QCD and Collider Physics:
Heavy Quarks, Fragmentation-Hadronization

- Resume from last lecture
- Heavy Quarks: fragmentation functions, massive/massless approach
- Dijet production in DIS (in LO and NLO)
- Approaches to even higher orders
  - Parton showers
  - unintegrated pdfs
- Fragmentation/hadronization

http://www-h1.desy.de/~jung/qcd.collider.physics_2005
From LO to NLO ...

- NLO for $F_2$: $O(\alpha_s)$
- NLO for dijets: $O(\alpha_s^2)$
- NLO for 3-jets: $O(\alpha_s^3)$

**NOTE:** NLO for dijets is **NOT** NNLO for $F_2$
Virtual (1-loop) corrections:
- UV
- IR
- collinear

UV corrections handled by renormalization procedure
soft/collinear singularities do not cancel within $d\sigma^V$
only with appropriate quantities from $d\sigma^R$
cancellation is guaranteed by: $F^{m+1} \to F^m$
both $d\sigma^V$ and $d\sigma^R$ are separately divergent.... need for regularization
massive gluon scheme
dimensional regularization

Computation very difficult:
use hybrid of analytical and numerical methods
- Phase space slicing method
- Subtraction method
define parameter $y_{\text{cut}}$ to separate soft + virtual from finite real emissions.

each contribution shows sensitivity

but sum of all contributions is independent of $y_{\text{cut}}$
Heavy Quark: NLO virtual corrs

- one-loop virtual corrections:

\[
|M|^2_{VB} = 2 \text{Re} \left( \tilde{M}_V \tilde{M}_B^* \right) = g_s^4 e^2 e_Q^2 \left[ 2 C_F \tilde{V}_{QED} + C_A \tilde{V}_{\text{non-abelian}} \right],
\]

Heavy Quarks: NLO real corrections

- IR singularities of virtual x-sections are canceled by soft part of the gluon bremsstrahlung.

\[
|\tilde{M}_R|^2 =\tilde{M}_R^* \tilde{M}_R = g_s^4 e^2 e_Q^2 \left[ 2C_F \tilde{R}_{QED} + C_A \tilde{R}_{\text{non-abelian}} \right]
\]
Heavy Quarks: NLO quark corrs

\[|\tilde{M}_q|^2 = \tilde{M}_q\tilde{M}_q^* = g_s^4 e^2 \frac{C_F}{2} \left[ e_Q^2 \tilde{A}_1 + e_q^2 \tilde{A}_2 + e_q e_Q \tilde{A}_3 \right]\]

\[\frac{d^2 \tilde{\sigma}_{q\gamma}^{(1)}}{dt_1 du_1} (\mu_f^2) = \frac{d^2 \tilde{\sigma}_{q\gamma}^{(1)}}{dt_1 du_1} (\mu^2) - \frac{\alpha_s}{2\pi} \int_0^1 dx_1 \left[ \tilde{P}_{gq}(x_1) \frac{2}{\varepsilon} + \tilde{F}_{gq}(x_1, \mu_f^2, \mu^2) \right] x_1 \left[ \frac{d^2 \tilde{\sigma}_{g\gamma}^{(0)}}{dt_1 du_1} \right] \left( s \to x_1 s \right) \left( t_1 \to x_1 t_1 \right) - \frac{\alpha}{2\pi} \int_0^1 dx_2 \left[ \tilde{P}_{q\gamma}(x_2) \frac{2}{\varepsilon} + \tilde{F}_{q\gamma}(x_2, \mu_f^2, \mu^2) \right] x_2 \left[ \frac{d^2 \tilde{\sigma}_{qq}^{(0)}}{dt_1 du_1} \right] \left( s \to x_2 s \right) \left( u_1 \to x_2 u_1 \right)\]
D* production

- transition from heavy quark to observable hadron by fragmentation function FF
- often Peterson FF is used:

\[ D_Q(z) \sim \frac{1}{z} \left[ 1 - \frac{1}{z} - \frac{\epsilon_Q}{1 - z} \right]^{-2} \]

- In massive mode, no collinear divergencies ... nothing to be resummed ... apply fragmentation function directly to parton level calculation.
- Only if \( p_t^2 \gg m^2 \) large logs could appear:

\[ \log \left( \frac{p_t^2}{m^2} \right) \]
all partons are treated massless
→ soft singularities cancel between real and virtual contributions
→ initial state collinear singularities are absorbed into PDFs
→ final state collinear singularities are absorbed into FF (not existing in massive case)

some additional diagrams compared to massive case

large logs appear here, which need to be resummed ... scale dependence of FF
Scaling violations of Frag. Fcts.

