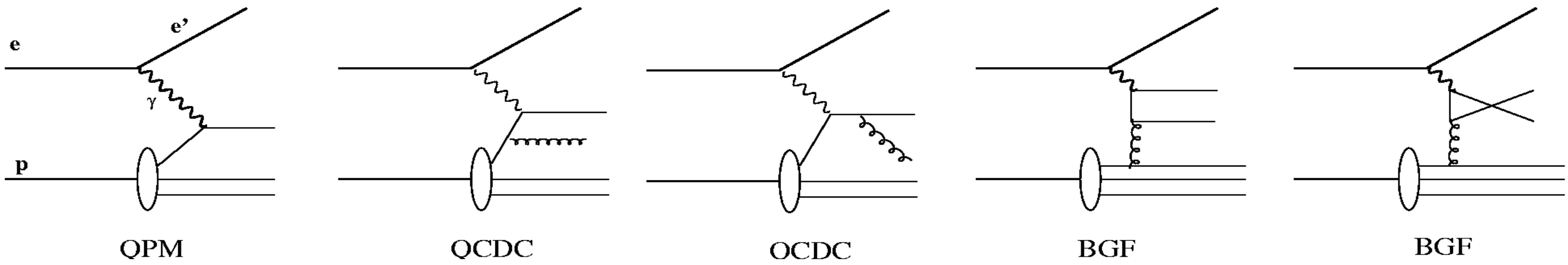
QCD and Collider Physics: DGLAP and all that ...

- Resume from last lecture
  - equivalent photon approximation
- Higher order corrections to DIS
  - evolution of parton densities
  - DGLAP
- Interpretation of DGLAP

http://www-h1.desy.de/~jung/qcd_collider_physics_2005

Higher order corrections to DIS



- lowest order: $e + q \to e' + q' \quad \mathcal{O}(\alpha_s^0)$
- higher order: $e + q \to e' + q' + g, \quad e + g \to e' + q + \bar{q} \quad \mathcal{O}(\alpha_s^1)$

- factorise electromagnetic vertex or calculate full $2 \to 3$ process
- use Weizsäcker(Z. Phys 88, 612 (1934)) -Williams (Phys Rev 45, 729 (1934)) (or Equivalent Photon(Budnev Phys Rep C15 , 181 (1974))) Approximation:

from:
$$\frac{d\sigma}{dydQ^2} = \frac{4\pi\alpha^2}{Q^4}\frac{x}{y}\left(1+(1-y)^2\right)\frac{1}{2}e_q^2\delta(x-z)$$

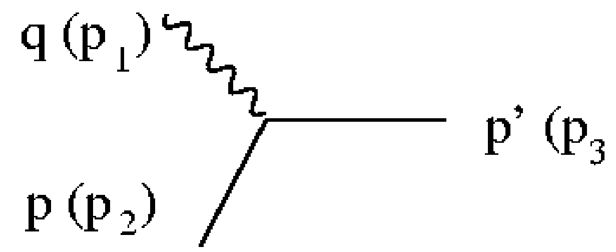
obtain:
$$\frac{d\sigma}{dydQ^2} = \frac{\alpha}{2\pi}\frac{1}{yQ^2}\left(1+(1-y)^2\right)\frac{4\pi^2\alpha}{Q^2}e_q^2 x\delta(x-z)$$

$$\frac{d\sigma}{dydQ^2} = F_{\gamma/e}(y,Q^2)\,\sigma(\gamma^* q \to q')$$

Equivalent Photon Approximation II

Halzen/Martin Exercise 10.2

- calculate: $|ME|^2 = \frac{1}{4} 2e_i^2 e^2 Tr(p' p) = 2e_i^2 e^2 p.q$



- use:

$$\begin{aligned} d\sigma &= \frac{1}{F} dLips |ME|^2 \\ F &= 4\sqrt{(p_1.p_2)^2 - m_1^2 m_2^2} = 4\sqrt{(p.q)^2} = 2Q^2 \\ dLips &= (2\pi)^4 \delta^4(-p_1 - p_2 + p_3) \frac{d^4 p_3}{(2\pi)^3} \delta(p_3^2 - m_3^2) \\ &= (2\pi)^4 \delta^4(-q - p + p') \frac{d^4 p'}{(2\pi)^3} \delta(p'^2) \end{aligned}$$

define $z = \frac{Q^2}{2pq}$

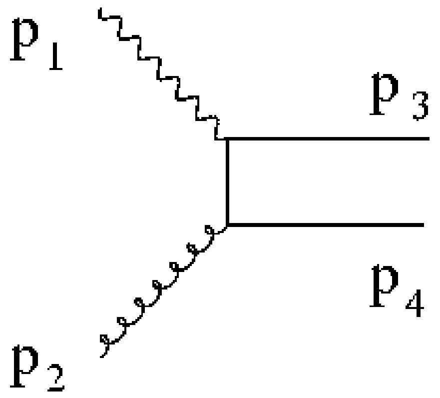
$$\begin{aligned} (q+p)^2 &= 2pq(1-z) \\ \delta(a(1-z)) &= \frac{1}{a}\delta(1-z) \end{aligned}$$

