta)^2$, but we'll take care of that later

What is the distribution in P_T?



We find a peak at $P_T^{max} \approx M_W/2$

Measurement of W

Fred Olness, CTEQ summerschool 2003

The Jacobian Peak

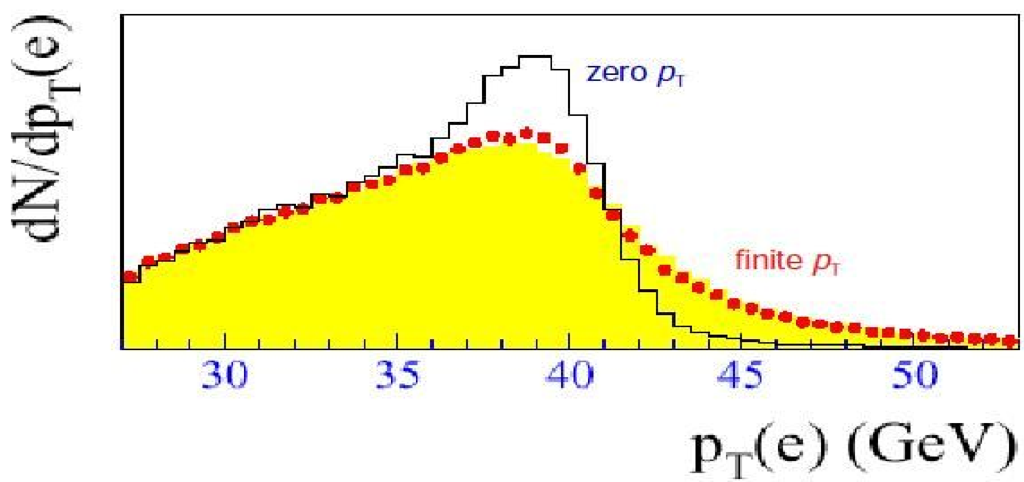
Now that we've got the picture, here's the math ... *(in the W CMS frame)*

$$p_T^2 = \frac{\hat{s}}{4}\sin^2\theta \qquad \cos\theta = \sqrt{1-\frac{4\,p_T^2}{\hat{s}}} \qquad \frac{d\cos\theta}{d\,p_T^2} = \frac{2}{\hat{s}}\,\frac{1}{\cos\theta}$$

So we discover the P_T distribution has a singularity at $\cos\theta=0$, or $\theta=\pi/2$

$$\frac{d\sigma}{d\,p_T^2} = \frac{d\sigma}{d\cos\theta}\times\frac{d\cos\theta}{d\,p_T^2} \approx \frac{d\sigma}{d\cos\theta}\times\frac{1}{\cos\theta}$$

singularity!!!



BUT !!!

Measuring the Jacobian peak is complicated if the W boson has finite P_T.

Transverse Momentum of W/Z

The complete P_T spectrum for the W boson

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summerschool 2003

The full P_T spectrum for the W-boson showing the different theoretical regions