QCD and Collider Physics: Renormalization, evolution, etc

- Resume from last lecture
- Structure functions
  - Parton distribution functions
  - Picture of the proton
  - Inelastic scattering – main results
- Renormalization
  - Coupling strength in QED
  - Coupling strength in QCD

http://www-h1.desy.de/~jung/qcd_collider_physics_2005

Space time picture: free partons ?

- Compare lifetime of proton fluctuation τ with time of interaction τ_0
- In CM frame P splits into $q1=xP$ and $q2=(1-x)P$, with k_t.

$$\tau = \frac{1}{\Delta E}$$

$$q_1 = (E_1, \vec{k}_t, xP)$$

$$q_2 = (E_2, -\vec{k}_t, (1-x)P)$$

$$E_1 \sim xP + \frac{1}{2}\frac{k_t^2}{xP}$$

$$E_2 \sim (1-x)P + \frac{1}{2}\frac{k_t^2}{(1-x)P}$$

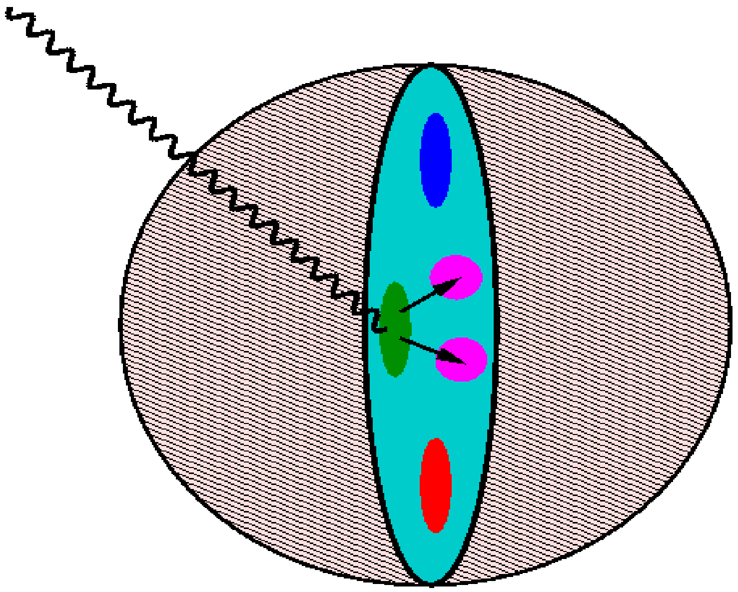
$$\Delta E = E_1 + E_2 - E_0 = \frac{k_t^2}{2x(1-x)P} \sim \frac{k_t^2}{2xP}$$

$$\gamma^* \text{ four-vector } q = (E_\gamma, \mathbf{q_t}, 0)$$

$$\text{with } x = \frac{Q^2}{2q.p} \text{ obtain } E_\gamma = \frac{Q^2}{2xP}$$

$$\tau_0 = \frac{1}{E_\gamma}$$

$$\frac{\tau}{\tau_0} \sim \frac{k_t^2}{Q^2}$$



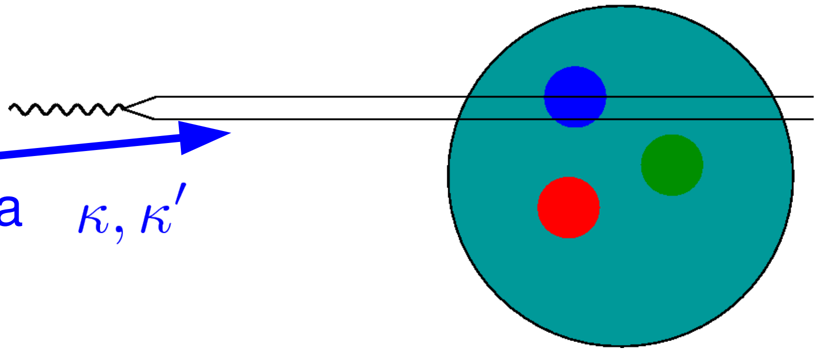
- Lifetime of proton fluctuation long compared to interaction time

Color Dipole Picture: formation time

Barone Predazzi p 270

- In target rest frame:

$$q = (q^+, q^-, \mathbf{0}) = (q^+, \frac{-Q^2}{2q^+}, \mathbf{0})$$



- Photon splits into $q\bar{q}$ with momenta κ, κ'

$$\kappa = (zq^+, \frac{\boldsymbol{\kappa}^2}{2zq^+}, \boldsymbol{\kappa})$$

$$\kappa' = ((1-z)q^+, \frac{\boldsymbol{\kappa}^2}{2(1-z)q^+}, -\boldsymbol{\kappa})$$

$$(\kappa + \kappa') = (q^+, \frac{\boldsymbol{\kappa}^2}{z(1-z)q^+,0}$$

$$M^2_{q\bar{q}} = \frac{\kappa^2}{z(1-z)}$$

$$\kappa^0 = \frac{1}{\sqrt{2}}\left(q^+ + \frac{\kappa^2}{2q^+ z(1-z)}\right)$$

$$q^0 = \frac{1}{\sqrt{2}}\left(q^+ - \frac{Q^2}{2q^+}\right)$$

- Formation time is

$$\Delta E = E_{pair} - E_\gamma = \frac{Q^2}{4\sqrt{2}q^+}\left(1 + \frac{M^2_{q\bar{q}}}{Q^2}\right) \sim mx$$

$$\tau_f \sim \frac{1}{\Delta E} \sim \frac{1}{mx}$$

- Transverse size of the dipole is frozen during the interaction with the proton !