- Similarity with evolution of parton density functions

\[ t \frac{\partial}{\partial t} D_i(x, t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ji}(z, \alpha_s) D_j \left( \frac{x}{z}, t \right) \]

- with splitting functions:

\[ P_{ji}(x, \alpha_s) = P_{ji}^{(0)} + \frac{\alpha_s}{2\pi} P_{ji}^{(1)} \]

- lowest order splitting functions are the same as in DIS case
- higher order \( P_{gg}, P_{qg} \) are more singular than in DIS

⇒ resummation of small x enhanced terms have different behavior...
Fragmentation Functions

\[ \frac{dD^h_q(z, \mu^2)}{d \log \frac{\mu^2}{\Lambda^2}} = \frac{\alpha_s}{2\pi} \int_0^1 \frac{dy}{y} \left( P_{qq}(y) D^h_q(z/y) + P_{gq}(y) D^h_g(z/y) \right) \]

\[ \frac{dD^h_g(z, \mu^2)}{d \log \frac{\mu^2}{\Lambda^2}} = \frac{\alpha_s}{2\pi} \int_0^1 \frac{dy}{y} \left( P_{qg}(y) \sum_i (D^h_q(z/y) + D^h_{q}(z/y)) + P_{gg}(y) D^h_g(z/y) \right) \]
D* production

- transition from heavy quark to observable hadron by fragmentation function FF
- often Peterson FF is used:

\[ D_Q(z) \sim \frac{1}{z} \left[ 1 - \frac{1}{z} - \frac{\epsilon_Q}{1 - z} \right]^{-2} \]

- apply FF to parton level calculation for comparison with measurement
- watch out for different FF in massive and massless approach!
Heavy Quark production: jets

- ensure proper cancellation of real and virtual corrections
- matching of
  \[ 2 \rightarrow 3 \text{ to } 2 \rightarrow 2 \]
- apply asymmetric pt-cuts for jets

Heavy Quark production

- ensure proper cancellation of real and virtual corrections
- matching of
  \[ 2 \rightarrow 3 \text{ to } 2 \rightarrow 2 \]
- apply asymmetric pt-cuts for jets
- average of transverse momenta of jets
- or....
- stay as inclusive as possible...
- define infrared safe observables!
- recently significant improvements by resummation of soft gluons

**k_t-factorization and collinear NLO**

- off-shell matrix elements ($k_t$ – factorization) includes part of NLO corrections:

  - even soft $k_t$ region is properly treated (not the case in part.level NLO calc)
  - in addition contributions to all orders are included
“Perfect” agreement of NLO(FMNR) calc with CASCADE on quark and hadron level for $x<0.01$
Jets in NLO: quark induced

- DIS: virtual corrs for QCDC
- real emissions
- also diagrams for BGF
- photo production:
  - even more diagrams contribute:
  - resolved photons ...

Klasen, Kleinwort, Kramer hep-ph/9712256
## Cancellation of individual contributions

<table>
<thead>
<tr>
<th>Process</th>
<th>Color Factor</th>
<th>NLO Correction</th>
<th>Singular Parts of Matrix Elements</th>
</tr>
</thead>
</table>
| $\gamma q \rightarrow gq$ | $C_F$ | Virtual Corr.  
Final State  
Initial State | $\begin{bmatrix} -\frac{2}{\varepsilon^2} - \frac{1}{\varepsilon} (3 - 2l(t)) \\ + \frac{1}{\varepsilon^2} + \frac{1}{2\varepsilon} (3 - 2l(t)) \\ + \frac{1}{\varepsilon^2} + \frac{1}{2\varepsilon} (3 - 2l(t)) \end{bmatrix} T_{\gamma q \rightarrow gq}(s, t, u)$ |
| $N_C$    | Virtual Corr.  
Final State  
Initial State | $\begin{bmatrix} -\frac{1}{\varepsilon^2} - \frac{1}{2\varepsilon} \left( \frac{11}{3} - 2l(s) + 2l(t) - 2l(u) \right) \\ + \frac{1}{\varepsilon^2} + \frac{1}{2\varepsilon} \left( \frac{11}{3} - l(s) + l(t) - l(u) \right) \\ + \frac{1}{2\varepsilon} \left( - l(s) + l(t) - l(u) \right) \end{bmatrix} T_{\gamma q \rightarrow gq}(s, t, u)$ |
| $N_f$    | Virtual Corr.  
Final State  | $\begin{bmatrix} +\frac{1}{3\varepsilon} T_{\gamma q \rightarrow gq}(s, t, u) \\ -\frac{1}{3\varepsilon} T_{\gamma q \rightarrow gq}(s, t, u) \end{bmatrix}$ |

Table 7: Cancellation of IR singularities from virtual, final state, and initial state NLO corrections for the direct partonic subprocesses and different color factors.
Reduced Scale Dependence in NLO

- Dependence of the specific choice of the scale for renormalization and factorization shows sensitivity to higher order contributions, which are not included.
- Scale is unphysical parameter.
- Physical observables must be independent of scale.
- In NLO scale dependence significantly reduced compared to lowest order.

Catani, Seymour hep-ph/9609521
Di-jet rates at LO?