$$\sigma = \frac{4\pi^2 \alpha e_i^2}{Q^2} \delta(1-z)$$

$$\frac{d\sigma}{dydQ^2} = \frac{4\pi^2\alpha^2}{Q^4} \frac{1}{y} \left(1 + (1-y)^2\right) \frac{1}{2} e_q^2 = \frac{\alpha}{2\pi} \frac{1}{yQ^2} \left(1 + (1-y)^2\right) \frac{4\pi^2 \alpha}{Q^2} e_q^2$$

$$\frac{d\sigma}{dydQ^2} = F_{\gamma/e}(y, Q^2)\, \sigma(\gamma^* q \to q')$$

Flux of virtual particles - cross sections



- Flux for virtual photons not unambiguously defined ....
- ONLY observable is cross section for real particles
  - use different conventions:
    - HAND convention: define flux as if it were real

$$F = 4\sqrt{(q.p_2)^2 + m_1^2 m_2^2} = 2\hat{s}$$

    - define flux including virtual mass (used here)

$$F = 4\sqrt{(q.p_2)^2 + m_1^2 m_2^2} = 2(\hat{s} + Q^2)$$

- **be consistent with x-section definition !**

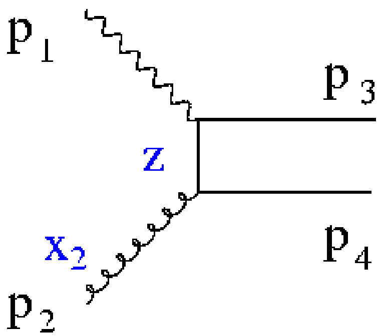
- x-section with virtual photons:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\frac{1}{\hat{s}}\frac{1}{\hat{s}}|ME|^2 \rightarrow \frac{1}{16\pi}\frac{1}{\hat{s}+Q^2}\frac{1}{\hat{s}}|ME|^2$$

real photons (Hand convention)

virtual photons

Kinematics



$$\hat{s} = (p_1 + p_2)^2 = Q^2 \frac{1-z}{z}$$

$$\hat{t} = k^2 = (p_1 - p_3)^2 = -\xi \frac{Q^2}{z} = \frac{-k_\perp^2}{1-\xi} - Q^2 \xi$$

$$\hat{u} = (p_2 - p_3)^2 = Q^2 \frac{\xi - 1}{z}$$

Define:

$$\xi = \frac{p_2 k}{p_1 p_2} = 1 - \frac{p_2 p_3}{p_1 p_2}$$

$$z = \frac{Q^2}{2 p_1 p_2}$$

$$x_{bj} = z x_2$$

- Using $s+t+u=-Q^2$ gives:

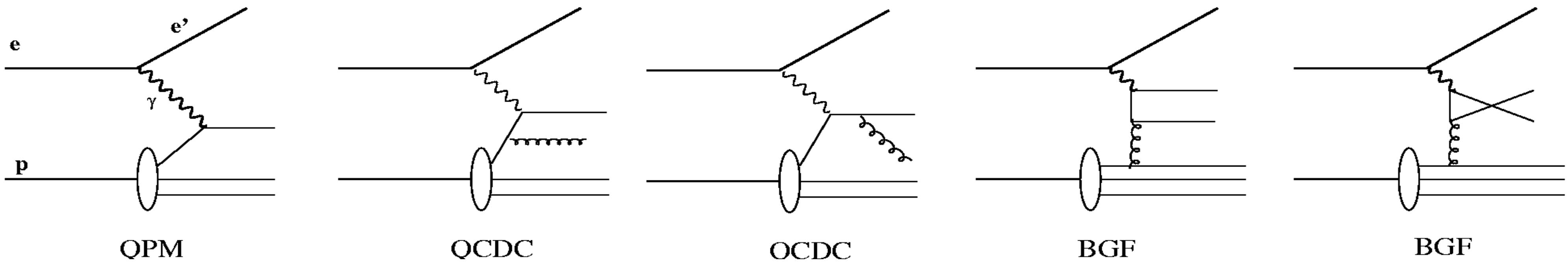
$$k_\perp^2 = \frac{\hat{t}\hat{u}\hat{s}}{\left(\hat{s} + Q^2\right)^2}$$

- and for $\hat{t} \ll \hat{s}$

$$k_\perp^2 = \frac{-\hat{t}\hat{s}}{\hat{s} + Q^2} = -t\,(1-z)$$

always neglecting masses

Higher order corrections to DIS



- lowest order: $e + q \rightarrow e' + q' \quad \mathcal{O}(\alpha_s^0)$
- higher order: $e + q \rightarrow e' + q' + g, \quad e + g \rightarrow e' + q + \bar{q} \quad \mathcal{O}(\alpha_s^1)$

- What is the dominant part of the x-section ?
  - Investigate full x-section of QCDC and BGF
  - dominant part comes from small transverse momenta ...
  - rewrite x-section in terms of $k_\perp$
  - use small *t* limit:

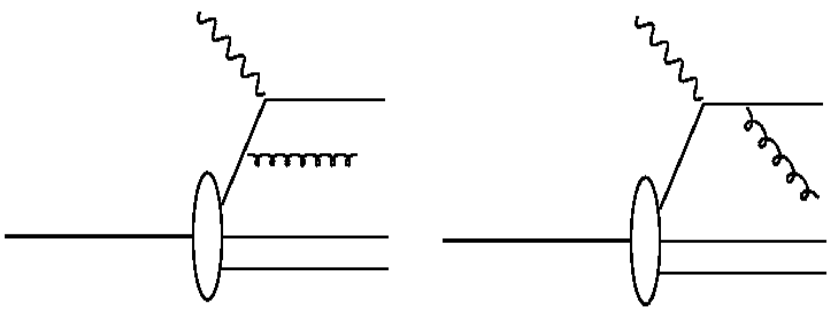
$$\frac{d\sigma}{dk_\perp} = \frac{d\sigma}{dt}\frac{1}{(1-z)} = \frac{1}{(1-z)}\frac{1}{F}dLips|ME|^2$$

$$= \frac{1}{(1-z)}\frac{1}{16\pi}\frac{1}{\hat{s}+Q^2}\frac{1}{\hat{s}}|ME|^2$$

QCDC - contribution

$$|ME|^2 = 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{4}{3} \left[\frac{-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}} \right]$$

$$= 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{4}{3} \frac{-1}{t} \left[\frac{Q^2(1+z^2)}{z(1-z)} + \cdots \right]$$



QCDC OCDC

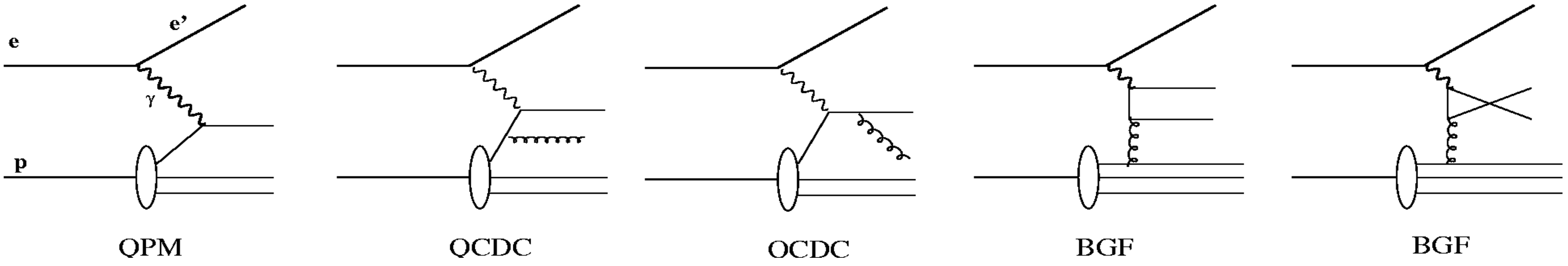
$$\frac{d\sigma}{dk_\perp^2} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \frac{1}{k_\perp^2} \left[P_{qq}(z) + \cdots \right]$$

$$P_{qq}(z) = \frac{4}{3} \frac{1+z^2}{1-z} \qquad \sigma_0 = \frac{4\pi^2 \alpha}{\hat{s}}$$

- integrate over *kt* generates *log,* BUT what is the lower limit

$$\sigma^{QCDC} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \left[P_{qq}(z) \log\left(\frac{Q^2(1-z)}{\kappa^2 z} \right) + \cdots \right]$$

Correction to cross section



$$\sigma^{QPM} = \sigma_0 \frac{1}{x_2} \delta\left(1 - \frac{x}{x_2}\right)$$

$$\sigma^{QCDC} = \sigma_0 \frac{1}{x_2} \otimes P_q(z) \otimes \log \ldots$$

$$\sigma^{BGF} = \sigma_0 \frac{1}{x_2} \otimes P_g(z) \otimes \log \ldots$$

x_2 is parton momentum fraction

$$\sigma_0 = \frac{4\pi^2\alpha}{2qP}$$

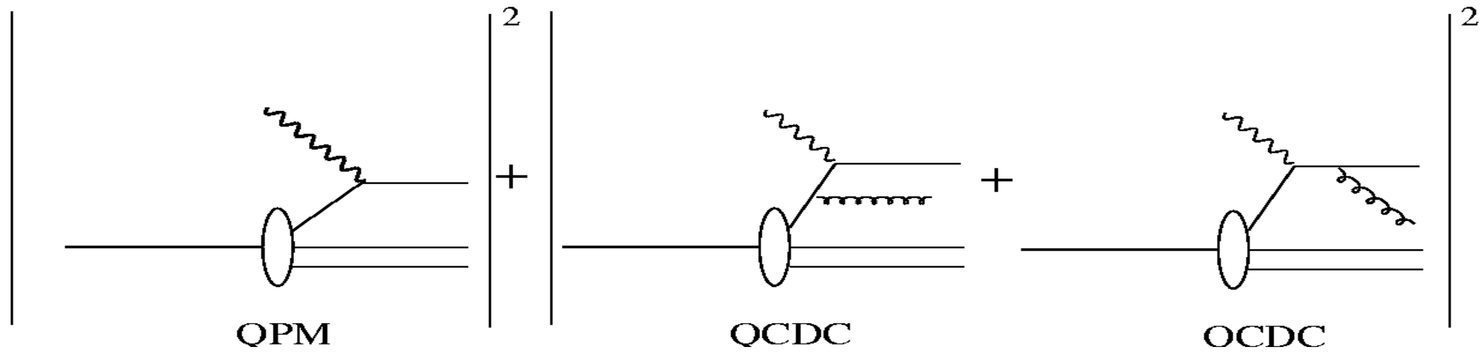
- Connect with F_2:

$$\sigma^{\gamma * p} = \frac{4\pi^2\alpha}{Q^2}\left(F_2(x,Q^2) - F_L(x,Q^2)\right) \sim \frac{4\pi^2\alpha}{Q^2} F_2(x,Q^2) = \frac{4\pi^2\alpha}{ys}\frac{F_2(x,Q^2)}{x}$$

$$\sigma^{\gamma * p} = \sigma_0 \frac{F_2(x,Q^2)}{x}$$

QCDC contribution to F_2

$\frac{F_2}{x} =$



$$\frac{F_2}{x} = \sum e_q^2 \int \frac{dx_2}{x_2} q_i(\xi) \left[\delta\left(1 - \frac{x}{x_2}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{x_2}\right) \left[\log\left(\frac{Q^2}{\kappa^2}\right) + \log\left(\frac{1-z}{z}\right) + \ldots \right] + C_q(z, ..) \right]$$

- again divergency for $k_\perp \to 0$ or $\kappa \to 0$
- but also for $z \to 1$

Collinear factorization (part 1)

$$F_2 = x \sum e_q^2 \left[q_0(x) + \int \frac{dx_2}{x_2} q_0(x) \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{x_2}\right) \log\left(\frac{Q^2}{\kappa^2}\right) + C_q(z,..) \right]$$

- bare distributions $q_0(x)$ are not measurable (like the bare charges ....)
- collinear singularities are absorbed into this bare distributions at a factorisation scale $\mu^2 \gg \kappa^2$, defining renormalized distributions

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left[q_i^0(\xi) P_{qq}\left(\frac{x}{\xi}\right) \log\left(\frac{\mu^2}{\kappa^2}\right) + C_q\left(\frac{x}{\xi}\right) \right] + \ldots$$

- now F_2 becomes:

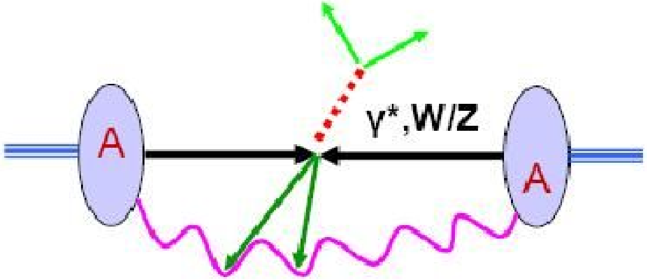
$$F_2 = x \sum e_q^2 \int \frac{dx_2}{x_2} q(x_2,\mu^2) \left[\delta\left(1 - \frac{x}{x_2}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{x_2}\right) \log\left(\frac{Q^2}{\mu^2}\right) + C \right]$$

- separating or factorising the long distance contributions to structure functions is a fundamental property of the theory
- factorisation provides a description for dealing with the logarithmic singularities, there is arbitariness in how the finite (non-logarithmic) parts are treated.
- **Be aware that factorisation is just an approximation to the full story ...**

Factorisation is an approximation !!!

Factorization is an approximation

- Drell-Yan cross section is **NOT** completely factorized!



$$\frac{d\sigma}{dQ^2} = f^{(2)} \otimes f^{(2)} \otimes \frac{d\hat{\sigma}^{(2)}}{dQ^2}$$

$$+ \frac{1}{Q^2} f^{(2)} \otimes f^{(4)} \otimes \frac{d\hat{\sigma}^{(4)}}{dQ^2}$$

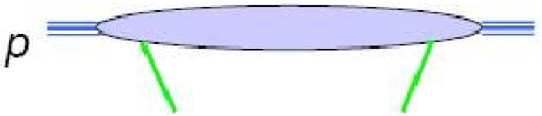
$$+ \frac{1}{Q^4} F\left(\frac{Q^2}{S}\right) + \ldots$$

Not factorized!

- There is **always** soft gluon interaction between two hadrons!
- Gluon field strength is **one power** more Lorentz contracted than ruler

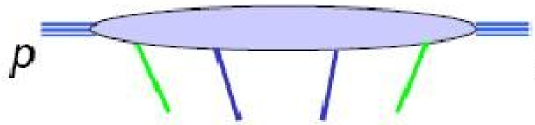
$$f^{(2)} \propto \langle p | \bar{\psi}(0) \gamma^+ \psi(y^-) | p \rangle,$$

$$\langle p | F^{+\alpha}(0) F_\alpha^{\ +}(y^-) | p \rangle$$



$$f^{(4)} \propto \langle p | \bar{\psi}(0) \gamma^+ F^{+\alpha}(y_1^-) F_\alpha^{\ +}(y_2^-) \psi(y^-) | p \rangle$$

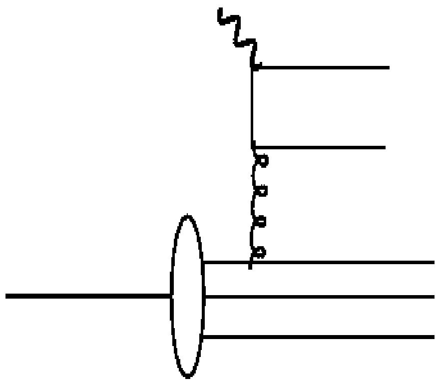
...