Cross section: example

- In CM system for $2 \to 2$process with $s = (p_1 + p_2)^2$

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{1}{|\mathbf{p}_i^{cm}|^2} |ME|^2$$

- neglecting masses of incoming particle

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |ME|^2$$

- using $$|ME(eq \to eq)|^2 = 2(4\pi\alpha)^2 \frac{s^2 + u^2}{t^2}$$

- gives for $eq \to eq$ $$\frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{t^2} \frac{s^2 + t^2}{s^2}$$

Inelastic Scattering: QPM

Ellis,Webber, Stirling, p 90 ff

- Infinite momentum frame: $p^\mu = (P, 0, 0, P)$ with $P \gg M$
- Virtual photon scatters off pointlike quark which moves parallel (collinear) to proton, with momentum fraction $p_q^\mu = \xi p^\mu$
- Using DIS variables gives for $eq \to eq$

$$|ME|^2 = \frac{2e_q^2 (4\pi\alpha)^2 \hat{s}^2}{Q^4} \left(1 + (1-y)^2\right)$$

- giving $$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4} \left(1 + (1-y)^2\right)$$
- Using mass shell condition for outgoing quark gives (with $\int_0^1 dx \delta(x-\xi) = 1$)

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left(1 + (1-y)^2\right) \frac{1}{2} e_q^2 \delta(x-\xi)$$

- Compare the with formula from lecture 1 (page 11), after some algebra

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 + (1-y)^2\right) F_1 + \frac{1-y}{x} (F_2 - 2xF_1) \right]$$

Inelastic Scattering QPM

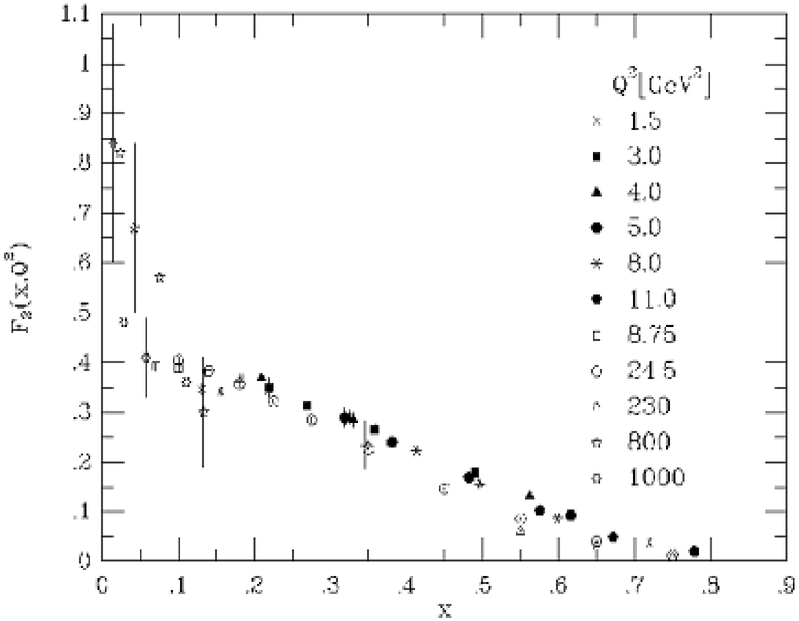
- Simple model with

$$\tilde{F}_2 = x e_q^2 \delta(x-\xi) = 2x\tilde{F}_1$$

- BUT structure function is a distribution. F_2 is a function of x:

  scaling, no Q^2 dependence

- $q(\xi)d\xi$ is probability to find q with momentum fraction $\xi ....\xi + d\xi$

