High–energy QCD near the unitarity limit:

k_T–factorization, color glass, and all that

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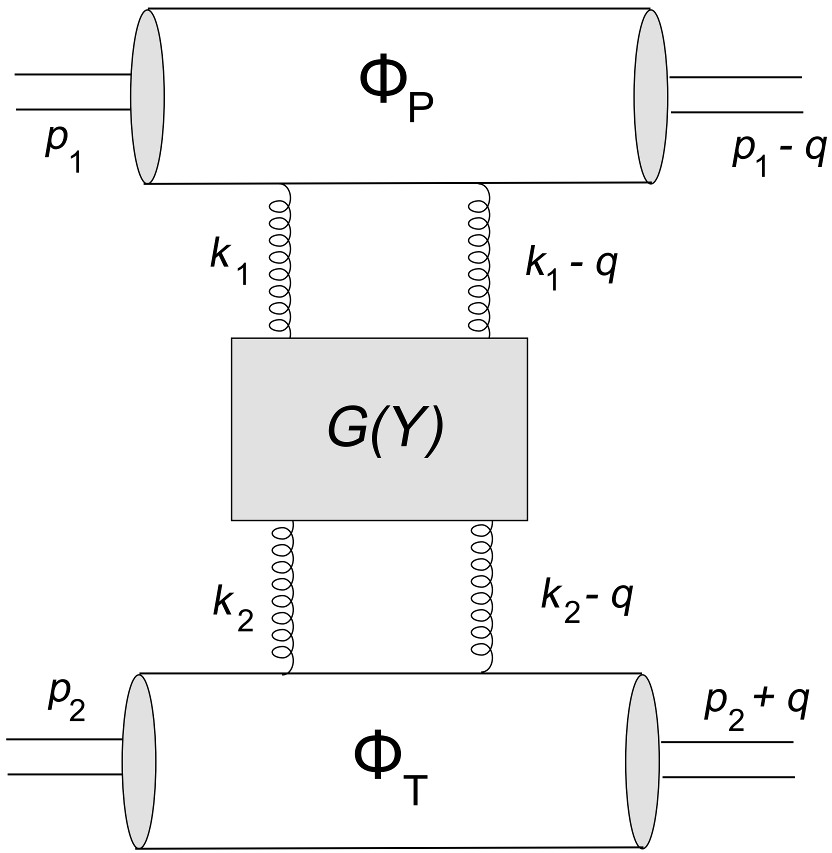
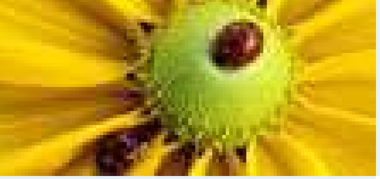
BFKL evolution & k_T–factorization

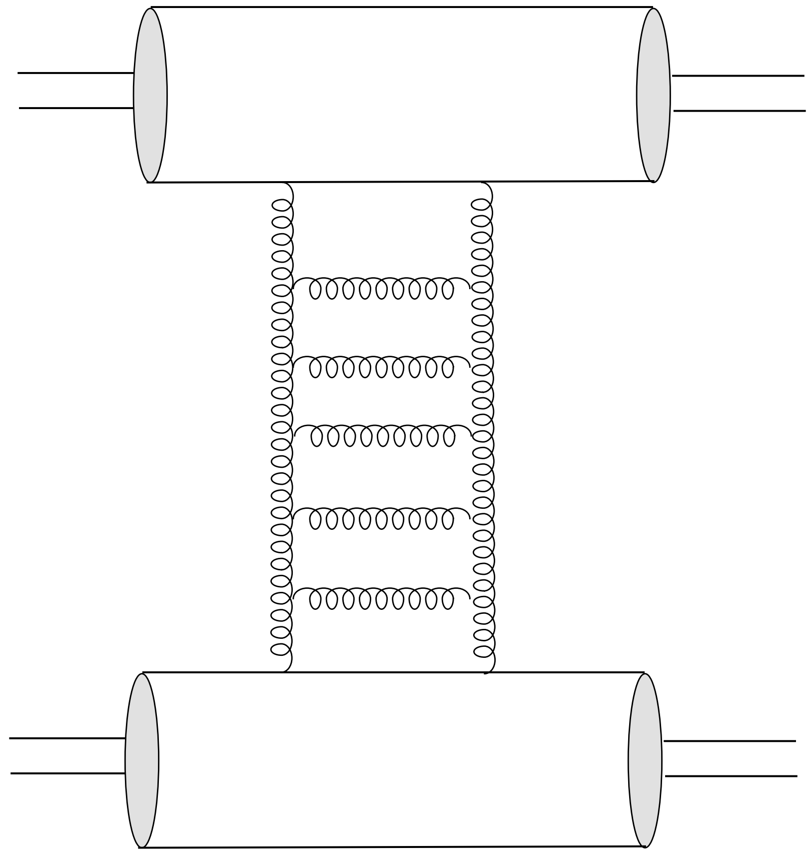


$$T(s,t) = \int \frac{d^2\boldsymbol{k}_1}{(2\pi)^2}\frac{d^2\boldsymbol{k}_2}{(2\pi)^2}\,\frac{\Phi_P(\boldsymbol{k}_1,\boldsymbol{q})\,\Phi_T(\boldsymbol{k}_2,\boldsymbol{q})}{\boldsymbol{k}_2^2(\boldsymbol{k}_1-\boldsymbol{q})^2}\,\mathcal{G}_{\mathrm{BFKL}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q};Y)$$

- $Y \sim \ln s$: rapidity, $t \sim -q^2$: momentum transfer



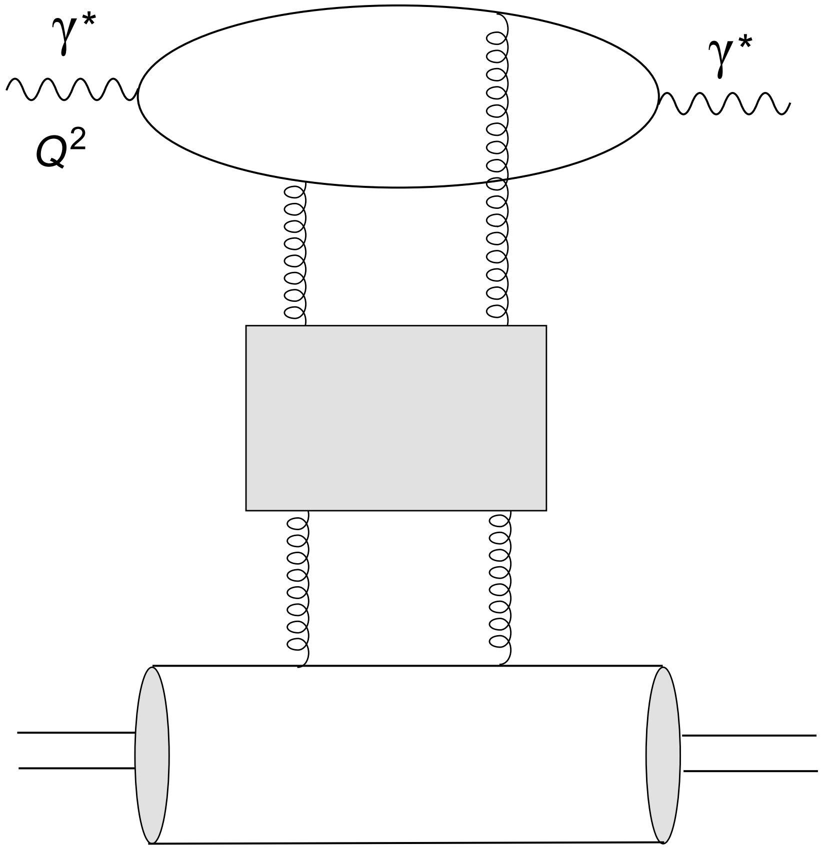
BFKL evolution & k_T–factorization



$$\mathcal{G}_{\rm BFKL}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{q}=0; Y) \simeq \frac{{\rm e}^{\omega_0 Y}}{(k_1^2\, k_2^2)^{1/2}} \frac{1}{\sqrt{\pi D_0 \tau}} \exp\left\{ -\frac{\ln^2(k_1^2/k_2^2)}{4 D_0 Y} \right\}$$

- $\omega_0 = 4 \ln 2 \bar{\alpha}_s$: ‘intercept’, $D_0 = 14\zeta(3)\bar{\alpha}_s$: ‘diffusion’