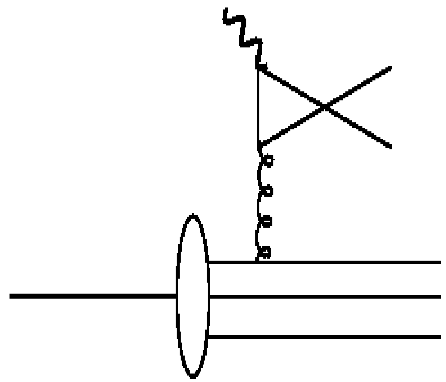
September 28, 2005 4 Jianwei Qiu, ISU

Boson gluon fusion

$$|ME|^2 = 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{1}{2} \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}} \right]$$



BGF



BGF

$$\frac{d\sigma}{dk_\perp^2} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \frac{1}{k_\perp^2} \left[P_{qg}(z) + \cdots \right]$$

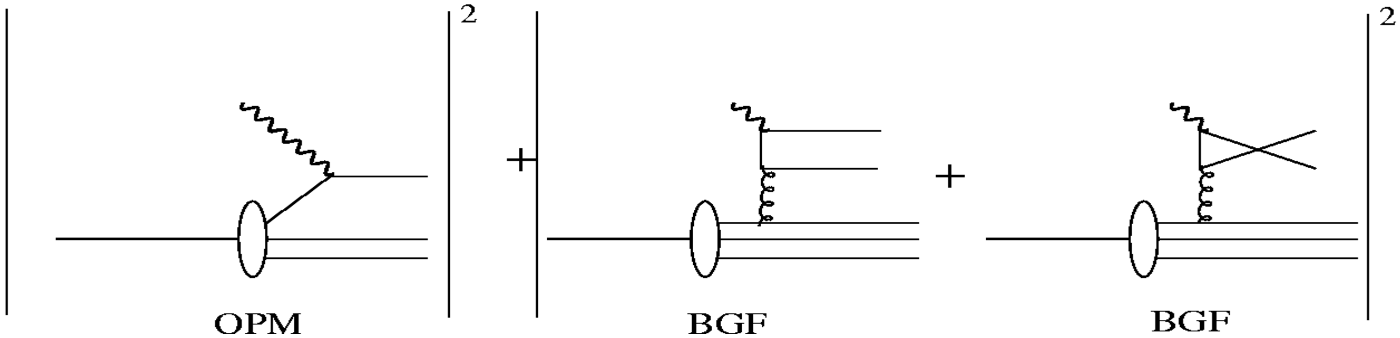
$$P_{qq}(z) = \frac{1}{2} \left(z^2 + (1-z)^2 \right)$$

- integrate over *kt* generates *log,* BUT what is the lower limit

$$\sigma^{BGF} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \left[P_{qg}(z) \log \left(\frac{Q^2(1-z)}{\kappa^2 z} \right) + \cdots \right]$$

BGF contribution to F_2

$$\frac{F_2}{x} =$$



$$\frac{F_2}{x} = \sum e_q^2 \int \frac{dx_2}{x_2} g(x_2)$$
$$\frac{\alpha_s}{2\pi}\left(P_{qg}\left(\frac{x}{x_2}\right)\left[\log\left(\frac{Q^2}{\kappa^2}\right) + \log\left(\frac{1-z}{z}\right) + \ldots\right] + C_g(z,\ldots)\right)$$

- again divergency for $k_\perp \to 0$ or $\kappa \to 0$
- but also for $z \to 1$

Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \kappa^2$ and include soft, non-perturbative physics into renormalised parton density:

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\kappa^2}\right)$$

- D okshitzer G ribov L ipatov A ltarelli P arisi equation (take derivative of the above eq):

  V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20,
  G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

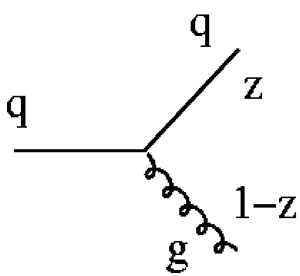
$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons....

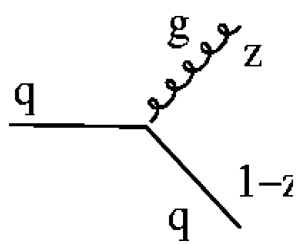
$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- DGLAP is the analgue to the beta function for running of the coupling

Splitting functions in lowest order

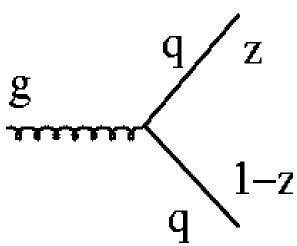


$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

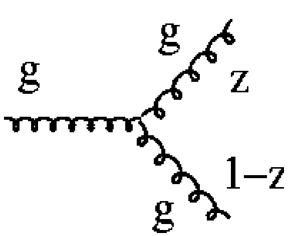


$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to EPA...



