### **QCD and Collider Physics: Heavy Quarks, Fragmentation-Hadronization**

- Resume from last lecture
- Heavy Quarks: fragmentation functions, massive/massless approach
- Dijet production in DIS (in LO and NLO)
- Approaches to even higher orders
  - Parton showers
  - unintegrated pdfs
- Fragmentation/hadronization

http://www-h1.desy.de/~jung/qcd\_collider\_physics\_2005

# From LO to NLO ...

• NLO for  $F_2: O(\alpha_s)$ 

- NLO for dijets:  $O(\alpha_s^2)$
- LO NLO  $\sim \alpha_s^{\alpha_s^0}$ F<sub>2</sub>  $\left| \sum_{k=1}^{\alpha_{s}^{1}} \right|^{2}$ di-jet I  $\alpha_{s}^{3}$ | | | ç. Ç. 3-jet



not included



• NLO for 3-jets:  $O(\alpha_s^{3})$ 

NOTE: NLO for dijets is **NOT** NNLO for  $F_{2}$ 

# **NLO calculations: principles**

$$\sigma = \int_{m} d\sigma^{\mathrm{B\,or\,n}} + \int_{m} d\sigma^{\mathrm{Virt\,ual}} + \int_{m+1} d\sigma^{\mathrm{R\,eal}}$$

- Virtual (1-loop) corrections:
  - UV
  - IR
  - collinear
- UV corrections handled by renormalization procedure
- soft/collinear singularities do not cancel within  $\,d\sigma^{V}$ 
  - only with appropriate quantities from  $d\sigma^R$
- cancellation is guaranteed by:  $F^{m+1} \to F^m$
- both  $d\sigma^V$  and  $d\sigma^R$  are separately divergent.... need for regularization
  - massive gluon scheme
  - dimensional regularization
- Computation very difficult:
  - use hybrid of analytical and numerical methods
    - Phase space slicing method
  - Subtraction method

# **Phase Space Slicing**

Klasen, Kleinwort, Kramer hep-ph/9712256

- define parameter y<sub>cut</sub> to separate soft + virtual from finite real emissions.
- each contribution shows sensitivity
- but sum of all contributions is independent of y<sub>cut</sub>



# Heavy Quark: NLO virtual corrs

• one-loop virtual corrections:

I. Bojak, M. Stratmann Nucl.Phys.B540:345-381,1999



# Heavy Quarks: NLO real corrections

I. Bojak, M. Stratmann Nucl.Phys.B540:345-381,1999

 IR singularities of virtual x-sections are canceled by soft part of the gluon bremsstrahlung
 No collinear



## Heavy Quarks: NLO quark corrs



H. Jung, QCD & Collider Physics, Lecture 11 WS 05/06

# **D**\* production

- transition from heavy quark to observable hadron by fragmentation function FF
- often Peterson FF is used:

$$D_Q(z) \sim \frac{1}{z} \left[ 1 - \frac{1}{z} - \frac{\epsilon_Q}{1 - z} \right]^{-2}$$

- In massive mode, no collinear divergencies ... nothing to be resummed ... apply fragmentation function directly to parton level calculation.
- Only if  $p_t^2 \gg m^2$  large logs could appear:  $\log\left(\frac{p_t^2}{m^2}\right)$



# Heavy Quarks in NLO: massless

- all partons are treated massless
  - soft singularities cancel between real and virtual contributions
  - initial state collinear singularities are absorbed into PDFs
  - ➔ final state collinear singularities are absorbed into FF (not existing in massive case)
- some additional diagrams compared to massive case
- large logs appear here, which need to be resummed ... scale dependence of FF



# Scaling violations of Frag. Fcts.

partons in hadrons
h
k

hadrons in jets (partons)



Similarity with evolution of parton density functions

$$t\frac{\partial}{\partial t}D_i(x,t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ji}(z,\alpha_s) D_j\left(\frac{x}{z},t\right)$$

- with splitting functions:  $P_{ji}(x, \alpha_s) = P_{ji}^{(0)} + \frac{\alpha_s}{2\pi} P_{ji}^{(1)}$
- lowest order splitting functions are the same as in DIS case
- higher order  $P_{gg}, P_{qg}$  are more singular than in DIS
- resummation of small x enhanced terms have different behavior...