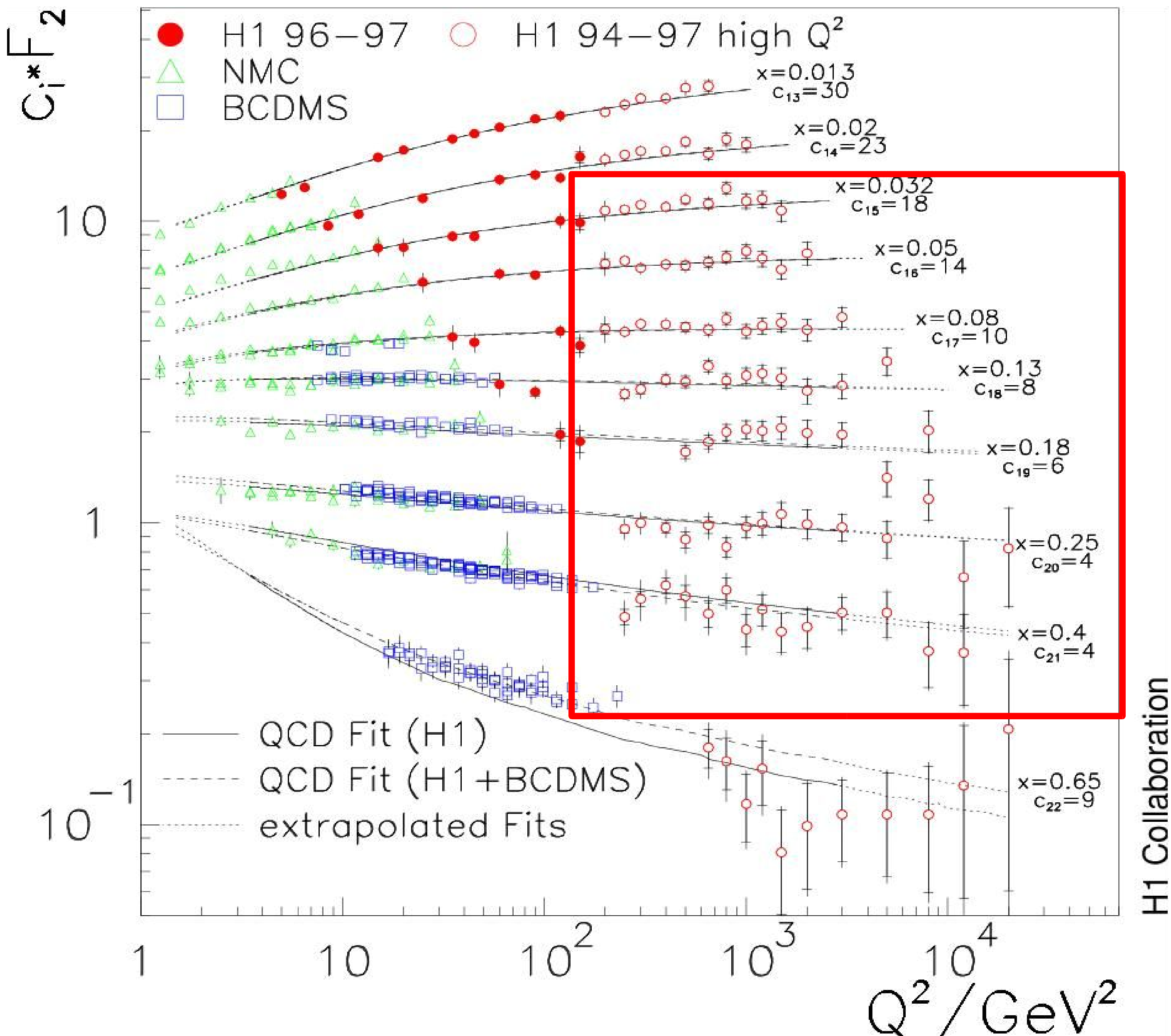


$$F_2(x) = 2xF_1(x) = \sum_{q,\bar{q}} \int_0^1 d\xi q(\xi) x e_q^2 \delta(x-\xi) = \sum_{q,\bar{q}} e_q^2 x q(x)$$

- Proton structure function is:

$$F_2^{em} = x\left[\frac{4}{9}\left(u(x)+\bar{u}(x)+c(x)+\bar{c}(x)\right)+\frac{1}{9}\left(d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right)\right]$$

Structure functions from HERA



- Proton structure function does not depend on Q^2 for large x
- F_2 scales ...
- Quarks are pointlike constituents of proton
- BUT things change at smaller x..... and smaller Q^2

Parton distribution functions (pdfs)

- $f_i(\xi)d\xi$ gives probability that parton *i* carries momentum fraction between ξ and $\xi + d\xi$ with $0 \leq \xi \leq 1$

- Number of partons *i*:

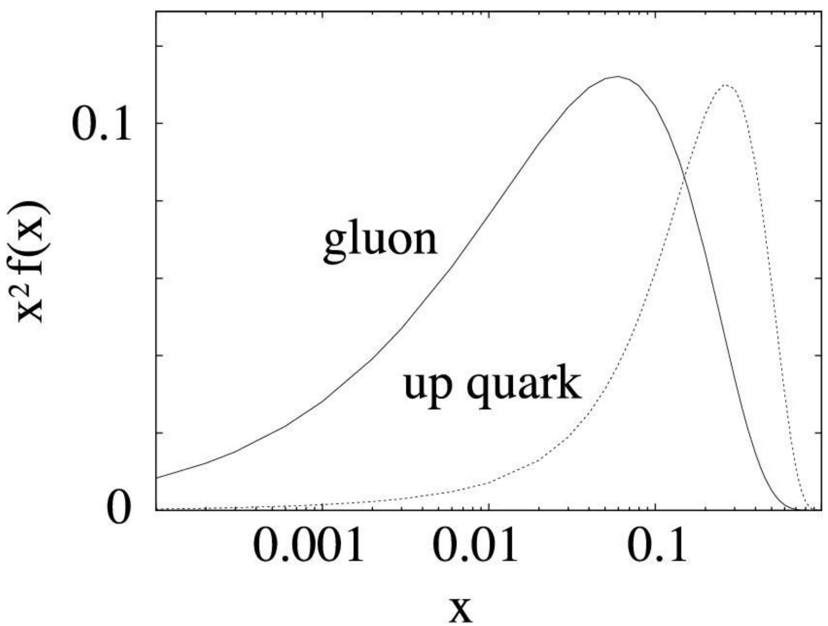
$$N_i = \int_0^1 d\xi f_i(\xi)$$

- Momentum fraction carried by partons *i*:

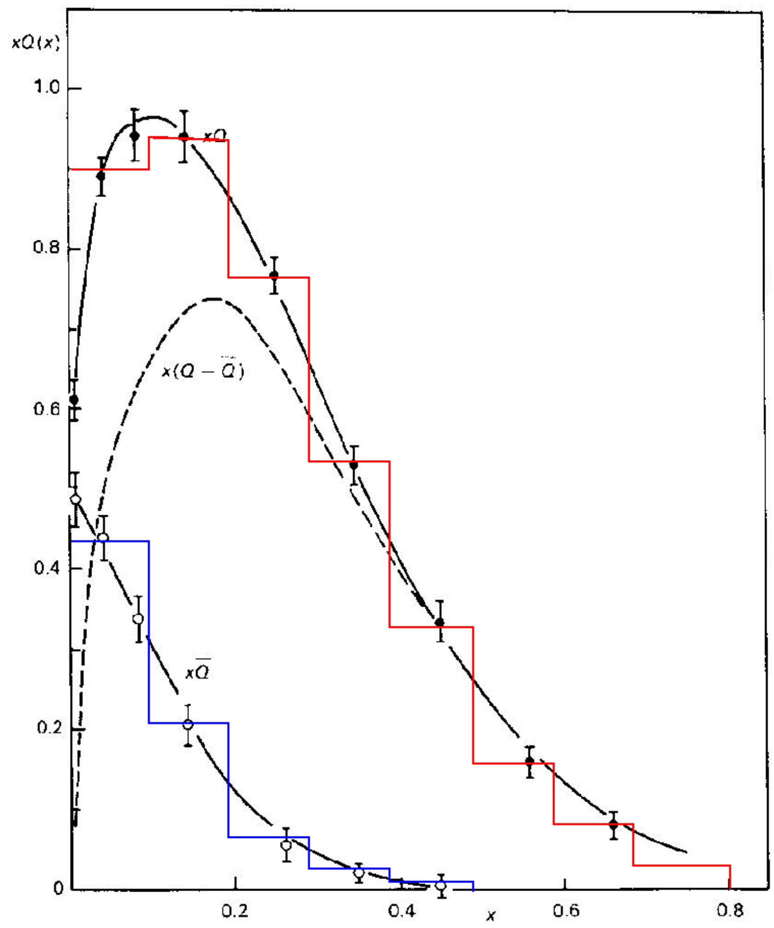
$$\frac{<p_i>}{P} = \int_0^1 d\xi \;\; \xi f_i(\xi) = \int_0^1 d\log\xi \;\; \xi^2 f_i(\xi)$$