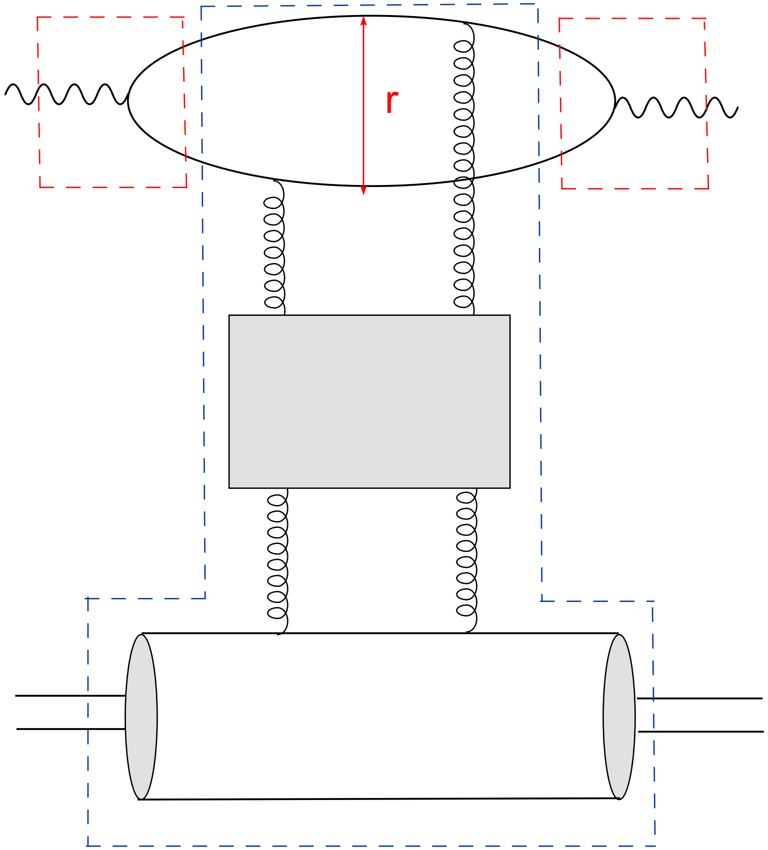
Deep inelastic scattering at small x



$$\sigma_{\gamma^* h}(Y, Q^2) = \int \frac{d^2\boldsymbol{k}_1}{(2\pi)^2} \frac{d^2\boldsymbol{k}_2}{(2\pi)^2} \frac{\Phi_{\gamma^*}(\boldsymbol{k}_1)\,\Phi_h(\boldsymbol{k}_2)}{\boldsymbol{k}_1^2\,\boldsymbol{k}_2^2}\,\mathcal{G}_{\rm BFKL}(\boldsymbol{k}_1, \boldsymbol{k}_2; Y)$$

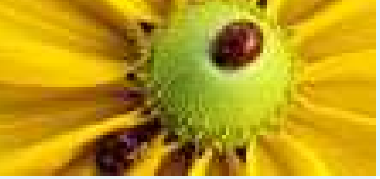
- $Y = \ln(1/x) \sim \ln(s/Q^2)$

Deep inelastic scattering at small x

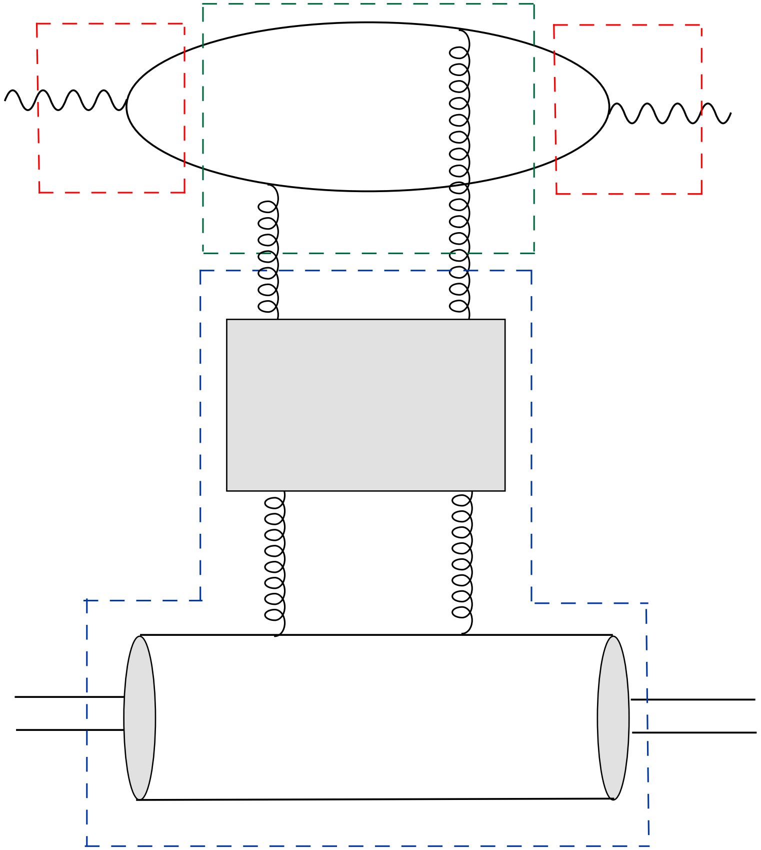


$$\sigma_{\gamma^* h}(Q^2, Y) = \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{r} \sum_{p=T,L} |\Psi_p(z, r)|^2 \, \sigma_{\text{dipole}}(r, Y)$$

- A $q\bar{q}$ color dipole with transverse size r



Deep inelastic scattering at small x

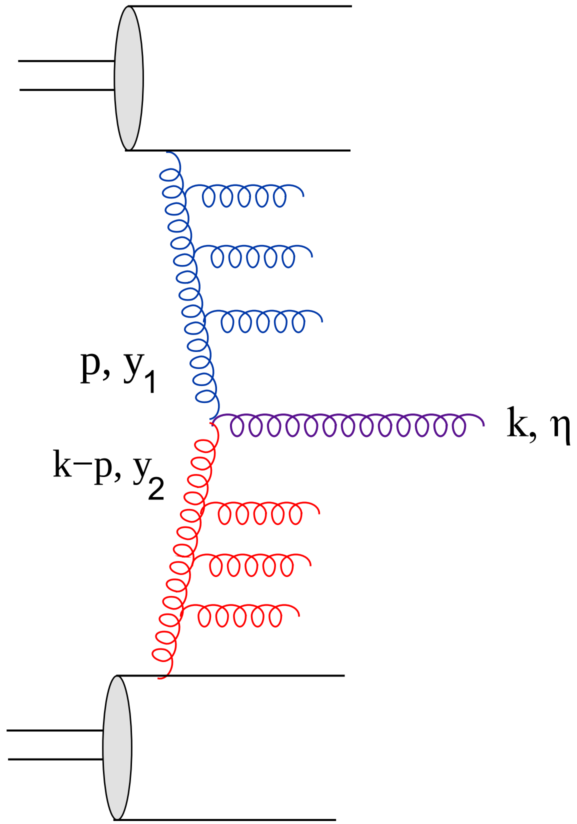


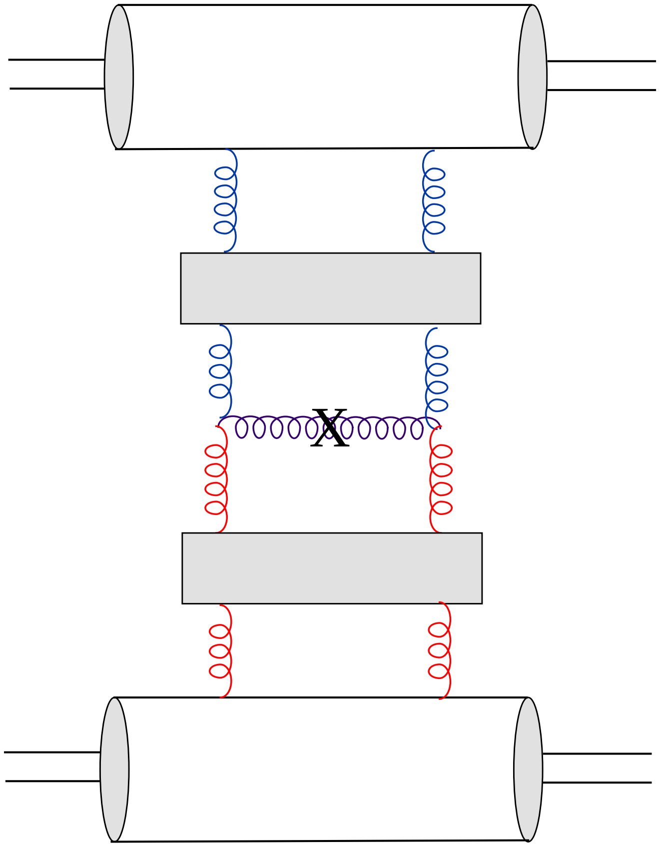
$$\sigma_{\rm dipole}(\boldsymbol{r}, Y) \approx \frac{4\pi}{N_c}\,\alpha_s \int \frac{{\rm d}^2\boldsymbol{k}}{\boldsymbol{k}^2}\,\varphi(\boldsymbol{k}, Y)\left(1 - {\rm e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right)$$

