QCD and Collider Physics: Jets and heavy quarks in LO/NLO

- Resume from last lecture
- F2 in LO and NLO
 - 1-jet inclusive x-section (LO and NLO)
 - reference frames
- heavy quark production in DIS (in LO and NLO)
- dijet production in DIS in LO and NLO
- approaches to even higher orders
 - parton showers
 - need for u-pdfs

http://www-h1.desy.de/~jung/qcd_collider_physics_2005

Catani Ciafaloni Fiorani Marchesini evolution

- Apply color coherence in form of angular ordering ۲ $\bar{q} > z_n q_n, q_n > z_{n-1} q_{n_1}, ..., q_1 > Q_0$
- with: ٩

$$\tilde{P}(z,q,k_t) = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{\mathsf{NS}}(z,q,k_t)$$

θ q E_{i-1} $\boldsymbol{\theta}_{i-1}$ \mathbf{q}_{i-1}

Ei

ZX

X

gives:

$$x\mathcal{A}(x,k_t,q) = x\mathcal{A}_0(x,k_t)\Delta_s(q) + \int dz \int \frac{d^2q'}{\pi q'^2}\Theta(\bar{q}-zq)$$
$$\cdot \Delta_s(q,zq')\tilde{P}(z,q',k_t)\frac{x}{z}\mathcal{A}\left(\frac{x}{z},k_t',q'\right)$$

integration much more complicated due to angular constraints ٩

k,-factorization

- need to couple gluons to photon
- use high energy (kt -) factorization:

(Catani, Ciafaloni, Hautmann NPB 366 (1991) 135, Gribov, Levin, Ryskin, Phys. Rep.100 ,(1983),1, Collins, Ellis, NPB 360 ,(1991) ,3)

$$\sigma(\mathbf{ep} \to \mathbf{e}'\mathbf{q}\bar{\mathbf{q}}) = \int \frac{dy}{y} d^2 Q \frac{dx_g}{x_g} \int d^2 k_t \hat{\sigma}(\hat{s}, k_t, \bar{q}) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

with

$$\int^{Q^2} d^2 k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$$

- t-channel gluon with virtuality $k^2 = -k_t^2$ dominates the process in the high energy limit $s \gg \hat{s}$
- collinear limit obtained by: $\hat{\sigma}(\hat{s},0,Q)\cdot\Theta(Q-k_t)$

"off-shell" matrix elements

- calculation using standard Feynman rules

$$\mathcal{M}(\gamma g \to c\bar{c}) = \bar{u}(p_c) \left(\frac{\not \epsilon_{\gamma} \left(\not p_c - \not k_{\gamma} + m_c \right) \not \epsilon_g}{k_{\gamma}^2 - 2k_{\gamma}p_c} + \frac{\not \epsilon_g \left(\not p_c - \not k_g + m_c \right) \not \epsilon_{\gamma}}{k_g^2 - 2k_g p_c} \right)$$

use high-energy polarization projection:

$$G^{\mu\nu} = \overline{\epsilon_g^{\mu}\epsilon_g^{*\nu}} = \frac{k_t^{\mu} k_t^{\nu} k_t^{\nu} g}{|k_t g|^2}$$

- ME is finite for $k_t
 ightarrow 0$
- ME has tail to large k,



 $u(p_{\bar{c}})$

LO BGF in collinear factorization

compare to R. Field, Appl. of pQCD, p 130 ff

- using simple polarization sum: $g_{\mu\nu}$
- \rightarrow then $F_2 = F_\Sigma + \frac{3}{2}F_L$
- BGF cross section (in Massive Gluon Scheme):

$$\begin{aligned} \frac{d\sigma_{\Sigma}(\gamma^*g \to q\bar{q})}{d\hat{t}} &= \frac{\pi\alpha\alpha_{\rm s}e_q^2 z^2}{Q^4} 2\left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2Q^2}{\hat{t}\hat{u}}(\hat{t} + \hat{u} + Q^2) \right. \\ &+ \frac{2m_g^2}{\hat{t}\hat{u}}(\hat{t} + \hat{u} + m_g^2) - Q^2m_g^2\left(\frac{1}{\hat{u}^2} + \frac{1}{\hat{t}^2} - \frac{4}{\hat{t}\hat{u}}\right) \right) \end{aligned}$$

• integrated over t with: $t_{\min}=-m_g^2 z$, $t_{\max}=-Q^2/z$ (compare R. Field, App. pQCD, p130ff)

$$\sigma_{\Sigma}(\gamma^*g \to q\bar{q}) = \frac{\pi e_q^2 \alpha \alpha_s z}{Q^2} 4 \left[\left(z^2 + (1-z)^2 \right) \log \frac{Q^2}{m_g^2 z^2} - 2 \right]$$

$$\sigma_L(\gamma^*g \to q\bar{q}) = \pi e_q^2 \alpha \alpha_s \left(\frac{4z^2}{Q^2} \right) 2(1-z)$$