- Define sum-rules for hadron target:
  - Number of valence partons
  - Momentum carried by partons
  - Flavor contents

From D. Soper hep-ph/9609018



Picture of the Proton



- Flavor sum rules for proton:

$$\int_0^1 dx u_V(x) = 2$$

$$\int_0^1 dx d_V(x) = 1$$

- Momentum sum of quarks:

$$\sum_q \int_0^1 dx x \left[q(x) + \bar{q}(x)\right] \sim 0.5$$

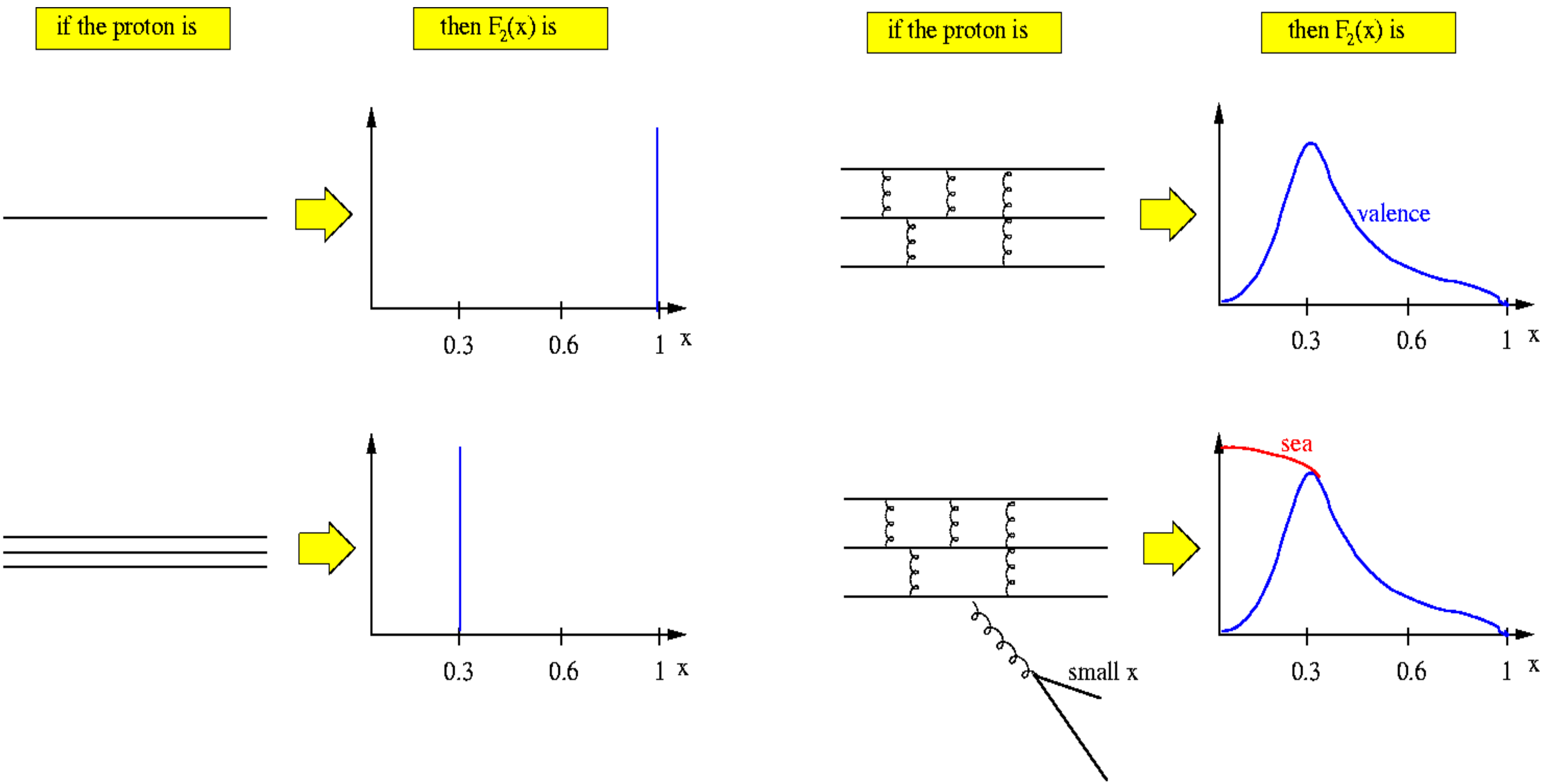
- Where are the other 50 % of the proton's momentum ?

$$\int dx x q(x) \sim 0.1 \left[0.9 + 0.95 + 0.85 + 0.7 + 0.35 + 0.15 + 0.1 + 0.05\right] = 0.1 \cdot 4.05 = 0.405$$

$$\int dx x \bar{q}(x) \sim 0.1 \left[0.42 + 0.2 + 0.06 + 0.03 + 0.01\right] = 0.1 \cdot 0.72 = 0.072$$