- $\varphi(\boldsymbol{k}, Y)$: the ‘unintegrated gluon distribution’

  ($\varphi \propto 1/k^2$ at very large k : bremsstrahlung)

Gluon production in hadron–hadron collisions

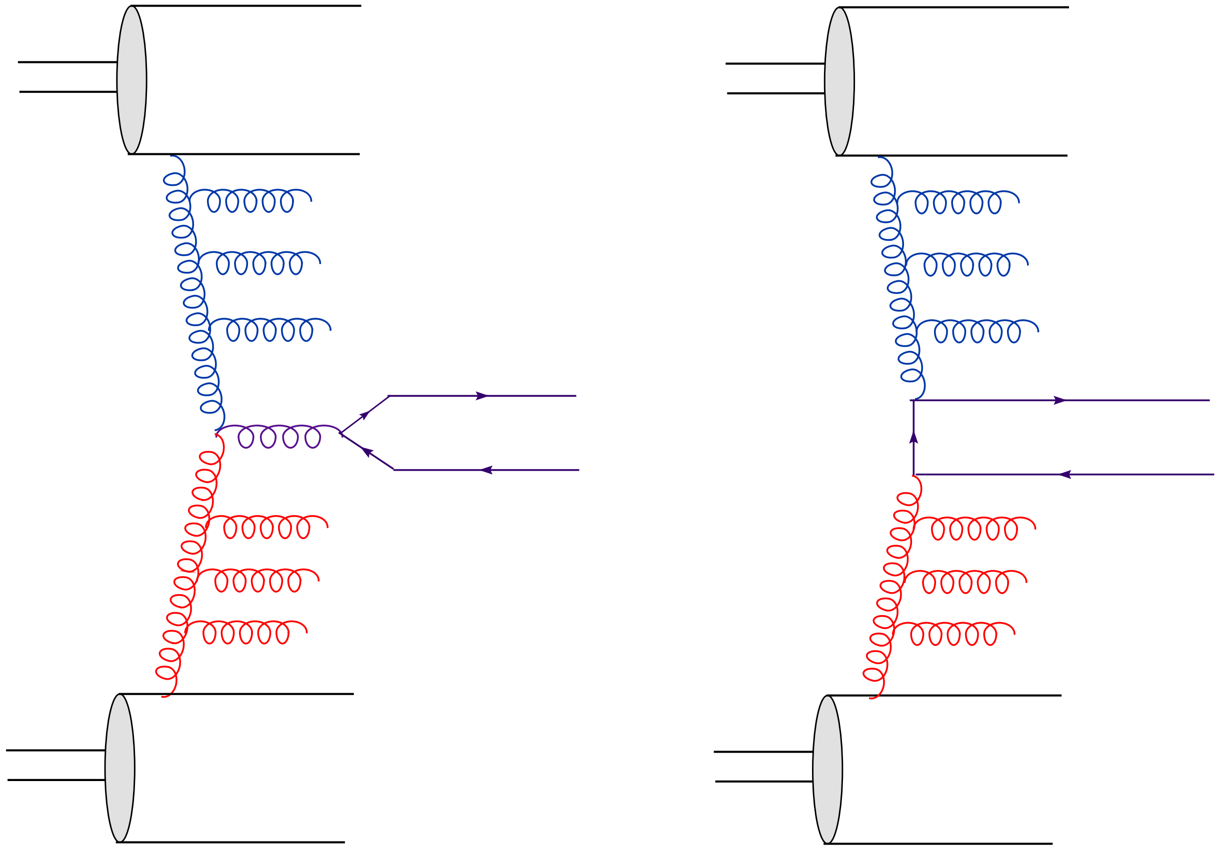




$$\frac{d\sigma}{d\eta d^2\boldsymbol{k}} = \frac{\bar{\alpha}_s}{k_\perp^2} \int \frac{d^2\boldsymbol{p}}{(2\pi)^2} \varphi_1(\boldsymbol{p}, y_1)\varphi_2(\boldsymbol{k} - \boldsymbol{p}, y_2)$$

- $y_i = \ln(1/x_i)$ where $x_{1,2} = (k_\perp/\sqrt{s})e^{\pm\eta}$

$q\bar{q}$ production in hadron–hadron collisions



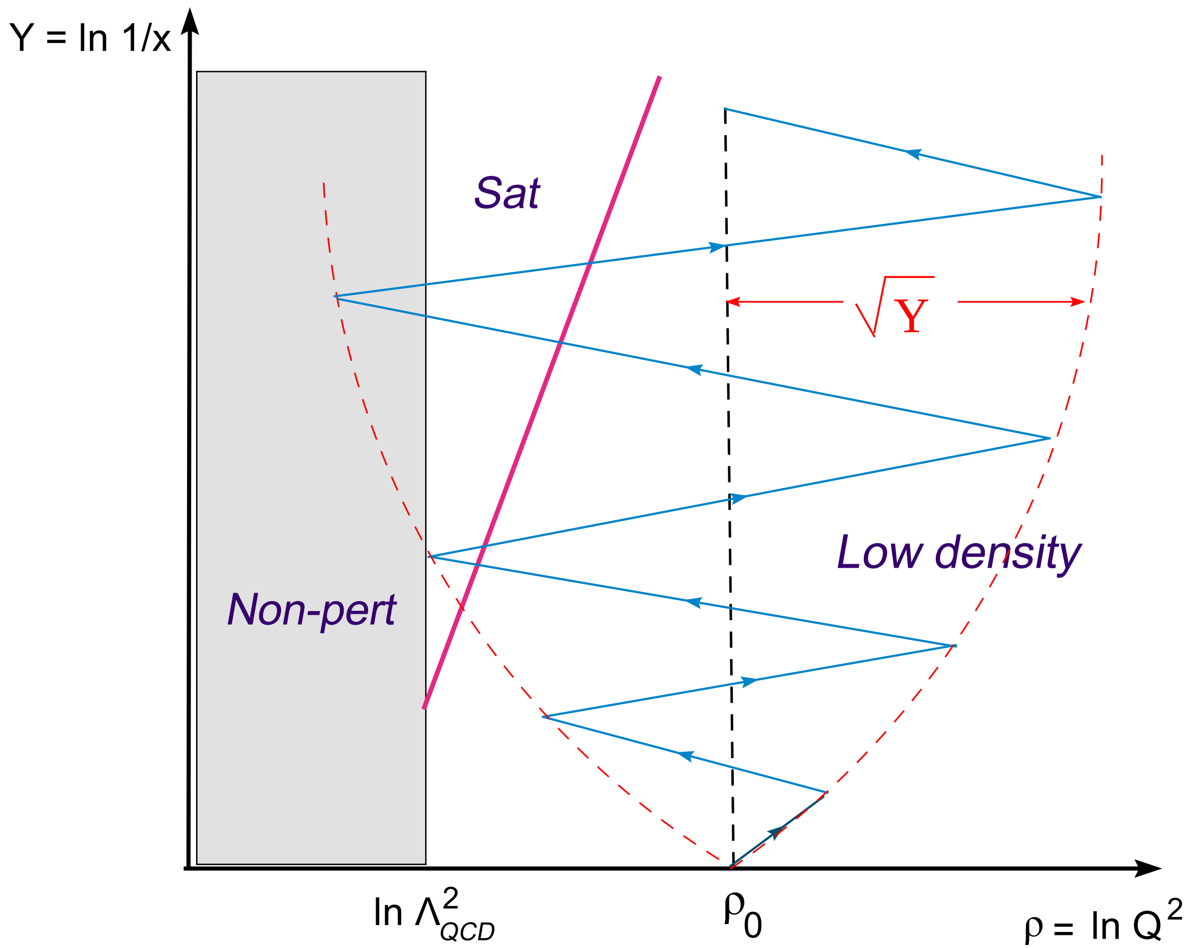
- The unintegrated gluon distribution is 'universal'

$$\varphi(\boldsymbol{k}, Y) \sim \int \mathrm{d}^2\boldsymbol{r}\, \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}} \left\langle F_a^{+i}(\boldsymbol{r}) F_a^{+i}(0) \right\rangle_Y$$