BGF contribution to F₂

• BGF contribution to F_2

$$F_2^g(x,Q^2) = 2e_q^2 \int_x^1 dz \frac{x}{z} g(\frac{x}{z},Q^2) \left(\frac{\alpha_s}{2\pi} P_{g \to q\bar{q}}(z) \log \frac{Q^2}{m_g^2} + \alpha_s f_{MG}^g(z)\right)$$

$$\alpha_{\rm s} f_{DIS}^g = \frac{\alpha_{\rm s}}{2\pi} \left[-(z^2 + (1-z)^2) \log(z) - 1 + 3z(1-z) \right]$$

QCDC in collinear factorization

- in QCDC $\gamma^* + q \rightarrow q + g$ collinear and soft divergency:
 - collinear divergency treated in Q² dependent PDF
 - soft divergency compensated by virtual corrections
 - need to match *n*-body phase space to *n-1* body phase space

• assume: $q_g^2 = m_g^2$ Massive Gluon scheme (calculate ME with FORM !!!)

$$\frac{d\sigma_{\Sigma}}{d\hat{t}} = \frac{\pi\alpha\alpha_{\rm s}e_q^2 z^2}{Q^4} \frac{16}{3} \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} - \frac{2(Q^2 - m_g^2)(\hat{s} + \hat{t} + Q^2 - m_g^2)}{\hat{s}\hat{t}} - \frac{m_g^2 Q^2 \left(\frac{1}{\hat{t}^2} + \frac{1}{\hat{s}^2}\right) \right]$$

with:

$$\hat{t}_{\min} = -\frac{\beta z Q^2}{1-z} , \quad \hat{t}_{\max} = -\frac{Q^2}{z} + \beta Q^2 , \quad \beta = \frac{m_g^2}{Q^2}$$

QCDC contribution

• integrated over t:

$$\frac{d\sigma_{\Sigma}}{dz} = \frac{\pi\alpha\alpha_{\rm s}e_q^2}{Q^2} \frac{16}{3} z \left[\frac{1+z^2}{1-z}\log\left(\frac{(1-z)Q^2}{z^2m_g^2}\right) -\frac{3}{2}\frac{1}{1-z} + z + 1 + \frac{5}{4}\delta(1-z)\right]$$

• Longitudinal part:
$$\sigma_L(real)_{DIS} = \pi \alpha \alpha_s e_q^2 \left(\frac{4z^2}{Q^2}\right) \left(\frac{4}{3}\right)$$

• QCDC contribution to F_2 :

$$F_2^q(x,Q^2) = e_q^2 \int_x^1 dz \frac{x}{z} q(\frac{x}{z},Q^2) \left[\delta(1-z) + \frac{\alpha_s}{2\pi} P_{q \to qg}(z) \log \frac{Q^2}{m_g^2} + \alpha_s f_{MG}^q(z) \right]$$

$$\begin{aligned} \alpha_{\rm s} f_{DIS}^q &= \frac{2\alpha_{\rm s}}{3\pi} \left[(1+z^2) \left(\frac{\log(1-z)}{1-z} \right)_+ + \frac{1+z^2}{1-z} (-2\log(z)) \right. \\ &\left. -\frac{3}{2} \frac{1}{91-z} \right)_+ + 4z + 1 - \left(\frac{2\pi^2}{3} + \frac{9}{4} \right) \delta(1-z) \right] \end{aligned}$$

H. Jung, QCD & Collider Physics, Lecture 9 WS 05/06

Virtual Corrections to QCDC

• Real emissions, integrated over z (R. Field, App. pQCD, p124ff):

$$\sigma_{\Sigma}(real)_{DIS} = \frac{2\alpha_{\rm s}}{3\pi}\sigma_0 \left[\log^2(\beta) + 3\log(\beta) + \frac{2\pi^2}{3} + 2\right]$$

- investigate virtual corrections to cancel $\beta = \frac{m_g^2}{Q^2}$ dependence.
 - virtual corrections relevant for $z \rightarrow 1$

Virtual corrections



amplitudes must be added:

 $|A_0 + A_v + B_v + C_v|^2 = |A_0|^2 + 2Re(A_0A_v^* + A_0A_v^* + A_0C_v^*) + |A_v + B_v + C_v|^2$

- enter again loop integrals which are divergent for $k \to \infty$ and $k \to 0$
- Adding vertex + self-energy diagrams
 - → UV divergencies cancel (similar to that in calc of α_{em})
 - only IR divergencies stay.... and can cancel real emissions