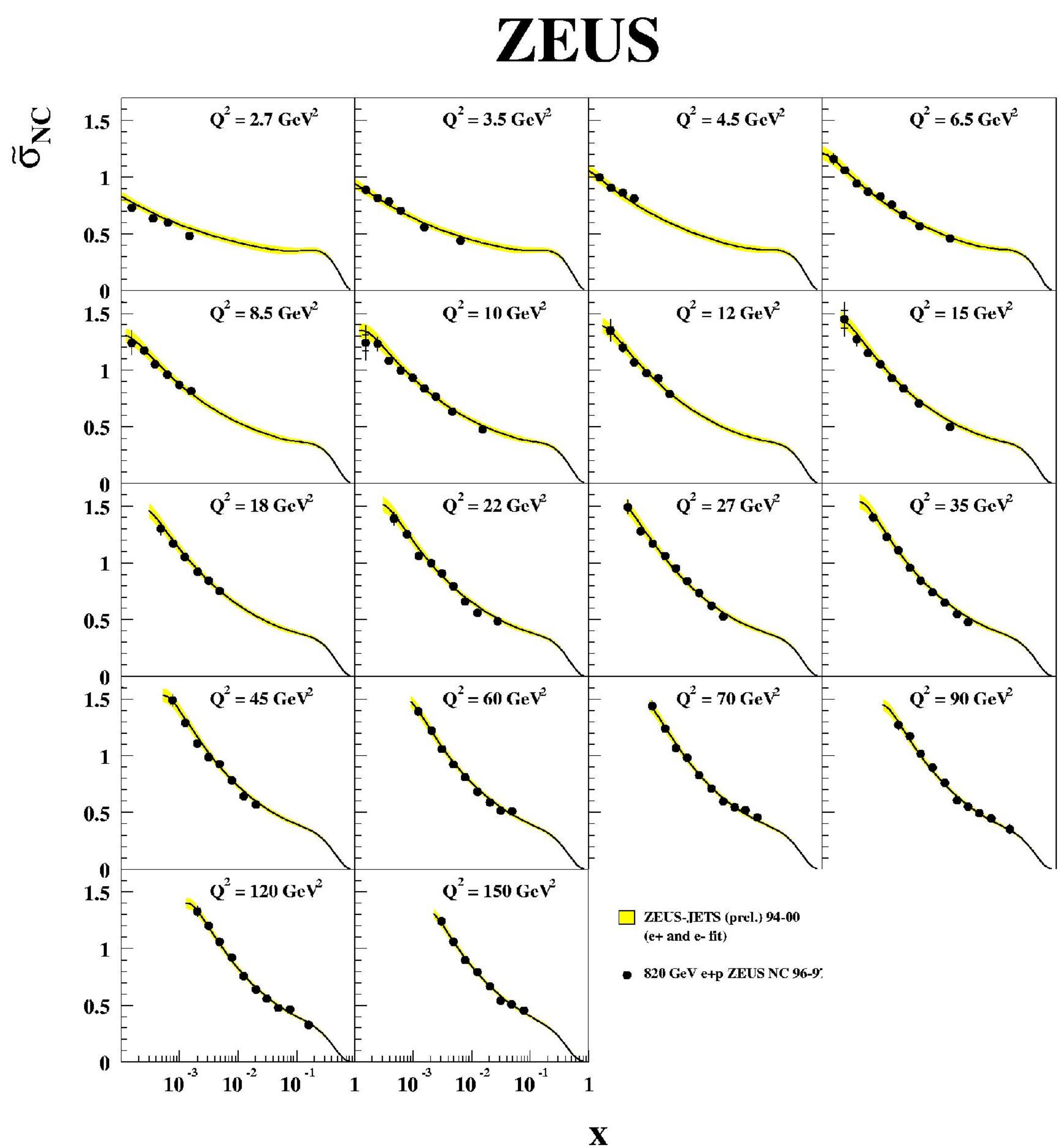
Naïve F_2 picture

From Halzen & Martin: Quarks & Leptons, p201



Inelastic Scattering: main results

- F_2 scaling at large x
- ~ 50 % gluons
- F_2 rise at small x
  - How can rising F_2 be understood ?
  - Does rise continue forever ?
  - What limits F_2 ?



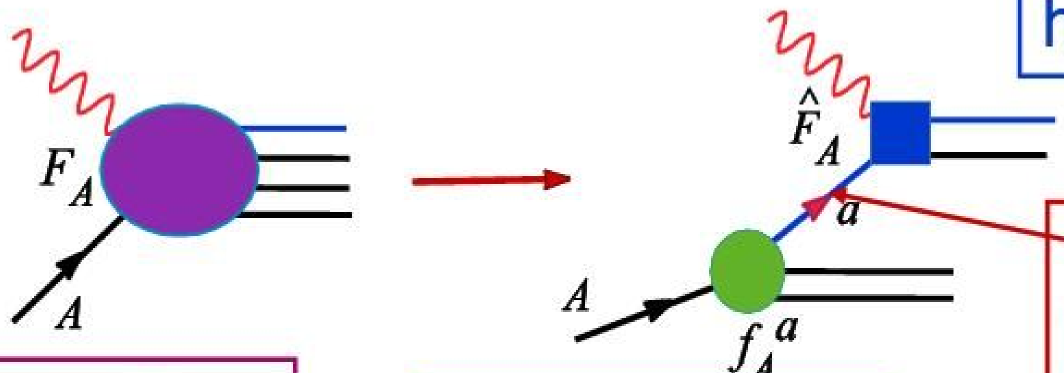
Structure functions and PDFs

W-K. Tung, CTEQ Summerschool 2005

A common misconception:

Parton distribution functions ≠ "Structure functions"

These are the hard Xsecs.



These are the (process-dep) S.F.s

These are the (universal) PDFs

There is a convolution integral and a summation over partons here!

