Homework Exercises for QCD and Collider Physics

2005/2006

Exercises for Lecture 11 (1. Feb 2006)

Kinematics:

- Calculate limits on z in case of a heavy quark: $Q \to Q' + g$, with $Q = (p^+, p^-, 0)$, $Q' = ((1-z)p^+, p^{-'}, k_t)$ and $g = (zp^+, k^-, -k_t)$. Prove that $z < 1 - \frac{m^2}{Q^2}$ with $Q'^2 = m^2$ and Q^2 being the invariant mass of the 4-vector Q' and Q.
- Calculate $k^2 = -\frac{k_t^2 + xm^2}{1-x}$ with m being the remnant mass and $q = (0, q^-, 0)$, $k = (xp^+, k^-, k_t)$ and $p = (p^+, 0, 0)$ and $p = k + p_{rem}$, with $p_{rem}^2 = m_{rem}^2$.

Crossing symmetries:

- use crossing symmetry arguments (Halzen/Martin p238) to show that the matrix-element for $\gamma^* \to q\bar{q}g$ can be obtained from the QCDC matrix element, and that it is:

$$|M|^2 \sim \left(\frac{t}{s} + \frac{s}{t} + \frac{2Q^2u}{st}\right)$$

Lund string model:

- assume constant force: $\frac{dp}{dt} = -\kappa$, of a relativistic particle with 4-vector $p = (E, p)$. Show that the velocity is $dx/dt = dE/dp$ and construct $\frac{dE}{dx} = -\kappa$. Use the solution to the time dependence of the equations for p and E to constrcut:

$$m^2 = \kappa^2 \left[(x_0 - x)^2 - (t_0 - t)^2\right]$$

  using $m^2 = E^2 - p^2$, and $p_0 = \kappa t_0$, $E_0 = \kappa x_0$.