$$P_{qg} = \frac{1}{2}\left(z^2+(1+z)^2\right)$$

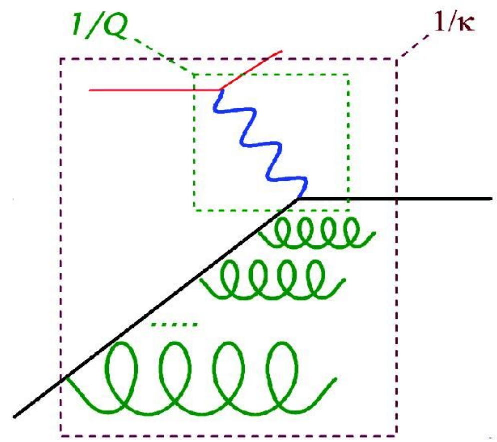


$$P_{gg} = 6\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

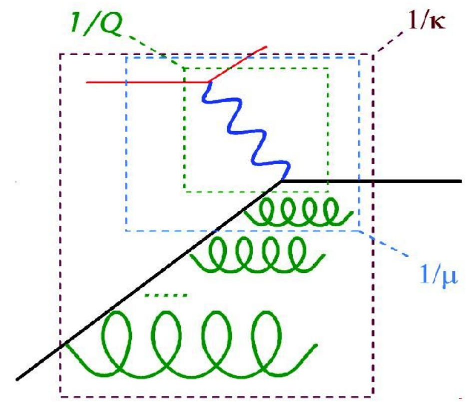
Meaning of evolution equations

- order emissions by "size"
- limits on the included emissions are provided by the scales (Q^2) for the short distance and κ for the long distance

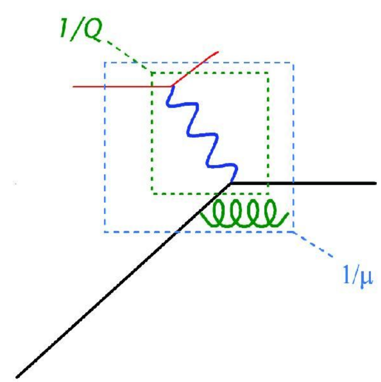
From S. Ellis, Lecture 2, 2003



- next separate contributions above and below factorisation scale μ



- factor scales κ to μ into renormalized parton distributions. All above μ can be calculated perturbatively



Collinear factorisation schemes

- DIS scheme: absorbing all finite contributions C_q into quark densities, with no finite $\mathcal{O}(\alpha_s)$ corrections:

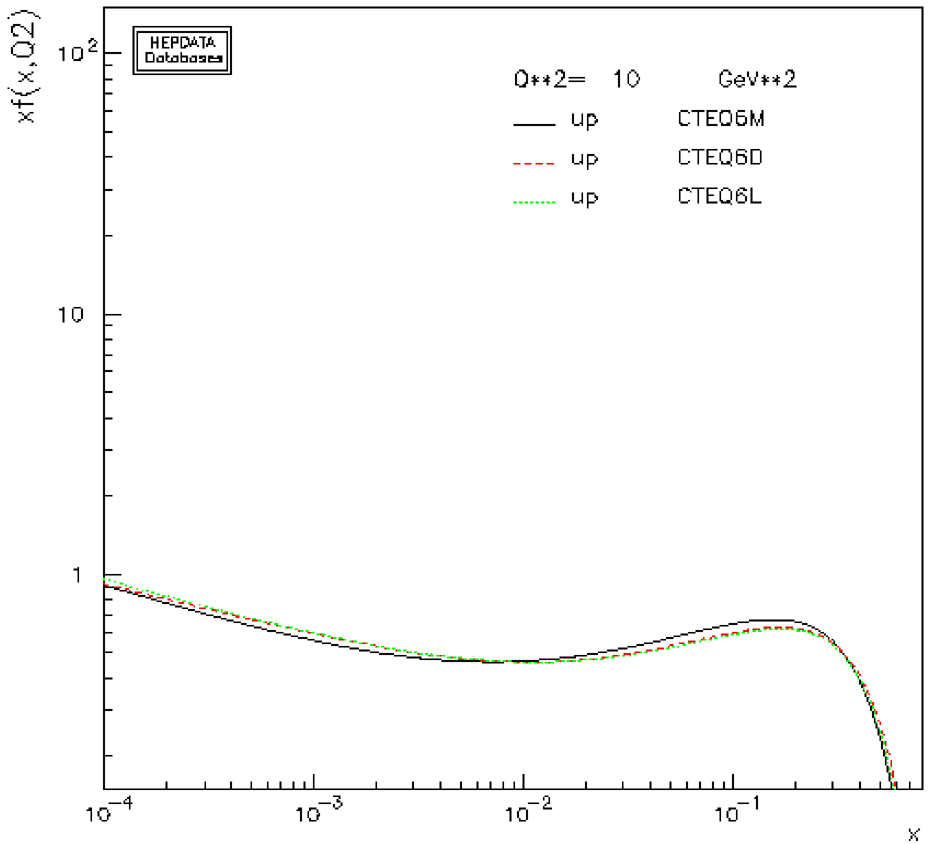
$$F_2^{DIS}(x,Q^2) = x\sum e_q^2 q(x,Q^2)$$