Inelastic Scattering: Z/W exchange

Ellis,Webber, Stirling, p 91 ff

- At large Q^2 also Z exchange and Z-gamma interference becomes important:

$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[xy^2F_1 + (1-y)F_2 + y(1-\frac{1}{2}y)F_3\right]$$

- With

$$F_2 = 2xF_1 = \sum_q x[q(x) + \bar{(}x)]C_q$$

$$xF_3 = \sum_q x[q(x) - \bar{(}x)]D_q$$

$$C_q(Q^2) = e_q^2 - 2e_qV_eV_qP_Z + (V_e^2 + A_e^2)(V_q^2 + A_q^2)P_Z^2$$

$$D_q(Q^2) = -2e_qA_eA_qP_Z + 4V_eA_eV_qA_q^2P_Z^2$$

$$P_Z = \frac{Q^2}{Q^2 + M_Z^2}$$

- Charged current (W) exchange

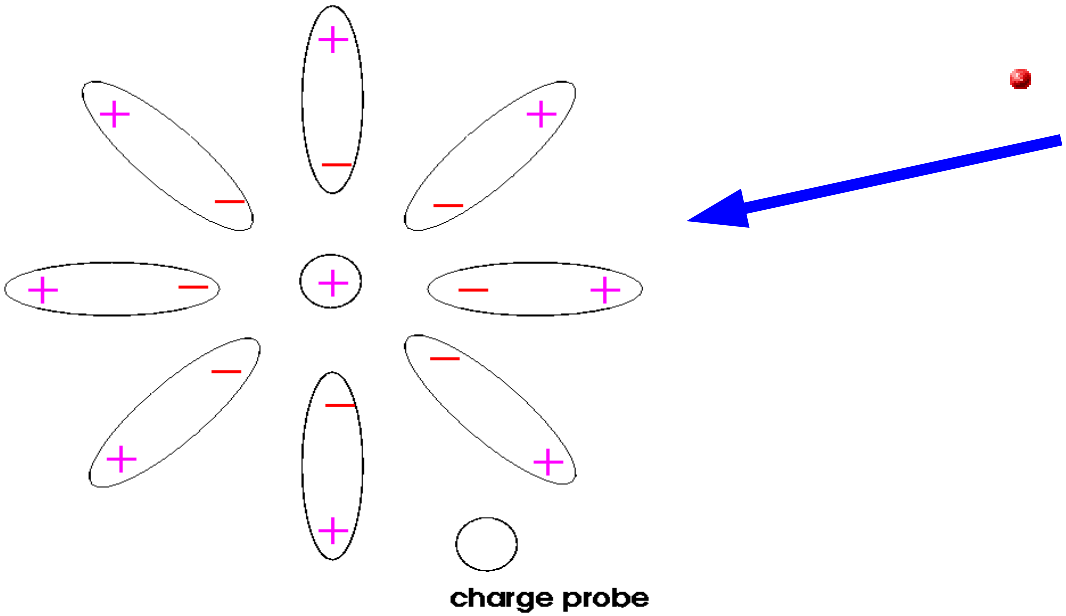
$$\frac{d\sigma_{CC}}{dxdQ^2} = \frac{(1-\lambda)\pi\alpha^2}{8sin^4\theta_W(Q^2+M_W^2)^2}\sum_{i,j}\left[|V_{u_id_j}|^2u_i(x) + (1-y)^2|V_{u_jd_i}|^2d_i(x)\right]$$

Inelastic Scattering: QPM

- Key factor in QPM explanation is that over a short time in which the hard scattering takes place, the quarks behave as if they are free, i.e. No interaction between them.
- In the asymptotic limit ($Q^2 \to \infty$) the theory should describe quarks as free particles
- Equivalent demanding that effective charge in theory should vanish as smaller and smaller distances are probed.
- Until 1973 in theories the reverse was true: because of screening of charge at larger distances coupling becomes smaller (QED)
- BREAK-THROUGH by 't Hooft (1972), Gross, Wilczek & Politzer (1973)
  non-Abelian theory describing asymptotic behaviour

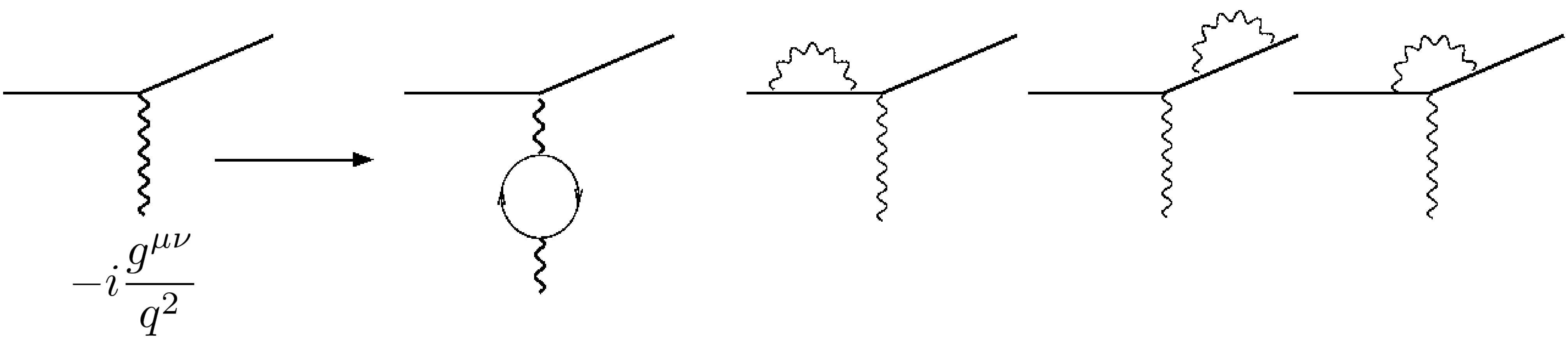
  QCD
- As in QED there is screening at large distances by the color charge of quarks and gluons, but this is more than compensated by anti-screening (splitting) of gluons. Thus for $Q^2 \to \infty$ the effective coupling tends to vanish !

