Homework Exercises for
QCD and Collider Physics IV
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summer term 2007

Exercises for Lecture 5 (11. June 2007)

- Heavy Quarks: Use $\Delta y = y_3 - y_4$ and

$$\begin{aligned} p^\mu &= (p_x, p_y, p_z, E) \\ &= (p_T \sin\phi, p_T \cos\phi, m_T \sinh y, m_T \cosh y) \\ x_1 &= \frac{m_T}{\sqrt{s}} \left(e^{y_3} + e^{y_4}\right) \\ x_2 &= \frac{m_T}{\sqrt{s}} \left(e^{-y_3} + e^{-y_4}\right) \end{aligned}$$

  to show that:

$$\begin{aligned} \hat{s} = & \; 2p_3p_4 + 2m^2 &= 2m_T^2 \left(1 + \cosh \Delta y\right) \\ \hat{t} - m^2 = & \; -2p_1p_3 &= -m_T^2 \left(1 + \exp\left(-\Delta y\right)\right) \\ \hat{u} - m^2 = & \; -2p_2p_3 &= -m_T^2 \left(1 + \exp\left(\Delta y\right)\right) \end{aligned}$$

- Heavy Quarks: calculate matrix elements for $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$ to obtain:

$$|M|^2(q\bar{q} \to Q\bar{Q}) = g^4 \frac{4}{9} \left(\tau_1^2 + \tau_2^2 + \frac{\rho}{2}\right)$$

$$|M|^2(gg \to Q\bar{Q}) = g^4 \left(\frac{1}{6\tau_1\tau_2} - \frac{3}{8}\right) \left(\tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2}\right)$$

  with $\tau_1 = \frac{2p_1p_3}{\hat{s}}, \tau_2 = \frac{2p_2p_3}{\hat{s}}, \rho = \frac{4m^2}{\hat{s}}, \hat{s} = (p_1 + p_2)^2$

- Heavy Quarks: show that in the limit $\rho \to 0$ the results from light quarks are recovered.
- Heavy Quarks: show that using $\Delta y = y_3 - y_4$:

$$|M|^2(q\bar{q} \to Q\bar{Q}) = \frac{4g^4}{9} \left(\frac{1}{1 + \cosh(\Delta y)}\right) \left(\cosh(\Delta y) + \frac{m^2}{m_T^2}\right)$$

$$|M|^2(gg \to Q\bar{Q}) = \frac{g^4}{24} \left(\frac{8\cosh(\Delta y) - 1}{1 + \cosh(\Delta y)}\right) \left(\cosh(\Delta y) + 2\frac{m^2}{m_T^2} - 4\frac{m^4}{m_T^4}\right)$$

- Heavy Quarks: show that for Δy large we obtain

$$|M|^2_{q\bar{q}} \sim \textit{constant}$$
$$|M|^2_{gg} \sim \exp(\Delta y)$$

- calculate $\bar{\mathcal{F}}_{ij}^{(1)}(\rho)$ from:

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{ij}(s, m^2, \mu^2)$$

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_{\rm s}^2(\mu^2)}{m^2} \mathcal{F}_{ij}\left(\rho, \frac{\mu^2}{m^2}\right)$$

and

$$\mathcal{F}_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = \mathcal{F}_{ij}^{(0)}(\rho) + 4\pi\alpha_{\rm s}(\mu^2)\left[\mathcal{F}_{ij}^{(1)}(\rho) + \bar{\mathcal{F}}_{ij}^{(1)}(\rho)\log(\frac{\mu^2}{m^2})\right] + \mathcal{O}(\alpha_{\rm s}^2)$$

to give:

$$\begin{aligned} \bar{\mathcal{F}}_{ij}^{(1)}(\rho) &= \frac{1}{8\pi^2}\left[4\pi b \mathcal{F}_{ij}^{(0)}(\rho) - \int_\rho^1 dz_1 \sum_k \mathcal{F}_{kj}^{(0)}\left(\frac{\rho}{z_1}\right) P_{ki}^{(0)}(z_1) \right. \\ & \left. - \int_\rho^1 dz_2 \sum_k \mathcal{F}_{ik}^{(0)}\left(\frac{\rho}{z_2}\right) P_{kj}^{(0)}(z_2)\right] \end{aligned}$$