Virtual Corrections

R. Field, Appl. of pQCD, p 32

$$\begin{split} \bar{q}_{j}, p_{1}, s_{1} & q_{l}, p_{2}, s_{2} \\ (-ig_{s}\gamma_{\alpha}T_{ji}^{a}) & a & (-ig_{s}\gamma_{\beta}T_{il}^{a}) \\ i, p_{a} = p_{1} - k & i, p_{b} = p_{2} + k \\ (-iee_{q}\gamma_{\mu}) \\ & \downarrow \\ Av & & \\ Av & & \\ A_{v} = & \bar{u}(p_{2}, s_{2})(-ig_{s}\gamma_{\beta}T_{il}^{a}) \left(\frac{ip_{b}'}{p_{b}^{b}}\right)(-iee_{q}\gamma_{\mu}) \left(\frac{ip_{a}}{p_{a}^{2}}\right) \\ & (-ig_{s}\gamma_{\alpha}T_{ji}^{a}) \left[\frac{-i(g_{\beta\alpha} + \eta k_{\beta}k_{\alpha}/k^{2})}{k^{2}}\right] v(p_{1}, s_{1}) \\ \sigma_{v}(\text{virtual}) = \int \frac{d^{4}k}{(2\pi)^{4}} (2A_{0}A_{v}^{*}) = \frac{4}{3}\sigma_{0}2g_{s}^{2}(-i) \int \frac{d^{4}k}{(2\pi)^{4}} \frac{N(p_{1}, p_{2}, k, q)}{(p_{1} - k)^{2}(p_{2} + k)^{2}k^{2}} \end{split}$$

Regularization schemes

R. Field, Appl. of pQCD, p 42

- Massive Gluon (MG) scheme:
 - give gluon fictitious mass, which then is removed
 - → regulate UV divergency by: $\frac{1}{k^2} \rightarrow \frac{1}{k^2} \frac{L}{L-k^2}$

→ regulate IR divergency by:

$$\frac{1}{k^2} \to \int_{m_a^2}^L \frac{dl}{k^2 - l^2}$$

- Dimensional Regularization (DR) scheme:
 - calculate in N rather than in 4 dimensions
 - add real and virtual corrections
 - set N=4

Virtual Corrections for QCDC

vertex correction:

$$\sigma_v(ext{virtual}) = rac{2lpha_s}{3\pi}\sigma_0\left[-\log^2(eta) - 3\log(eta) - rac{7}{2} - rac{2\pi^2}{3} + \log(L/m_g^2)
ight]$$

- self-energy correction: $\sigma_s(\text{virtual}) = \frac{2\alpha_s}{3\pi}\sigma_0 \left[-\log(L/m_g^2)\right]$
- Summing up vertex and self-energy:

$$\sigma_{MG}(\text{virtual}) = \sigma_v(\text{virtual}) + \sigma_s(\text{virtual})$$
$$= \frac{2\alpha_s}{3\pi}\sigma_0 \left[-\log^2(\beta) - 3\log(\beta) - \frac{7}{2} - \frac{2\pi^2}{3} \right]$$
$$m_s^2$$

- with $\beta = \frac{m_g^2}{Q^2}$
- ➔ which contains only infrared divergencies
- Ultraviolet divergencies dropped out by summing over vertex and self-energy corrections

Virtual Corrections for QCDC II

Sum over real

$$\sigma_{\Sigma}(real)_{DIS} = \frac{2\alpha_{\rm s}}{3\pi}\sigma_0 \left[\log^2(\beta) + 3\log(\beta) + \frac{2\pi^2}{3} + 2\right]$$

and virtual corrections:

$$\sigma_{MG}(\text{virtual}) = \frac{2\alpha_s}{3\pi}\sigma_0 \left[-\log^2(\bar{\beta}) - 3\log(\bar{\beta}) - \frac{7}{2} - \frac{2\pi^2}{3} \right]$$

→ gives:

$$\sigma_{\Sigma}(real)_{DIS} + \sigma_{\Sigma}(virtual) = \frac{2\alpha_{\rm s}}{3\pi}\sigma_0\left(-\frac{3}{2}\right) = -\frac{\alpha_{\rm s}}{\pi}$$