Coupling strength in QED



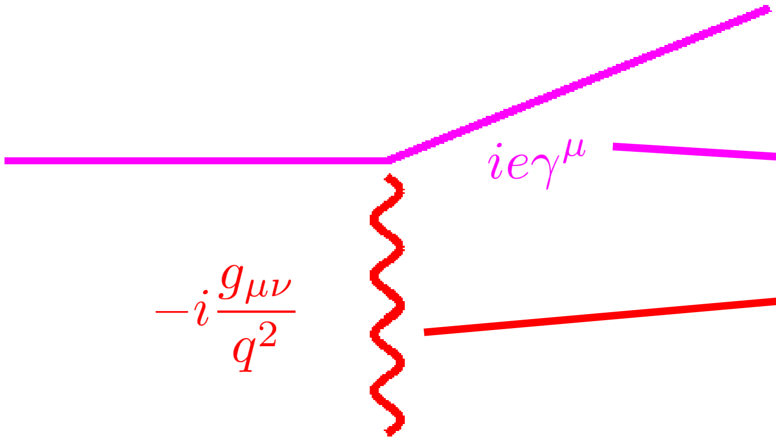
- In QED effective charge increases from large to small distances due to vacuum fluctuations

- Contributions of higher order α_{em} in perturbation series:



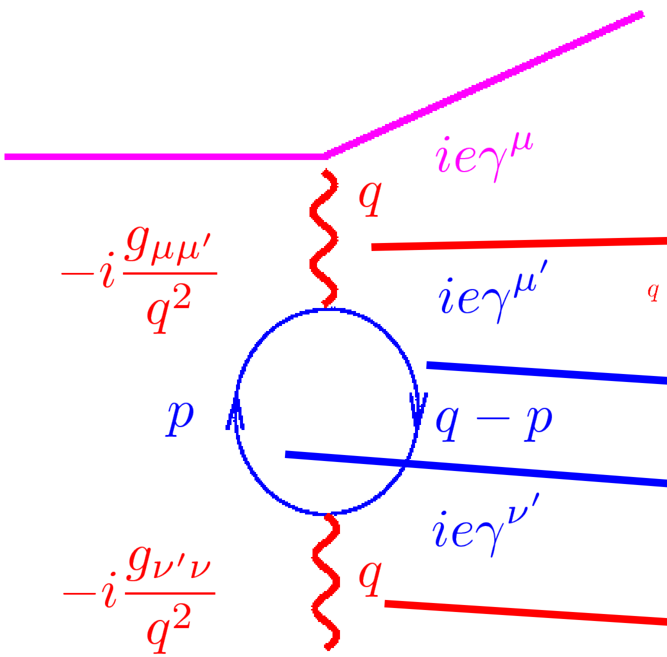
Higher order corrections

From Halzen & Martin: Quarks & Leptons, p157 ff



- Use Feynman rules to calc ep scattering:

$$-i\mathcal{M}_1 = (ie\bar{u}_f\gamma^\mu u_i)\left(\frac{-ig_{\mu\nu}}{q^2}\right)(-ij^\mu(\mathbf{q}))$$



- Apply same rules for loop:

l fermion loops

$$-i\mathcal{M}_2 = (-1)^l (ie\bar{u}_f\gamma^\mu u_i)\left(\frac{-ig_{\mu\nu}}{q^2}\right)$$
$$\times \int \frac{d^4p}{2\pi^4}\left[\left(ie\gamma^{\mu'}\right)\frac{i(\not{p}+m)}{p^2-m^2}\left(ie\gamma^{\nu'}\right)\frac{i(\not{p}-\not{q}+m)}{(p-q)^2-m^2}\right]$$
$$\times\left(\frac{-ig_{\mu\nu}}{q^2}\right)(-ij^\mu(\mathbf{q}))$$

Loops, Loops and Loops

$$
\begin{aligned}
-i\mathcal{M} &= -i\mathcal{M}_1 - i\mathcal{M}_2 \\
\left(\frac{-ig_{\mu\nu}}{q^2}\right) &\to \left(\frac{-ig_{\mu\nu}}{q^2}\right) + \left(\frac{-i}{q^2}\right) I_{\mu\nu}(q^2) \left(\frac{-i}{q^2}\right) \\
I_{\mu\nu} &= (-1)^l \int \frac{d^4p}{2\pi^4} \mathrm{Tr}\left[(ie\gamma^\mu)\frac{i(\not p + m)}{p^2 - m^2}(ie\gamma^\nu)\frac{i(\not p - \not q + m)}{(p-q)^2 - m^2}\right] \\
N^{\mu\nu} &= \gamma^\mu(\not p + m)\gamma^\nu(\not p - \not q + m)
\end{aligned}
$$

- Use *FORM* to calculate trace....

$$
N^{\mu\nu}(p,q) = 4\left[(p^\mu + q^\mu)p^\nu + (p^\nu + q^\nu)p^\mu + [m^2 - p(p+q)]g^{\mu\nu}\right]
$$