# **Fragmentation Functions**



$$\frac{dD_q^h(z,\mu^2)}{d\log\frac{\mu^2}{\Lambda^2}} = \frac{\alpha_s}{2\pi} \int_0^1 \frac{dy}{y} \left( P_{qq}(y) D_q^h(z/y) + P_{gq}(y) D_g^h(z/y) \right)$$

$$\frac{dD_g^h(z,\mu^2)}{d\log\frac{\mu^2}{\Lambda^2}} = \frac{\alpha_{\rm s}}{2\pi} \int_0^1 \frac{dy}{y} \left( P_{qg}(y) \sum_i \left( D_q^h(z/y) + D_{\bar{q}}^h(z/y) \right) + P_{gg}(y) D_g^h(z/y) \right)$$

# **D**\* production

- transition from heavy quark to observable hadron by fragmentation function FF
- often Peterson FF is used:

$$D_Q(z) \sim \frac{1}{z} \left[ 1 - \frac{1}{z} - \frac{\epsilon_Q}{1 - z} \right]^{-2}$$

- apply FF to parton level calculation for comparison with measurement
- watch out for different FF in massive and massless approach !



# Heavy Quark production: jets

S. Frixione, G. Ridolfi Nucl.Phys.B507:315-333,1997

- ensure proper cancellation of real and virtual corrections
- matching of

 $2 \rightarrow 3$  to  $2 \rightarrow 2$ 

 apply asymmetric pt-cuts for jets



# **Heavy Quark production**

S. Frixione, G. Ridolfi Nucl. Phys. B507:315-333,1997

- ensure proper cancellation of real and virtual corrections
- matching of

 $2 \rightarrow 3 \ to \ 2 \rightarrow 2$ 

- apply asymmetric pt-cuts for jets
- average of transverse momenta of jets
- or....
- stay as inclusive as possible...
  - define infrared safe observables !
- recently significant improvements by resummation of soft gluons



# k,-factorization and collinear NLO

• off-shell matrix elements ( $k_t$  – factorization) includes part of NLO corrections:



- even soft k, region is properly treated (not the case in part.level NLO calc)
- in addition contributions to all orders are included

## **Beauty at HERA**



# Jets in NLO: quark induced

Klasen, Kleinwort, Kramer hep-ph/9712256

- DIS: virtual corrs for QCDC
  - real emissions
  - also diagrams for BGF
- photo production:
  - even more diagrams contribute:
  - resolved photons ...



### **Cancellation of individual contributions**

Klasen, Kleinwort, Kramer hep-ph/9712256

Process	Color Factor	NLO Correction	Singular Parts of Matrix Elements
$\gamma q \rightarrow g q$	$C_F$	Virtual Corr.	$\left[-\frac{2}{\varepsilon^2} - \frac{1}{\varepsilon}(3 - 2l(t))\right] T_{\gamma q \to gq}(s, t, u)$
		Final State	$\left[ +\frac{1}{\varepsilon^2} + \frac{1}{2\varepsilon} (3 - 2l(t)) \right] T_{\gamma q \to gq}(s, t, u)$
		Initial State	$\left[ +\frac{1}{\varepsilon^2} + \frac{1}{2\varepsilon} (3 - 2l(t)) \right] T_{\gamma q \to gq}(s, t, u)$
	$N_C$	Virtual Corr.	$\left[-\frac{1}{\varepsilon^2} - \frac{1}{2\varepsilon}\left(\frac{11}{3} - 2l(s) + 2l(t) - 2l(u)\right)\right] T_{\gamma q \to gq}(s, t, u)$
		Final State	$\left[ \left[ +\frac{1}{\varepsilon^2} + \frac{1}{2\varepsilon} \left( \frac{11}{3} - l(s) + l(t) - l(u) \right) \right] T_{\gamma q \to gq}(s, t, u) \right]$
		Initial State	$\left[\begin{array}{ccc} +\frac{1}{2\varepsilon} \left( & - l(s) + l(t) - l(u) \right) \right] T_{\gamma q \to gq}(s, t, u)$
	$N_f$	Virtual Corr.	$+\frac{1}{3\varepsilon}T_{\gamma q \to gq}(s,t,u)$
		Final State	$-\frac{1}{3\epsilon}T_{\gamma q \to gq}(s,t,u)$

Table 7: Cancellation of IR singularities from virtual, final state, and initial state NLO corrections for the direct partonic subprocesses and different color factors.