Full NLO corrections for F₂

adding everything together:

$$F_{2}(x,Q^{2}) = e_{q}^{2} \int_{x}^{1} dz \frac{x}{z} q(\frac{x}{z},Q^{2}) \left[\delta(1-z) + \frac{\alpha_{s}}{2\pi} P_{q \to qg}(z) \log \frac{Q^{2}}{m_{g}^{2}} + \alpha_{s} f_{MG}^{q}(z) \right] \\ + 2e_{q}^{2} \int_{x}^{1} dz \frac{x}{z} g(\frac{x}{z},Q^{2}) \left(\frac{\alpha_{s}}{2\pi} P_{g \to q\bar{q}}(z) \log \frac{Q^{2}}{m_{g}^{2}} + \alpha_{s} f_{MG}^{g}(z) \right)$$

$$\alpha_{s} f_{DIS}^{q} = \frac{2\alpha_{s}}{3\pi} \left[(1+z^{2}) \left(\frac{\log(1-z)}{1-z} \right)_{+} + \frac{1+z^{2}}{1-z} (-2\log(z)) - \frac{3}{2} \frac{1}{91-z} + 4z + 1 - \left(\frac{2\pi^{2}}{3} + \frac{9}{4} \right) \delta(1-z) \right]$$

$$\alpha_{\rm s} f_{DIS}^g = \frac{\alpha_{\rm s}}{2\pi} \left[-(z^2 + (1-z)^2) \log(z) - 1 + 3z - 3z^2 \right]$$

Reference Frames

- Laboratory system: transverse momenta originate also from transverse momentum of scattered electron... not only QCD effects..
- hadronic center of mass system (HCM) $\vec{p} + \vec{q} = 0$

- Breit Frame (Brick Wall Frame) from $\vec{q} + x\vec{p} = -x\vec{p}$
 - → get $2x\vec{p} + \vec{q} = 0$
 - photon is space-like: q = (0, 0, 0, Q)
 - incoming quark reverses direction
 - $x\vec{P} = (Q/2, 0, 0, -Q/2)$



Jet Cross Sections in $O(\alpha_s)$

S. Schilling desy-thesis-00-040

From differential x-section

$$\begin{aligned} \frac{d\sigma(\gamma g \to q\bar{q})}{dx \, dy \, d\hat{t}} &= K \sum_{\text{quarks } a} e_a^2 \frac{\alpha_s(Q^2)}{4\pi^2} \frac{2z}{Q^2} z f_g(\frac{x}{z}, Q^2) \times \\ & \times \left\{ \frac{1}{4} \left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - 2\frac{\hat{s}Q^2}{\hat{u}\hat{t}} + 4\frac{\hat{s}Q^2}{(\hat{s} + Q^2)^2} \right) \right\} \end{aligned}$$

→ obtain:

J. Collins JHEP 0005:004,2000

$$\begin{split} \frac{d\sigma(\gamma g \to q\bar{q})}{dx \, dy \, dz \, d\cos\theta \, d\phi} &= K \sum_{\text{quarks } a} e_a^2 \frac{\alpha_s(Q^2)}{4\pi^2} z f_g(\frac{x}{z}, Q^2) \times \\ & \times \left\{ P(z) \left[\frac{1}{1 - \cos\theta} + \frac{1}{1 + \cos\theta} \right] - \frac{1}{2} + 3z(1 - z) \right\}, \end{split}$$

BGF: LO and subtraction



➔ problem: kinematics ... avoid to subtract too much

$$\frac{d\sigma_{\text{hard}}}{dx \, dy \, dz \, d\cos\theta \, d\phi} = K \sum_{\text{quarks } a} e_a^2 \frac{\alpha_s(Q^2)}{4\pi^2} z f_g(\frac{x}{z}, Q^2) \times \left\{ \frac{P(z)(1 - C(-t))}{1 - \cos\theta} + \frac{P(z)(1 - C(-u))}{1 + \cos\theta} - \frac{1}{2} + 3z(1-z) \right\}$$

• with
$$C(a) = \Theta(Q^2 - a)$$

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BGF x-section

collinear contribution $C(a) = \Theta(Q^2 - a)$ x 10 ⁻⁻ -2 x 10 do_{unsubtracted}/ dxdydx₃dcos@d¢ agenower axayax_aacosoaa 0.14 0.14 x= 0.001 x= 0.001 0.12 0.12 x,=10 x x_=10 x 0.1 $Q^2 = 10 \text{ GeV}^2$ $Q^2 = 10 \text{ GeV}^2$ 0.1 0.08 0.08 0.06 0.06 0.04 0.04 0.02 0.02 0 0 0.6 0.8 -1 -0.8 -0.6 -0.4 -0.2 ο 0.2 0.4 1 -1 -0.8 -0.6 -0.4 -0.2 0.2 0.4 0.6 o cos @

full BGF contribution

0.8

1

cos @

Inclusive jet Cross Section

Inclusive jets selected in Lab frame

ZEUS PLB 632 (2006) 13





From LO to NLO ...

LO

 $\sim \alpha_s^0$

NLO for F_2 : $O(\alpha_2)$ ۵

- NLO for dijets: $O(\alpha_{s}^{2})$ ۵
- $\left| \sum_{k=1}^{\alpha_{s}^{1}} \right|^{2}$ di-jet NLO for 3-jets: $O(\alpha_{s}^{3})$ ļ 3-jet

F₂









NOTE: NLO for dijets is **NOT** NNLO for F_{2}

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