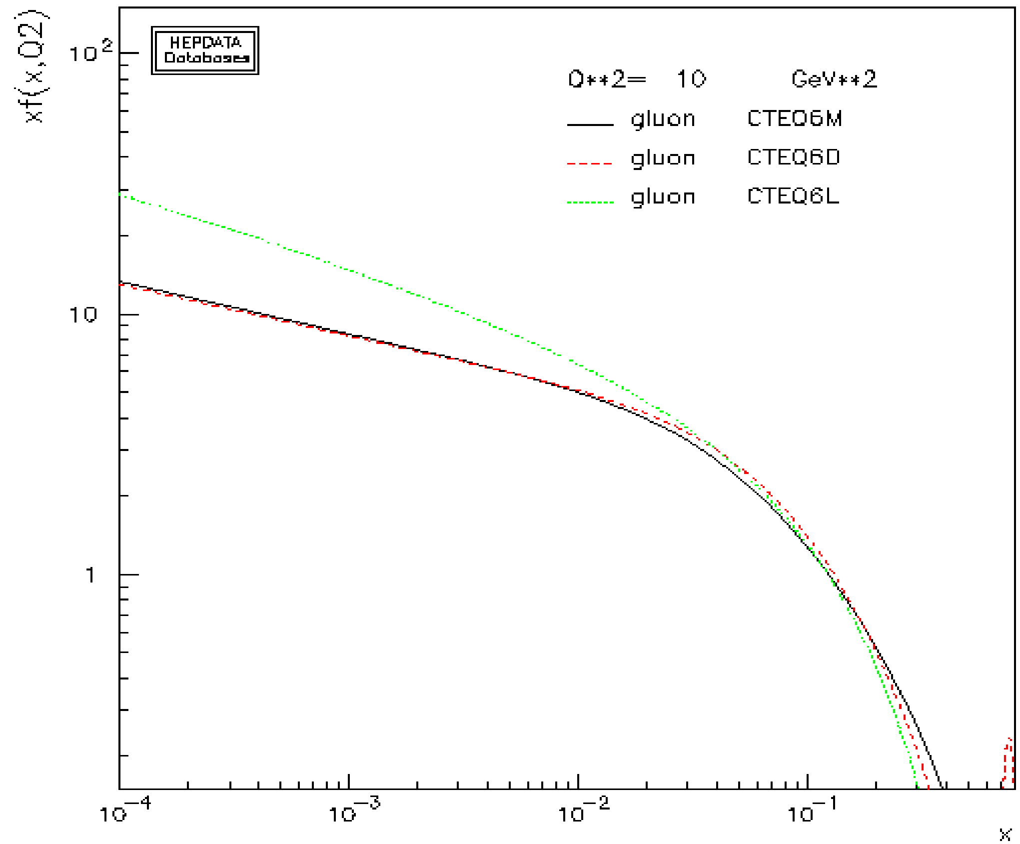
- $\overline{\mathrm{MS}}$ scheme, where only minimal contributions from the finite parts are absorbed in the quark distributions:

$$F_2^{\overline{MS}}(x,Q^2) = x\sum e_q^2 \int \frac{dx_2}{x_2} q^{\overline{MS}}(x,Q^2)\left[\delta\left(1-\frac{x}{x_2}\right) + \frac{\alpha_s}{2\pi} C^{\overline{MS}}\left(\frac{x}{x_2}\right) + \ldots\right]$$

- once the scheme is chosen, it MUST be used in all other cross section calculations
- higher order corrections will of course depend on the chosen scheme...
- **BUT....** there are still other contributions to be included... gluon induced processes

pdfs in different fact. schemes





- differences between DIS and $\overline{MS}$ scheme in quark and gluon densities
- can make significant effects for x-sections

Evolution kernels – splitting fcts

- Splitting functions have perturbative expansion in the running coupling:

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + ...$$

- including more and more loops .... as for running coupling...
- some of the splitting functions are also divergent... $\frac{1}{1-z}$
- use *plus-distribution* to avoid dangerous region:

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{1-z}$$

- divergence cancelled by virtual corrections ...
- use splitting fcts with *plus-distribution*

Splitting functions at higher orders

S. Moch, HERA-LHC workshop, June 2004

The calculation (in a nut shell)

– Calculate anomalous dimensions (Mellin moments of splitting functions)
$\longrightarrow$ divergence of Feynman diagrams in dimensional regularization $D = 4 - 2\varepsilon$

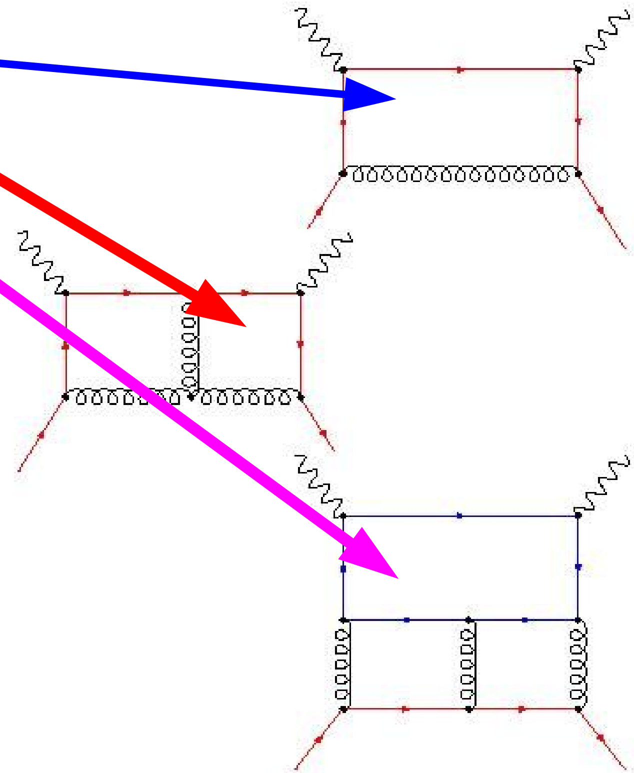
$$\gamma_{\rm ij}^{(n)}(N) = -\int_0^1 dx\, x^{N-1} P_{\rm ij}^{(n)}(x)$$

