Homework Exercises for
QCD and Collider Physics II

Summer 2006

Exercises for Lecture 1 (19. April 2006)

- The variance is defined as:

  $$V(f) = \int (f - E(f))^2 \, dG = \left(\frac{1}{b-a} \int_a^b (f(u) - E(f))^2 \, du \right)$$

  Show that one obtains for

  if x, y uncorrelated

  $V(cx + y) = c^2 V(x) + V(y)$

  if x, y correlated

  $V(cx + y) = c^2 V(x) + V(y) + 2cE\left[(y - E(y))(x - E(x))\right]$

- Write a small program, that simulates radioactive decay: $dN = -N\alpha dt$ which results in $N = N_0 e^{-\alpha t}$.
  The probability that nucleus undergoes radioactive decay in time Δt is p:
  $p = \alpha\Delta t$ (for $\alpha\Delta t \ll 1$)
  show results after 3000 sec for:
  $N_0 = 100$, $\alpha = 0.01 s^{-1}$, $\Delta t = 1s$
  $N_0 = 5000$, $\alpha = 0.03 s^{-1}$, $\Delta t = 1s$

- The expectation value is defined as:

  $$E(f) = \int f(u) dG(u) = \left(\frac{1}{b-a} \int_a^b f(u) du \right) = \frac{1}{N} \sum_{i=1}^{N} f(u_i).$$

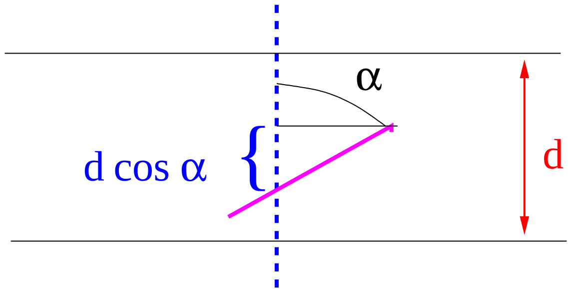
  A simple example for $dG(u)$ is uniformly distributed u in $[a, b]$: $dG(u) = du/(b - a)$. Using the definition of $E(x)$, show that one obtains:

  $$E(cx + y) = cE(x) + E(y)$$

- Write a small program, that simulates Buffon's needle (Buffon 1777):

  pattern of parallel lines with distance d, randomly throw needle with length d onto stripes, count hit, when needle crosses stripes and count miss, if not. The probability for hit is: $\frac{d\cos(\alpha)}{d} = \cos(\alpha)$ and all angles are equally likely, giving a probability:

$$\frac{\int_0^{\pi/2}\cos(\alpha)d\alpha}{\pi/2} = \frac{2}{\pi}$$



- Write a small program simulation Compton scattering, using

$$\frac{d\sigma}{d\Omega} = \frac{\alpha_{em}^2}{2m^2}\left(\frac{k'}{k}\right)^2\left(\frac{k'}{k} + \frac{k}{k'} - \sin^2\theta\right)$$

with $k' = \frac{k}{1+(k/m)(1-\cos\theta)}$. The angular distribution of the photon is:

$$\sigma(\theta,\phi)d\theta d\phi = \frac{\alpha_{em}^2}{2m^2}\left(\left(\frac{k'}{k}\right)^3 + \left(\frac{k}{k'}\right) - \left(\frac{k'}{k}\right)^2\sin^2\theta\right)\sin\theta d\theta d\phi.$$

The azimuthal angle is generated by: $\phi = 2\pi R_1$. Use now the approximate function:

$$d\sigma^a d\theta d\phi = \frac{\alpha_{em}^2}{2m^2}\left(1 + \frac{k}{m}u\right)^{-1} du d\phi$$

with $u = (1 - \cos\theta)$, and obtain:

$$u = \frac{m}{k}\left[\left(1 + 2\frac{k}{m}\right)^{R_2} - 1\right].$$

At the weight (accept/reject) events with $\frac{\sigma}{\sigma^a}$. Plot the photon scattering angle for 2 different values of the incoming photon energy, $E_\gamma = 0.0005$ GeV and $E_\gamma = 1$ GeV