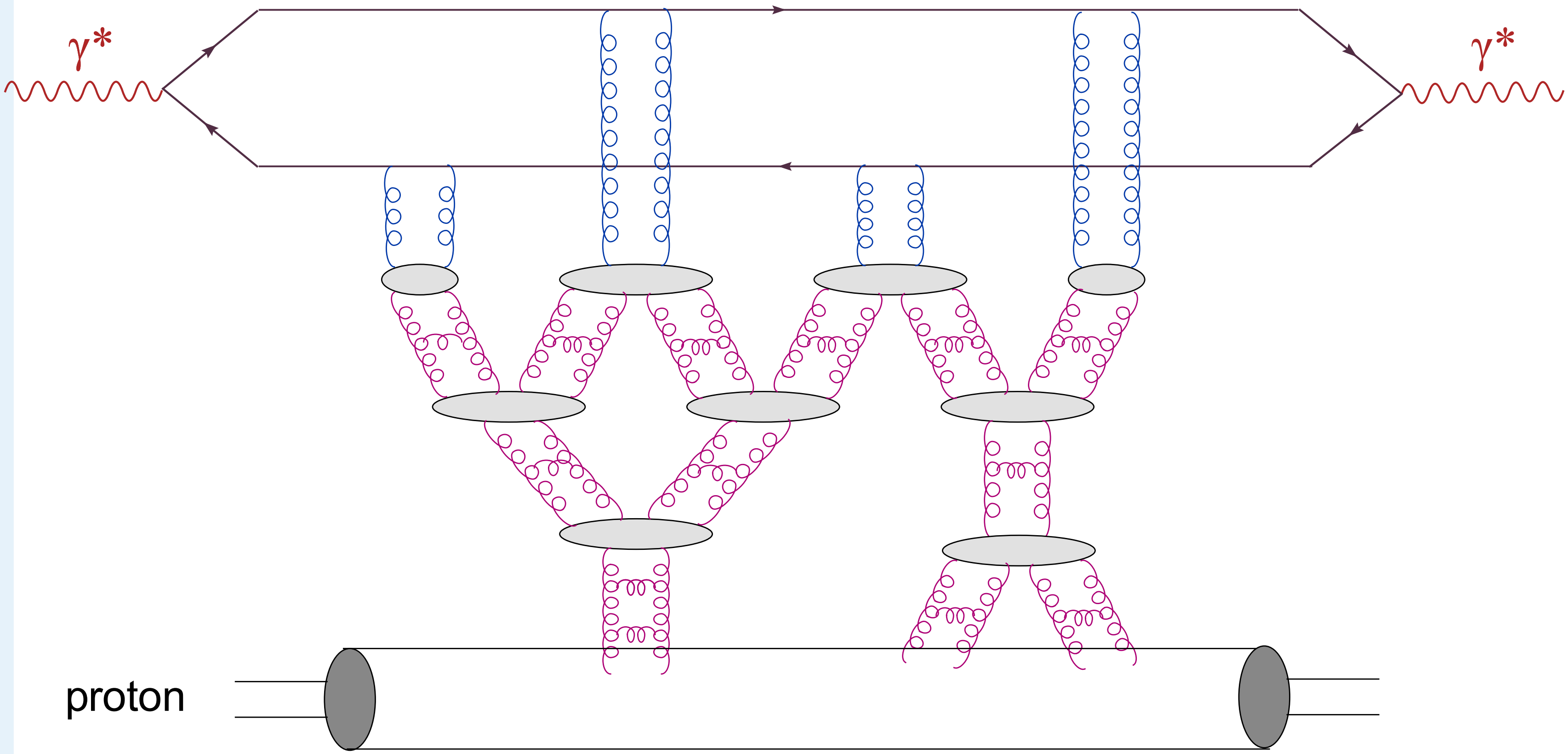
- A single scattering measures just the gluon density

Conceptual problems at high energy

- Unitarity violation : $T(Y) \propto e^{\omega_0 Y}$
- Infrared diffusion :

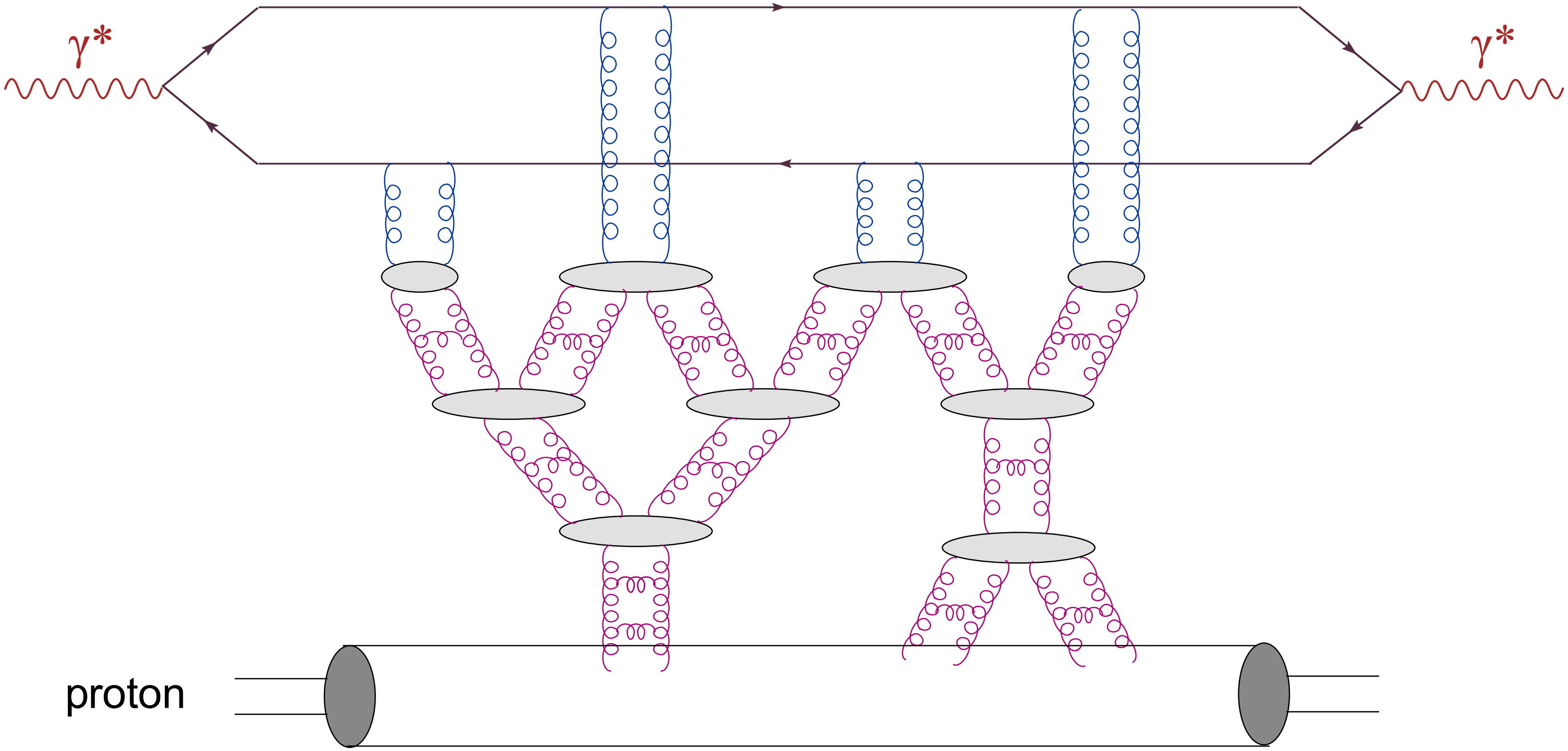


DIS at **very** small values of x



- Gluon recombination/splitting in the hadron wavefunction
- Leading–order effects at sufficiently high energy !
  - Recombination requires $\varphi(\boldsymbol{k}, Y) \sim 1/\alpha_s$: $\alpha_s\varphi \sim (\alpha_s\varphi)^2$
  - Splitting important when $\varphi(\boldsymbol{k}, Y) \sim \alpha_s$: $\alpha_s^2\varphi \sim \alpha_s\varphi^2$