loops again:
1-loop
2-loops
3-loops

– **One-loop** Feynman diagrams
$\longrightarrow$ in total 18 for $\gamma_{\rm ij}^{(0)}$ / $P_{\rm ij}^{(0)}$
(pencil + paper)

– **Two-loop** Feynman diagrams
$\longrightarrow$ in total 350 for $\gamma_{\rm ij}^{(1)}$ / $P_{\rm ij}^{(1)}$
(simple computer algebra)

– **Three-loop** Feynman diagrams
$\longrightarrow$ in total 9607 for $\gamma_{\rm ij}^{(2)}$ / $P_{\rm ij}^{(2)}$
(cutting edge technology $\longrightarrow$ computer algebra system FORM Vermaseren '89-'04)



Splitting functions (cont'd)

S. Moch, HERA-LHC workshop, June 2004

LO and NLO singlet splitting functions

$$
\begin{aligned}
P^{(0)}_{\mathrm{ps}}(x) &= 0 \\
P^{(0)}_{\mathrm{qg}}(x) &= 2 n_f \, p_{\mathrm{qg}}(x) \\
P^{(0)}_{\mathrm{gq}}(x) &= 2 C_F \, p_{\mathrm{gq}}(x) \\
P^{(0)}_{\mathrm{gg}}(x) &= C_A \Big(4 p_{\mathrm{gg}}(x) + \frac{11}{3}\delta(1-x) \Big) - \frac{2}{3} n_f \, \delta(1-x)
\end{aligned}
$$

$$
P^{(1)}_{\mathrm{ps}}(x) = 4 C_F n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2 \Big[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9} \Big] + (1+x) \Big[5\mathrm{H}_0 - 2\mathrm{H}_{0,0} \Big] \Big)
$$

$$
\begin{aligned}
P^{(1)}_{\mathrm{qg}}(x) &= 4 C_A n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2 p_{\mathrm{qg}}(-x)\mathrm{H}_{-1,0} - 2 p_{\mathrm{qg}}(x)\mathrm{H}_{1,1} + x^2 \Big[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9} \Big] \\
&+ 4(1-x) \Big[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1 \Big] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0 \Big) + 4 C_F n_f \Big(2 p_{\mathrm{qg}}(x) \Big[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 \\
&- \zeta_2 \Big] + 4x^2 \Big[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2} \Big] + 2(1-x) \Big[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4} \Big] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0 \Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\mathrm{gq}}(x) &= 4 C_A C_F \Big(\frac{1}{x} + 2 p_{\mathrm{gq}}(x) \Big[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1 \Big] - x^2 \Big[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9} \Big] + 4\zeta_2 - 2 \\
&- 7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x) \Big[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9} \Big] - 2 p_{\mathrm{gq}}(-x)\mathrm{H}_{-1,0} \Big) - 4 C_F n_f \Big(\frac{2}{3}x \\
&- p_{\mathrm{gq}}(x) \Big[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9} \Big] \Big) + 4 C_F^2 \Big(p_{\mathrm{gq}}(x) \Big[3\mathrm{H}_1 - 2\mathrm{H}_{1,1} \Big] + (1+x) \Big[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0 \Big] - 3\mathrm{H}_{0,0} \\
&+ 1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x \Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\mathrm{gg}}(x) &= 4 C_A n_f \Big(1 - x - \frac{10}{9} p_{\mathrm{gg}}(x) - \frac{13}{9}\Big(\frac{1}{x} - x^2\Big) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x) \Big) + 4 C_A^2 \Big(27 \\
&+ (1+x) \Big[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2} \Big] + 2 p_{\mathrm{gg}}(-x) \Big[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2 \Big] - \frac{67}{9}\Big(\frac{1}{x} - x^2\Big) - 12\mathrm{H}_0 \\
&- \frac{44}{3}x^2\mathrm{H}_0 + 2 p_{\mathrm{gg}}(x) \Big[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2 \Big] + \delta(1-x) \Big[\frac{8}{3} + 3\zeta_3 \Big] \Big) + 4 C_F n_f \Big(2\mathrm{H}_0 \\
&+ \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x) \Big[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0} \Big] - \frac{1}{2}\delta(1-x) \Big) .
\end{aligned}
$$

Splitting functions (cont'd)

S. Moch, HERA-LHC workshop, June 2004

NNLO singlet splitting functions

Gluon distribution at higher orders

- using different approximations to splitting fcts results in different behaviour of parton distributions
- observe negative gluon distribution at small x
- higher order corrections are important
- behaviour at small/medium Q2 changes significantly when using higher order corrections ....

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