- (2+remnant) jets in DIS for $Q^2 > 5 \text{ GeV}^2$, $p_t^{\text{jets}} > 5 \text{ GeV}$
- $O(\alpha_s)$ processes not enough
  ➔ need higher order contributions
Diet production at NLO

- lowest order NOT enough to describe dijet rates!
- NLO for dijets needed
- BUT require asymmetric pt to ensure cancellation of real and virtual corrs

Limitations in fixed order NLO calculations

- NEED asymmetric $p_t$ cuts: $p_{t1} \neq p_{t2}$
  for proper cancellation of real and virtual emissions....
  ➔ loose most of the data... !!!

- improvements by resummations:
  A. Banfi et al hep-ph/0508096
  - soft gluon radiation.... like parton showers... resummed to all orders
  check dijets:
  \[ \Delta p_t = p_{t1} - p_{t2} \]

- resummed result at agrees with MC using parton showers...
From LO to NLO ...

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NOTE: NLO for dijets is NOT NNLO for $F_2$
The need for unintegrated PDFs

- using integrated pdfs ignores proper kinematics
- large NLO corr comes from wrong kinematics in LO

**LO**

\[ k_t = 0 \]

**NLO**

\[ k_t = 0 \]

\[ k_t \neq 0 \]

\[ \text{uPDFs} \]

- collinear factorization is wrong if details of final state are investigated
- Need for fully unintegrated PDFs

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Collins, Zu, JHEP 03, 059 (2005)
Need for uPDFs

Define:

- $p_T^{q\bar{q}}$
- $x_\gamma = \sum_{i=q,\bar{q}} (E_i - p_{z_i}) / 2yE_e = \frac{p_{q\bar{q}}}{q^-}$
- parton kinematics
Need for uPDFs

Define:

- $pTq\bar{q}$
- $x_\gamma = \frac{\sum_{i=q,\bar{q}} (E_i - p_z i)}{2yE_e} = \frac{p_{q\bar{q}}}{q^-}$
- parton kinematics
- uPDFs
Define:

- $p_T \bar{q} q$

$$x_\gamma = \frac{\sum_{i=q, \bar{q}} (E_i - p_{z_i})}{2 y E_e} = \frac{p_{q\bar{q}}}{q^-}$$

- parton kinematics

- uPDFs

- full kinematics
Need for double uPDFs

\[ k^2 = -\frac{k_t^2}{1 - x} \]
Need for double uPDFs

\[ k^2 = - \frac{k_t^2}{1 - x} \]

\[ k^2 = - \frac{k_t^2}{1 - x} \left( 1 + x \frac{m_{\text{rem}}^2}{k_t^2} \right) \]
Explicit parton evolution: parton showers

- use LO matrix elements
- for light quarks, cutoffs are needed
- apply initial and final state parton showers (PS)
- matching of cutoff in ME with parton showers
- apply synchronization
- obtain cross sections fully differential in any observable

**BUT:**
- only in LO (attempts to include NLO: Collins et al, MC@NLO, etc)
DGLAP evolution again and again...

- Differential form:
  \[
  t \frac{\partial}{\partial t} f(x, t) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z}, t\right)
  \]

- Differential form using \( f / \Delta_s \) with
  \[
  \Delta_s(t) = \exp\left(- \int_x^{z_{\text{max}}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi} \frac{dt'}{t'} \tilde{P}_2(z)\right)
  \]
  with \( \tilde{P}_2 \sim \frac{1}{1 - z} \)

- Integral form
  \[
  f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)
  \]

  no – branching probability from \( t_0 \) to \( t \)
DGLAP for parton showers

\[ f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f \left( \frac{x}{z}, t' \right) \]

- solve integral equation via explicit iteration:

\[
\begin{align*}
f_0(x, t) & = f(x, t_0) \Delta(t) \\
from \text{t' to t} & \quad \text{w/o branching} \\
branching \text{at } t' & \\
f_1(x, t) & = f(x, t_0) \Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f \left( x / z, t_0 \right) \Delta(t') \int_{t_0}^{t} dt \left( f_0 - f_1 \right) \end{align*}
\]

\[ z = \frac{x}{x_0} \quad t' \quad \tilde{P}(z) \]
spacelike $(Q<0)$ parton shower evolution

- starting from hadron (fwd evolution)
  
  or from hard scattering (bwd evolution)

- select $q_1$ from Sudakov form factor

- select $z_1$ from splitting function

- select $q_2$ from Sudakov form factor

- select $z_2$ from splitting function

- stop evolution if $q_2 > Q_{\text{hard}}$
Parton showers to solve DGLAP evolution

- for fixed $x$ and $Q^2$ chains with different branchings contribute
- iterative procedure, spacelike parton showering

\[ f(x, t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0) \Delta_s(t) \]
Parton showers for the final state

timelike $(Q>0)$ parton shower evolution
- starting with hard scattering

- select $q_1$ from Sudakov form factor

- select $z_1$ from splitting function

- select $q_2$ from Sudakov form factor

- select $z_2$ from splitting function
- stop evolution if $q_2 < q_0$