Saturation & Unitarization for DIS



- Saturation momentum: $\varphi(\boldsymbol{k}, Y) \sim 1/\alpha_s$ when $k_\perp \sim Q_s(Y)$

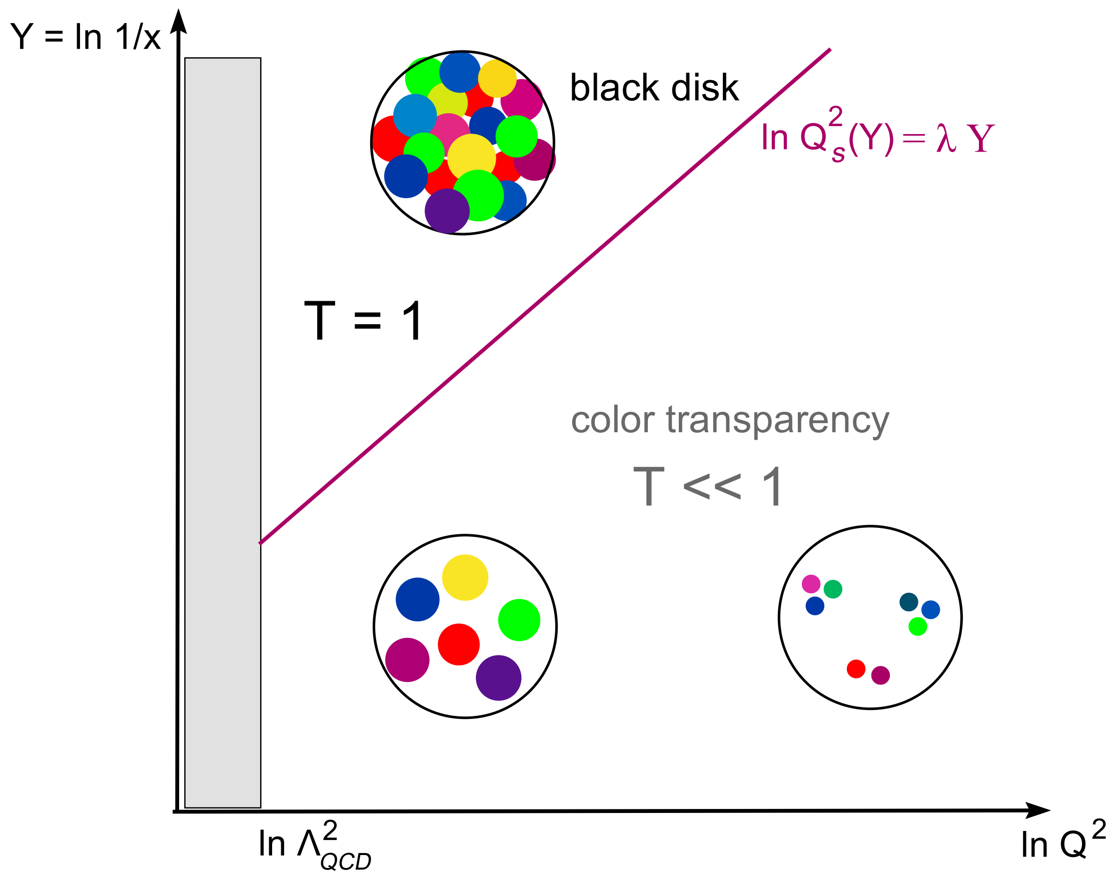
LO BFKL : $\varphi(\boldsymbol{k}, Y) \sim \left(\frac{\Lambda^2}{\boldsymbol{k}^2}\right)^{1/2} \mathrm{e}^{\omega_0 Y} \implies Q_s^2(Y) \sim \alpha_s^2 \Lambda^2 \, \mathrm{e}^{2\omega_0 Y}$

- NLO BFKL : $2\omega_0 \sim \mathcal{O}(1) \implies \lambda \approx 0.3$

Saturation & Unitarization for DIS

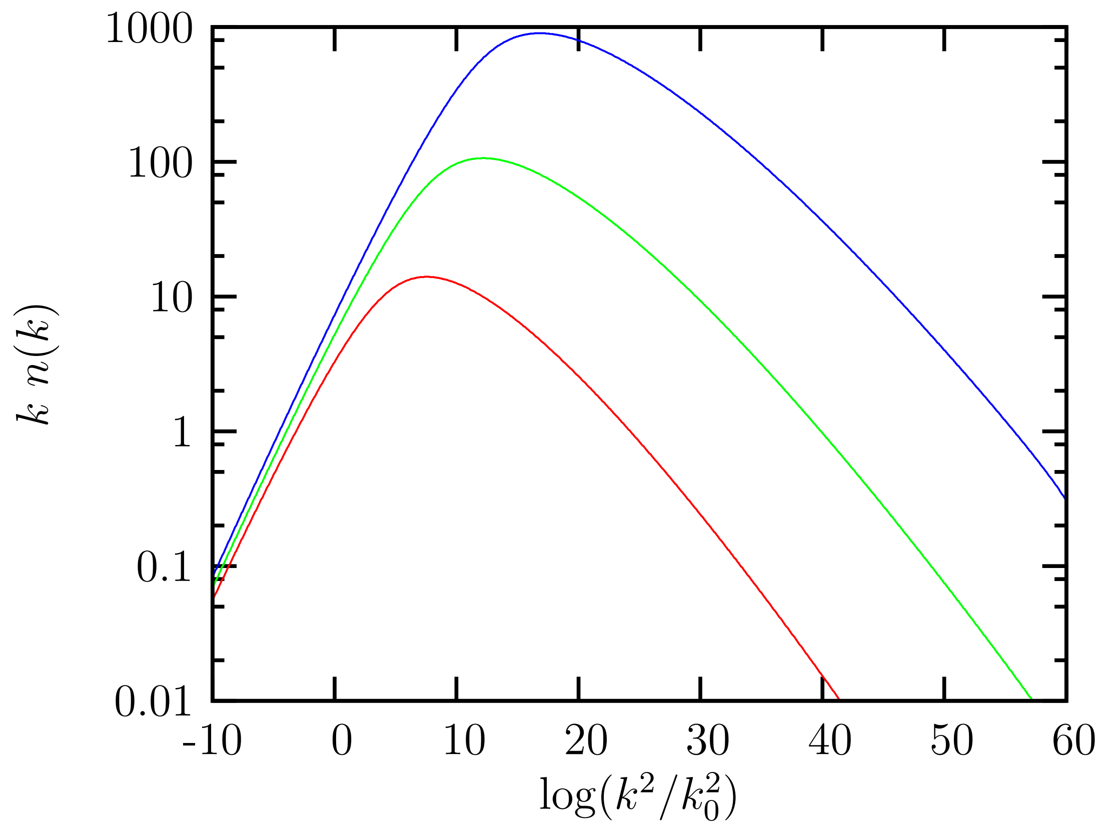
- The unitarization scale for dipole scattering (hence, for DIS)

$$T_{\rm dipole}(r, Y) \;=\; 1 \quad \text{for} \quad r \gtrsim 1/Q_s(Y) \qquad\qquad (r^2 \longleftrightarrow 1/Q^2)$$