# **Reduced Scale Dependence in NLO**

Catani, Seymour hep-ph/9609521

- dependence of the specific choice of the scale for renormalization and factorization shows sensitivity to higher order contributions, which are not included.
- scale is unphysical parameter
- physical observables must be independent of scale
- in NLO scale dependence significantly reduced compared to lowest order



## Di-jet rates at LO?



- (2+remnant) jets in DIS for  $Q^2 > 5 \text{ GeV}^2$ ,  $p_t^{\text{jets}} > 5 \text{ GeV}$
- $\mathcal{O}(\alpha_s)$  processes not enough
  - ➔ need higher order contributions

# **Diet production at NLO**



- lowest order NOT enough to describe dijet rates !
- NLO for dijets needed
  - BUT require asymmetric pt to ensure cancellation of real and virtual corrs



### Limitations in fixed order NLO calculations



# From LO to NLO ...

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not included

• NLO for 3-jets:  $O(\alpha_s^{3})$ 

NOTE: NLO for dijets is **NOT** NNLO for  $F_2$ 

# The need for unintegrated PDFs



## **Need for uPDFs**

J. Collins, H. Jung, hep-ph/0508280

#### Define:

•  $p_{Tq\bar{q}}$ 

• 
$$x_{\gamma} = \frac{\sum_{i=q,\bar{q}} (E_i - p_{z\,i})}{2yE_e} = \frac{p_{q\bar{q}}}{q^-}$$

• parton kinematics





# **Need for uPDFs**

#### Define:

•  $p_{Tq\bar{q}}$ 

• 
$$x_{\gamma} = \frac{\sum_{i=q,\bar{q}} (E_i - p_{z\,i})}{2yE_e} = \frac{p_{q\bar{q}}}{q^-}$$

- parton kinematics
- uPDFs





J. Collins, H. Jung, hep-ph/0508280

## **Need for uPDFs**

#### Define:

•  $p_{Tq\bar{q}}$ 

• 
$$x_{\gamma} = \frac{\sum_{i=q,\bar{q}} (E_i - p_{z\,i})}{2yE_e} = \frac{p_{q\bar{q}}^-}{q^-}$$

- parton kinematics
- uPDFs
- full kinematics





J. Collins, H. Jung, hep-ph/0508280

## Need for double uPDFs





## Need for double uPDFs





## **Explicit parton evolution: parton showers**



- use LO matrix elements
  - for light quarks, cutoffs are needed
- apply initial and final state parton showers (PS)
  - matching of cutoff in ME with parton showers
- apply synchronization
- obtain cross sections fully differential in any observable
- BUT:
  - only in LO (attempts to include NLO: Collins et al, MC@NLO, etc.)

## **DGLAP** evolution again and again...

- differential form:  $t\frac{\partial}{\partial t}f(x,t) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z},t\right)$
- differential form using  $f/\Delta_s$  with

$$\Delta_{s}(t) = \exp\left(-\int_{x}^{z_{max}} dz \int_{t_{0}}^{t} \frac{\alpha_{s}}{2\pi} \frac{dt'}{t'} \tilde{P}_{2}(z)\right) \quad \text{with} \quad \tilde{P}_{2} \sim \frac{1}{1-z}$$
$$t \frac{\partial}{\partial t} \frac{f(x,t)}{\Delta_{s}(t)} = \int \frac{dz}{z} \frac{\alpha_{s}}{2\pi} \frac{\tilde{P}(z)}{\Delta_{s}(t)} f\left(\frac{x}{z},t\right)$$

integral form

٩

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z},t'\right)$$
  
no – branching probability from  $t_0$  to  $t$ 

## **DGLAP** for parton showers

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

solve integral equation via explicit iteration:

$$f_{0}(x,t) = f(x,t_{0})\Delta(t)$$

$$f_{0}(x,t) = f(x,t_{0})\Delta(t) + \int_{t_{0}}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z)f(x/z,t_{0})\Delta(t')$$

$$x \ t$$

$$z = x/x_{0} \ t'$$

$$F(z)$$

# Parton showers for the initial state

#### spacelike (Q<0) parton shower evolution

- starting from hadron (fwd evolution) or from hard scattering (bwd evolution)
- select  $q_1$  from Sudakov form factor
- select  $z_1$  from splitting function

- select q<sub>2</sub> from Sudakov form factor
- select  $z_2$  from splitting function
- stop evolution if  $q_2 > Q_{hard}$



хo





### Parton showers to solve DGLAP evolution

- for fixed x and  $Q^2$  chains with different branchings contribute
- iterative procedure, spacelike parton showering



# Parton showers for the final state

timelike (Q>0) parton shower evolution

- starting with hard scattering
- select q<sub>1</sub> from Sudakov form factor
- select  $z_1$  from splitting function

• select  $q_2$  from Sudakov form factor

- select  $z_2$  from splitting function
- stop evolution if  $q_2 < q_0$

