

Homework Exercises for QCD and Collider Physics IV Lecturer: H. Jung

summer term 2007

Exercises for Lecture 3 (14. May 2007)

- leftover from last week:

- show that a four-vector can be expressed in terms of $m_t = \sqrt{p^2 + t^2 + m^2}$ and rapidity $y = \frac{1}{2} \log \frac{E+p_z}{E-p_z}$ as:

$$p^\mu = (p_x, p_y, p_z, E) = (p_t \sin \phi, p_t \cos \phi, m_t \sinh y, m_t \cosh y)$$

- show that for massless parton we have:

$$p_t^2 = \frac{\hat{u}\hat{t}}{\hat{s}}$$

- calculate the Matrix element for

$$q\bar{q} \rightarrow q\bar{q}$$

$$q\bar{q}' \rightarrow q\bar{q}'$$

$$gg \rightarrow q\bar{q}$$

- show that the cross section for jet production in hadron collision can be written as:

$$\frac{d^2\sigma}{dyd^2P_T} = \frac{1}{16\pi^2s} \sum \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} f_i(x_1, \mu^2) f_j(x_2, \mu^2) \overline{\sum} |\mathcal{M}(ij \rightarrow kl)|^2 \frac{1}{1 + \delta_{kl}} \delta(\hat{s} + \hat{t} + \hat{u})$$

express phase space in terms of y and p_t .

Show that using the laboratory rapidities of the outgoing partons y_3, y_4 we obtain:

$$\frac{d^2\sigma}{dy_3 dy_4 d^2P_T} = \frac{1}{16\pi^2s} \sum \int \frac{1}{x_1} \frac{1}{x_2} f_i(x_1, \mu^2) f_j(x_2, \mu^2) \overline{\sum} |\mathcal{M}(ij \rightarrow kl)|^2 \frac{1}{1 + \delta_{kl}}$$

- Multiparton Interactions: use $p(x_t) = \frac{1}{\sigma_{nd}} \frac{d\sigma}{dx_t}$ and

$$P_1 = p(x_{t1}) \exp\left(-\int_{x_{t1}}^1 p(x') dx'\right)$$

to show that

$$P_N = p(x_{tN}) \frac{1}{(N-1)!} \left(\int_{x_{tN}}^1 p(x') dx'\right)^{N-1} \exp\left(-\int_{x_{t2}}^{x_{t1}} p(x') dx'\right)$$