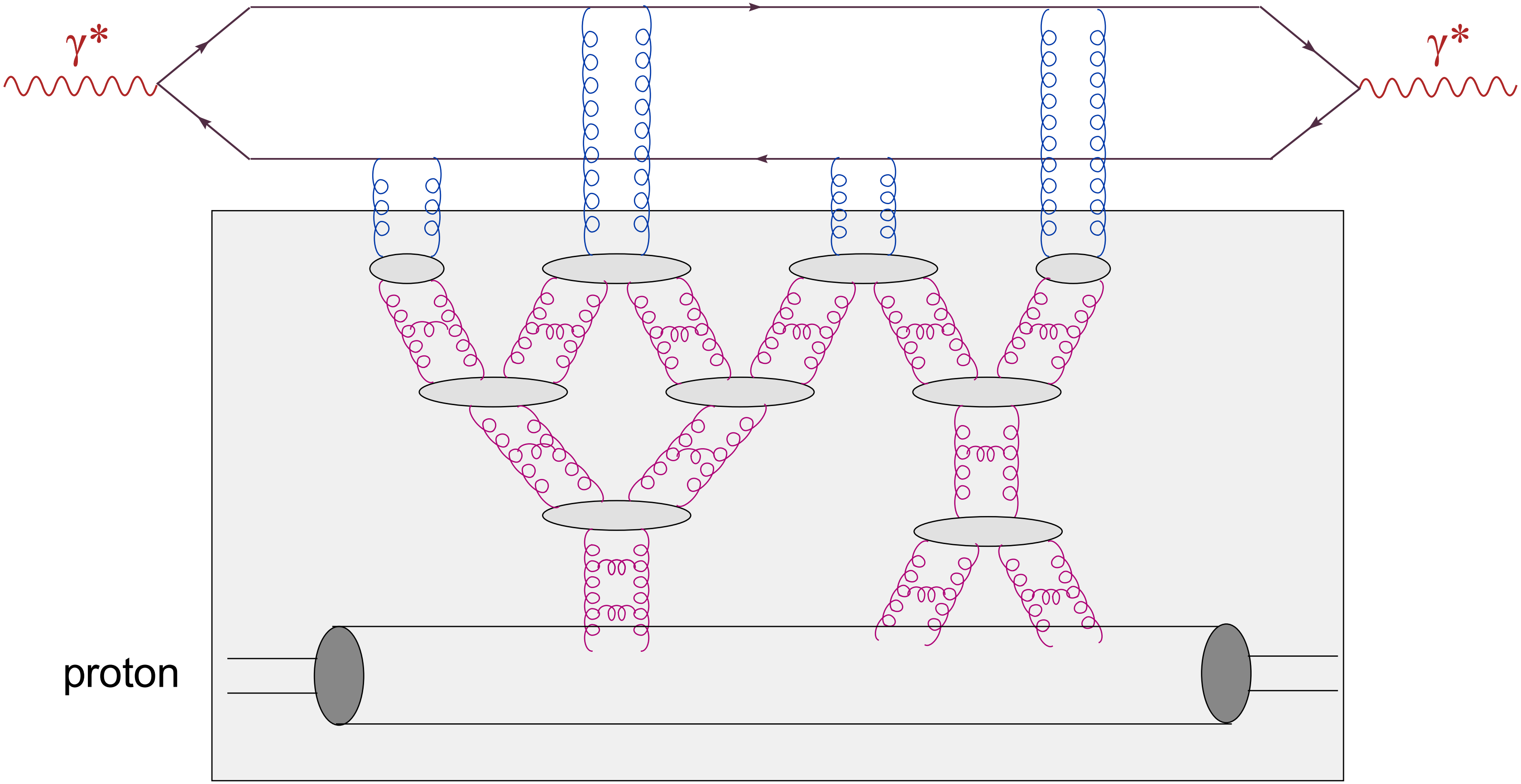
Saturation & Unitarization for DIS

- The typical transverse momentum of the gluon distribution



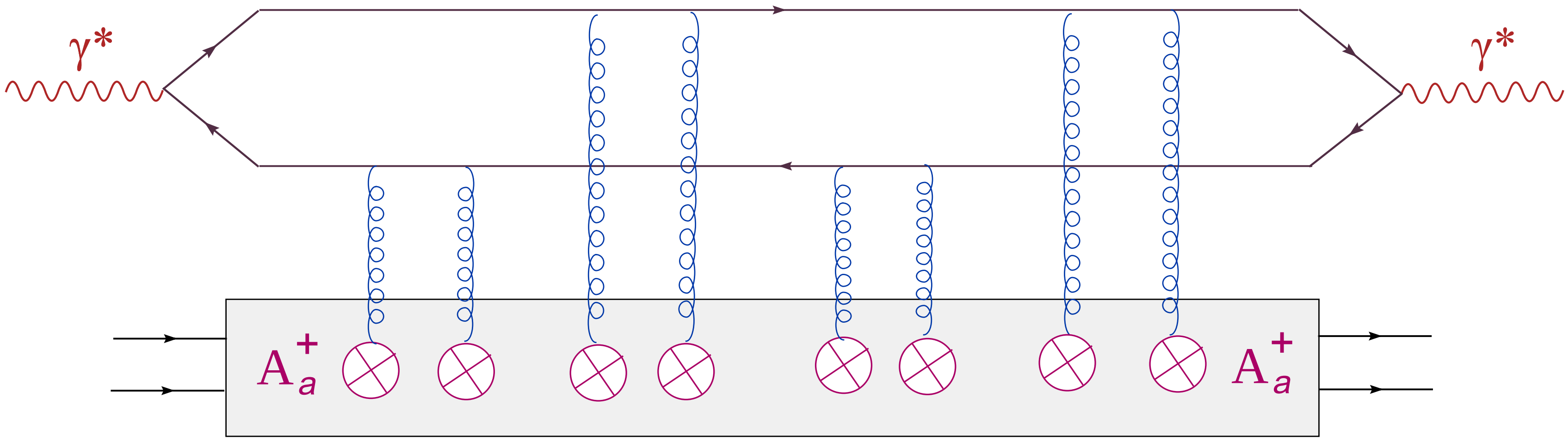
$$xG(x,Q^2) = \int^Q \mathrm{d}k\; k\,\varphi(k,Y)$$

CGC factorization of DIS



- Include all the evolution effects in the target wavefunction
- Replace the target by a random 'background color field' off which the dipole scatters in the eikonal approximation

CGC factorization of DIS



- Multiple scattering off a given configuration of target fields A:

$$S(\boldsymbol{x},\boldsymbol{y})[A] = \frac{1}{N_c}\mathrm{tr}\big(V_{\boldsymbol{x}}^\dagger V_{\boldsymbol{y}}\big)$$

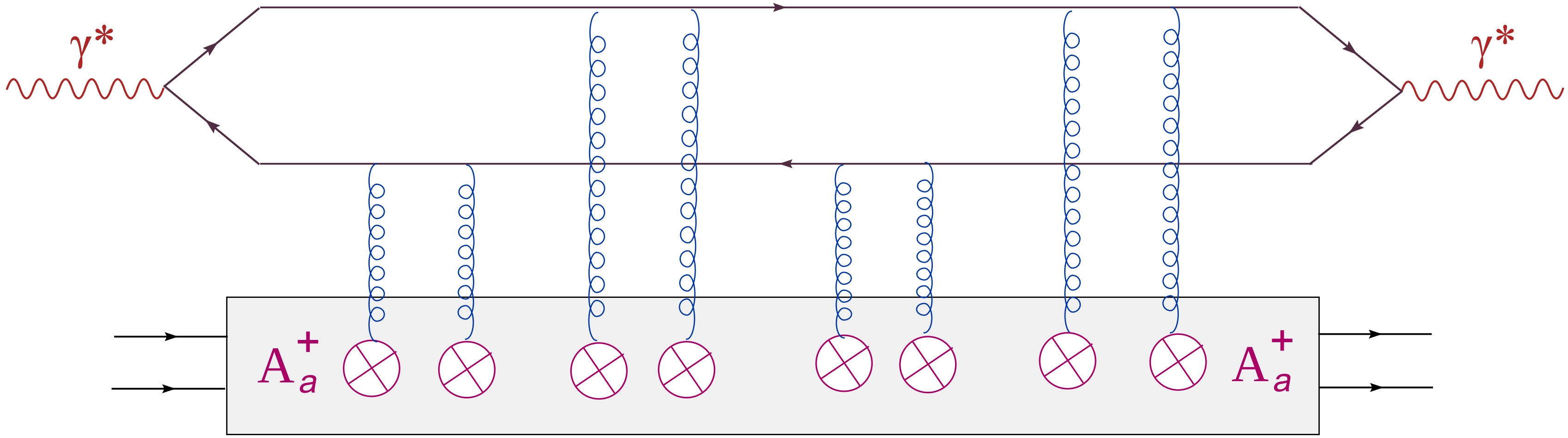
$$V(\boldsymbol{x}) \equiv \mathrm{P}\exp\Big(\mathrm{i}g\int dx^- A_a^+(x^-,\boldsymbol{x})T^a\Big) \qquad \text{(Wilson line)}$$

- Average over A^+ with weight function $W_Y[A]$ ("$|\Psi[A]|^2$")

$$\langle S(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \int \mathrm{D}[A]\; W_Y[A]\; S(\boldsymbol{x},\boldsymbol{y})[A]$$

- Functional evolution equation for $W_Y[\rho]$: 'JIMWLK'