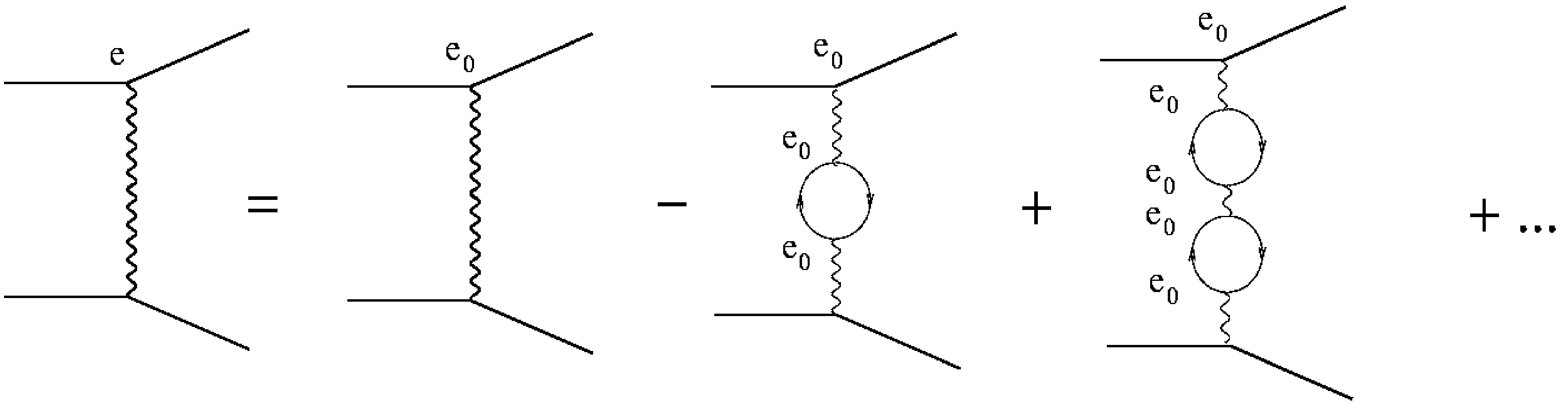
- Loop integral divergent: p momentum is not restricted
  note: $\int \frac{dp^2}{p^2} = \log p^2$
- Use tricks to evaluate integral.... (Mandl,Shaw Quantum field theory, p229)

$$
I(q^2) = \frac{\alpha}{3\pi}\int_{m^2}^{\infty}\frac{dp^2}{p^2} - \frac{\alpha}{3\pi}\log\left[\frac{-q^2}{m^2}\right] = \frac{\alpha}{3\pi}\log\left[\frac{M^2}{q^2}\right]
$$

for - q^2 large

Renormalization and α_{em}

- The electric charge which can be measured is:



- The relation between e and e_0 has to be specified at a particular value of the photon virtuality Q^2

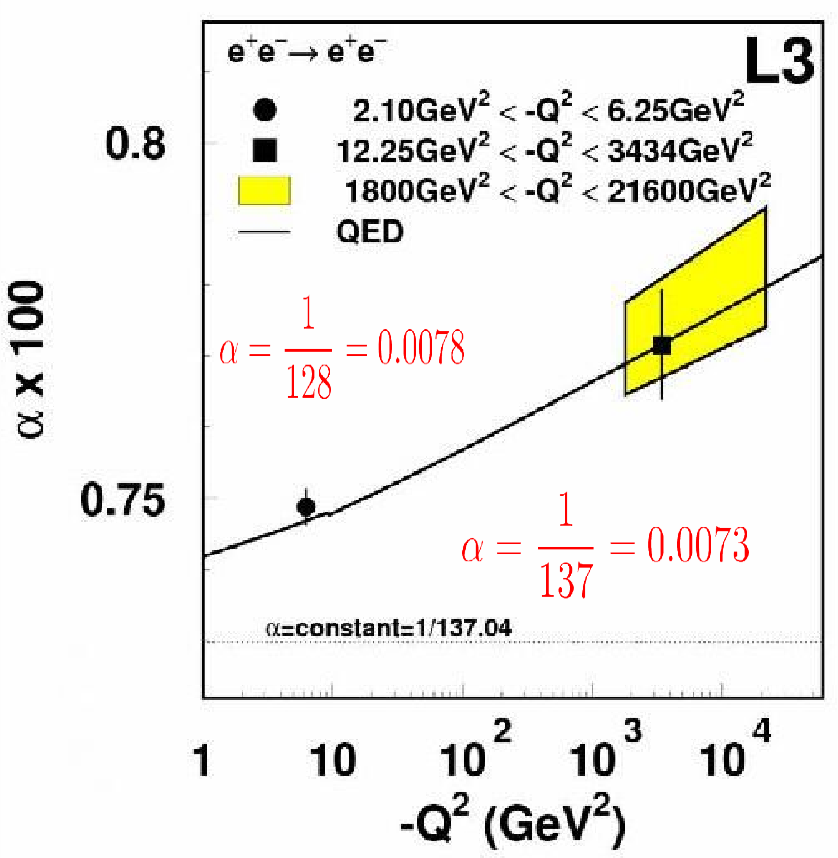
$$e^2 = e_0^2 \left[1 - I(q^2) + \left(I(q^2) \right)^2 - \cdots \right] = e_0^2 \frac{1}{(1 + I(q^2))}$$

- Replacing α_0 with $\alpha(\mu^2)$:

$$\alpha_{em}(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{Q^2}{\mu^2} \right)}$$

Measurement of α_{em}

Combination of the three L3 measurements



$$\alpha(Q^2) = \frac{\alpha_0}{1 - C \cdot \Delta\alpha(Q^2)}$$

$C = 1.05 \pm 0.15$ for $1800\,GeV^2 < -Q^2 < 21600\,GeV^2$
$26° < \theta < 90°$ $\sqrt{s} = 189 - 209\,GeV$ hep-ex/0507078

$\alpha^{-1}(-2.10\,GeV^2) - \alpha^{-1}(-6.25\,GeV^2) = 0.78 \pm 0.26$
$1.8° < \theta < 3.1°$ $\sqrt{s} \approx 91\,GeV$ PLB **476**(2000)40

$\alpha^{-1}(-12.25\,GeV^2) - \alpha^{-1}(-3434\,GeV^2) = 3.80 \pm 1.29$
$20° < \theta < 36°$ $\sqrt{s} = 189$ PLB **476**(2000)40 (not discussed here)

The last two measurements only give the difference of two values of $\alpha(Q^2)$: need to anchor these numbers to plot the evolution of $\alpha(Q^2)$!

Use the measured value of C to fix $\alpha(-2.10\,GeV^2)$ and $\alpha(-12.25\,GeV^2)$ and therefore extract $\alpha(-6.25\,GeV^2)$ and $\alpha(-3434\,GeV^2)$.

QED predictions from Burkhardt&Pietrzyk PLB **513**(2001)46