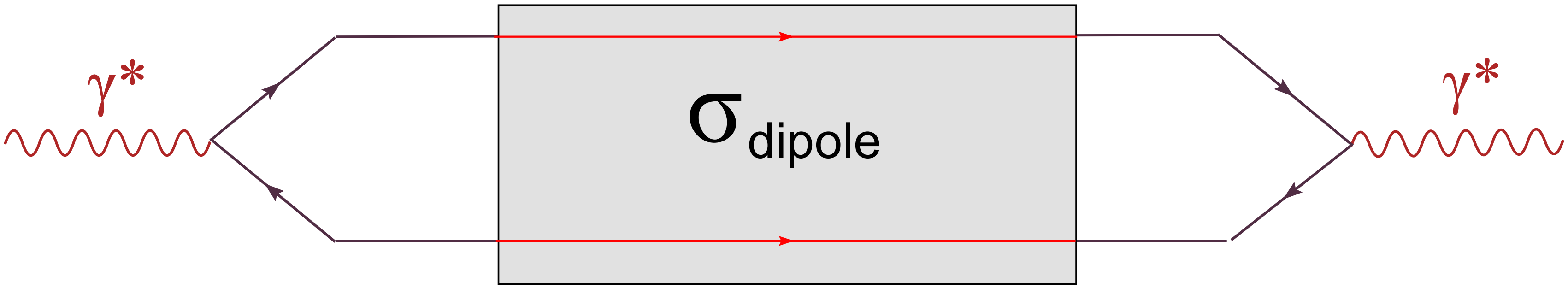
CGC factorization of DIS



$$\langle S(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \int \mathrm{D}[A]\; W_Y[A]\; S(\boldsymbol{x},\boldsymbol{y})[A]$$

- $W_Y[A]$: a kind of “super gluon distribution”
  - information about all the n–point correlations of A^+
  - gluon evolution up to Y (LLA) : JIMWLK eq.
  - long range correlations in rapidity and transverse space
  - color correlations

CGC factorization of DIS



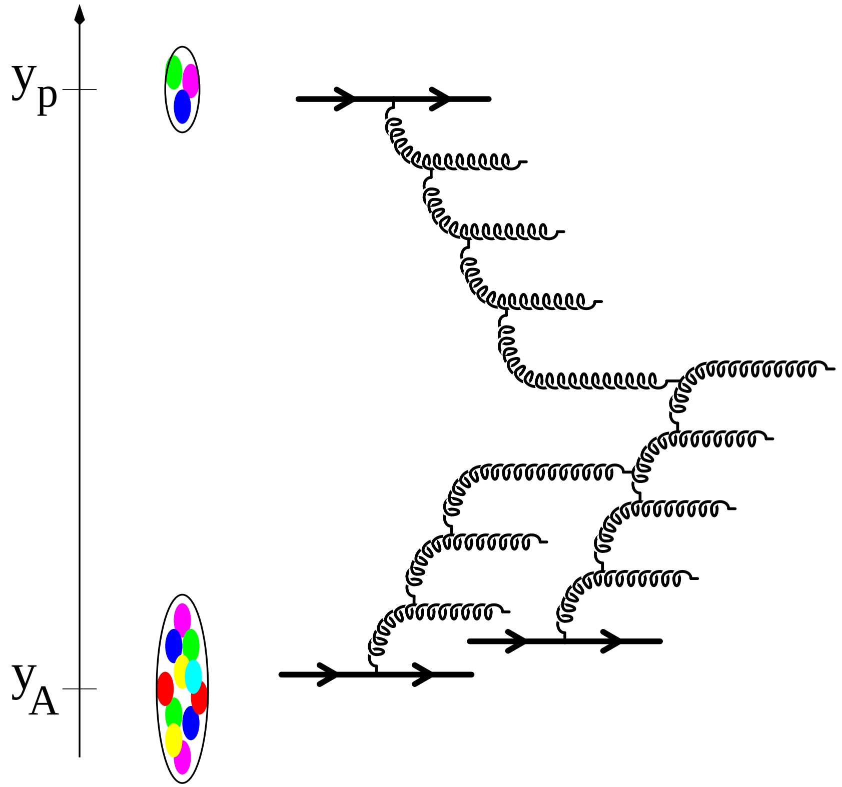
$$\sigma_{\gamma^* h}(Q^2, Y) = \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{r} \sum_{p=T,L} |\Psi_p(z, r)|^2 \, \sigma_{\mathrm{dipole}}(r, Y)$$

$$\sigma_{\mathrm{dipole}}(\boldsymbol{r}, Y) = 2 \int \mathrm{d}^2\boldsymbol{b} \left[1 - \mathrm{Re}\, \langle S(\boldsymbol{x}, \boldsymbol{y})\rangle_Y\right]$$

- Formally similar to the standard k_T factorization (LO BFKL) ... except that, now, σ_{dipole} includes a lot more information !
- Only useful provided we know how to compute σ_{dipole}.

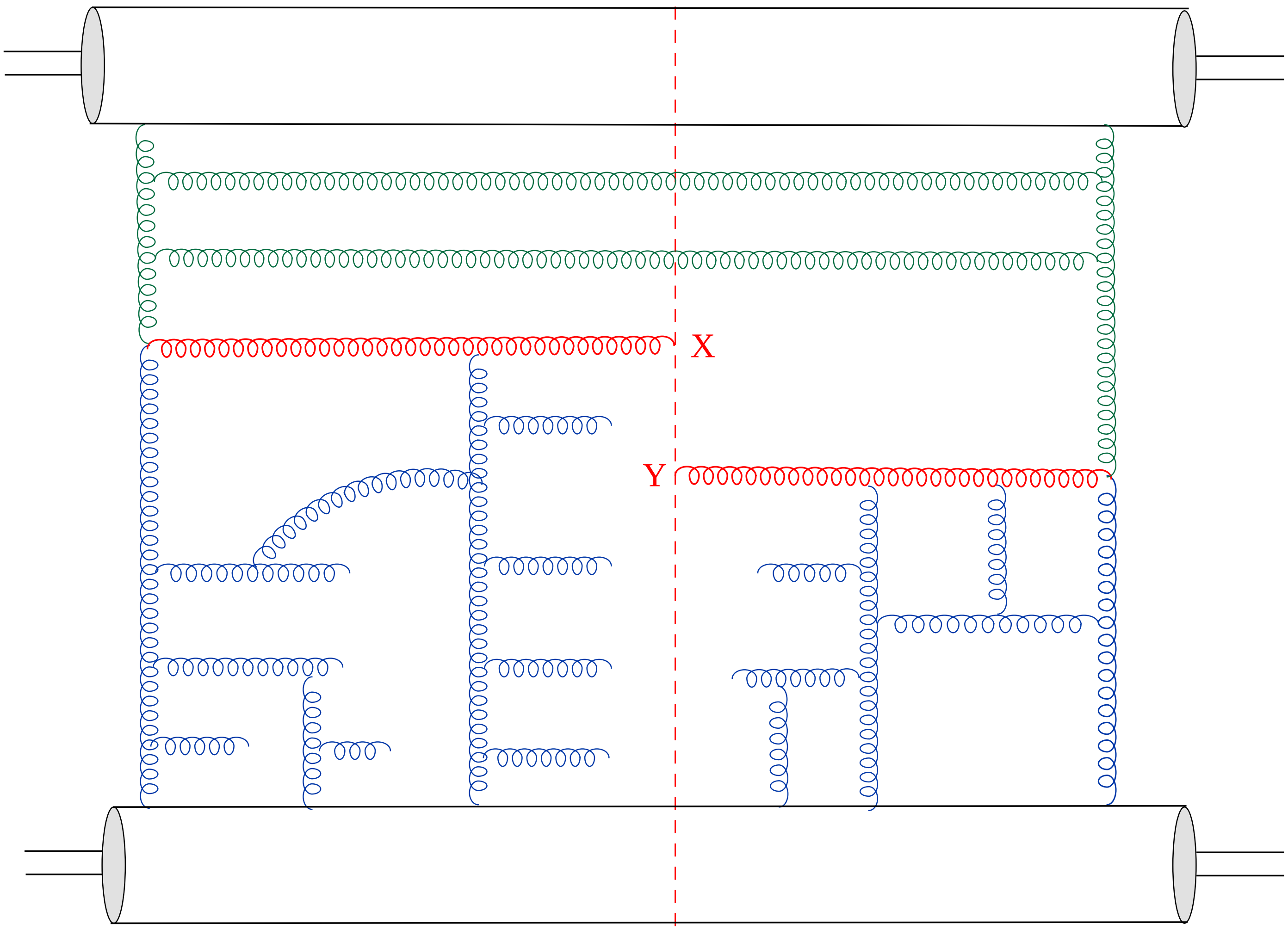
CGC factorization of particle production

- Dense–dilute scattering
  - pA collisions (RHIC, LHC)
  - pp collisions at forward rapidity (LHC)
- Only one parton from the dilute projectile gets involved



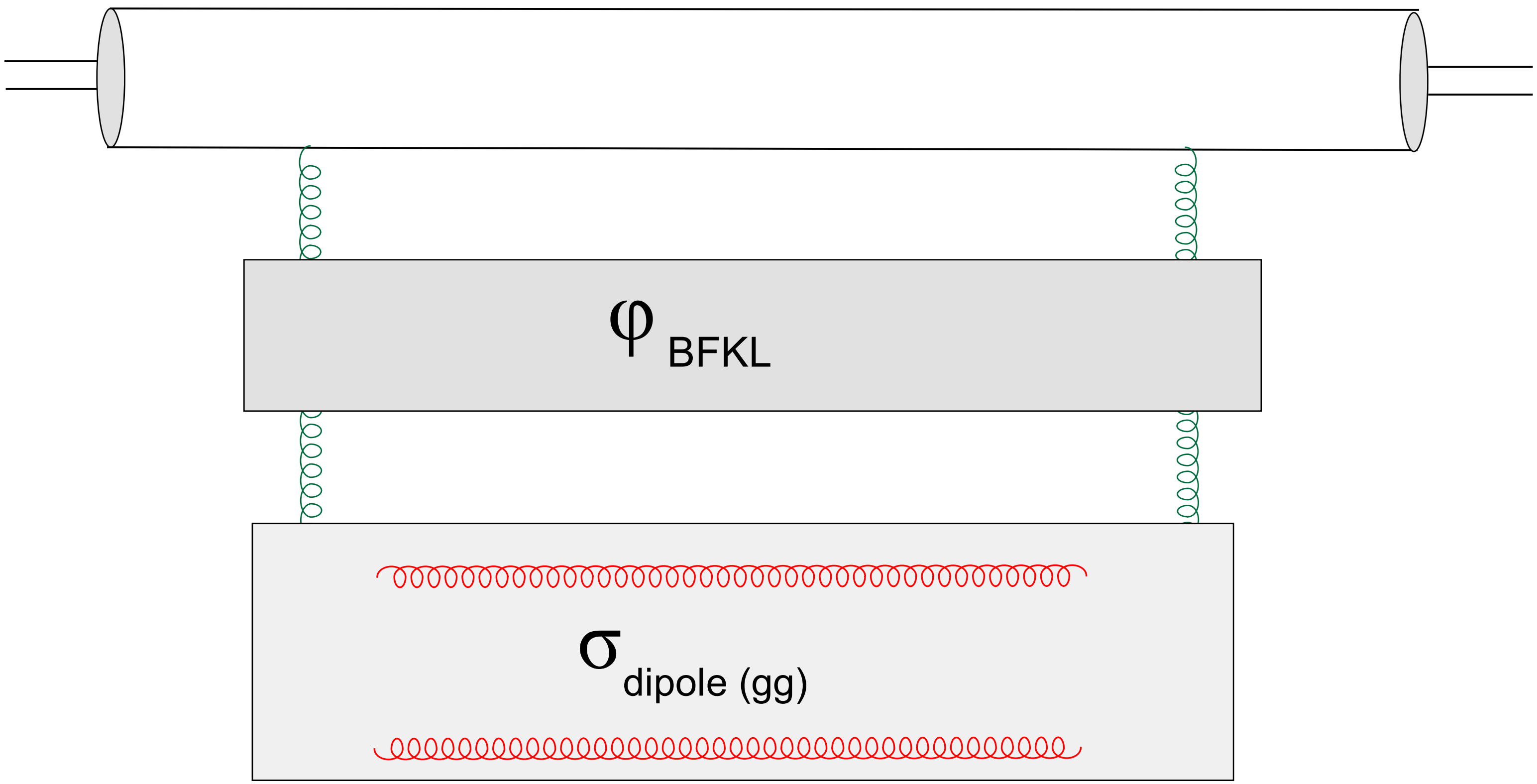
- The particles in the final state undergo multiple scattering

CGC factorization of particle production



- An effective gluon–gluon dipole which multiply scatters

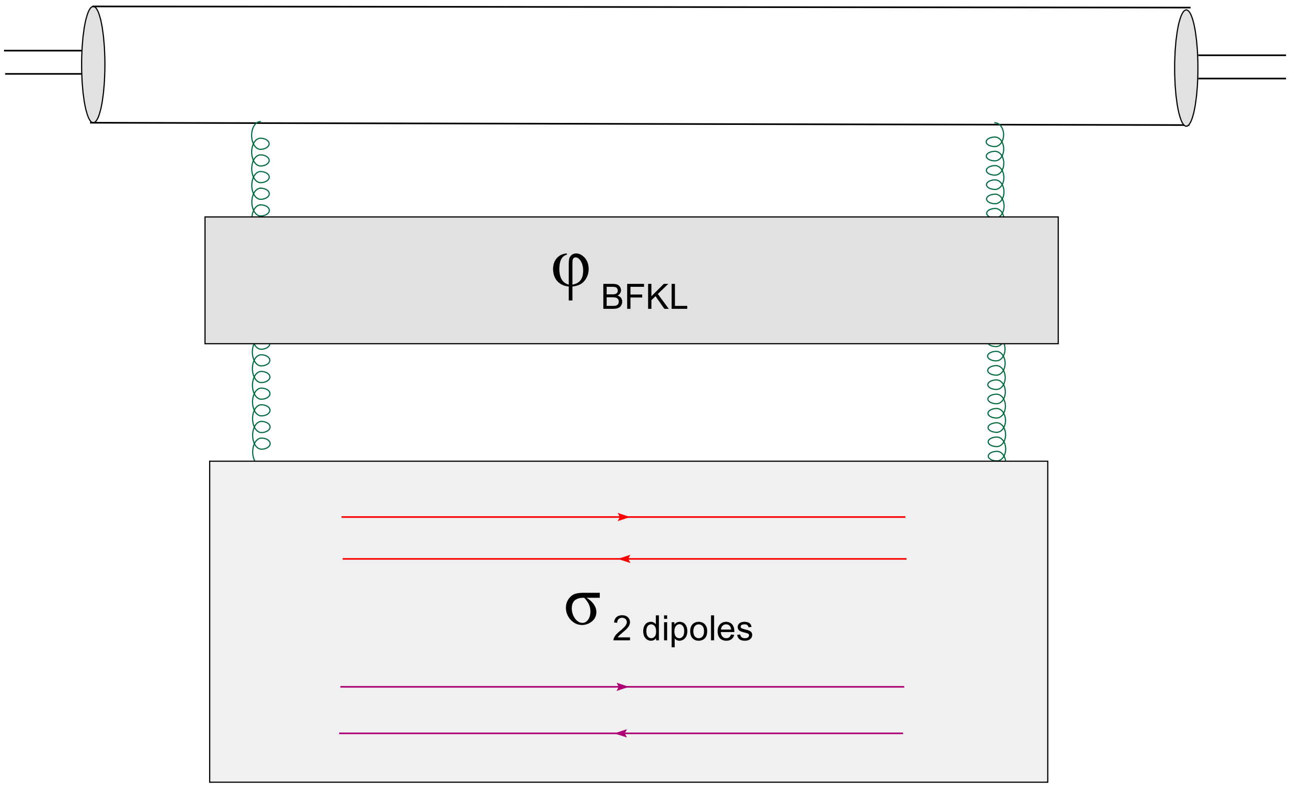
CGC factorization of particle production



$$\frac{d\sigma}{d\eta d^2\boldsymbol{k}} = \frac{\bar{\alpha}_s}{k_\perp^2} \int \frac{d^2\boldsymbol{p}}{(2\pi)^2} \varphi_1(\boldsymbol{p}, y_1)\Phi_2(\boldsymbol{k}-\boldsymbol{p}, y_2)$$

$$\Phi(\boldsymbol{k}, y) \equiv \int d^2\boldsymbol{r}\; e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\; \nabla_{\boldsymbol{r}}^2 \sigma_{\mathrm{dipole}}^{gg}(\boldsymbol{r}, y)$$

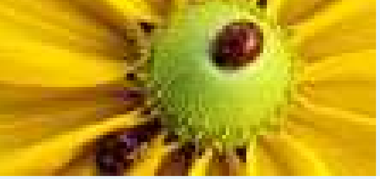
CGC factorization of particle production



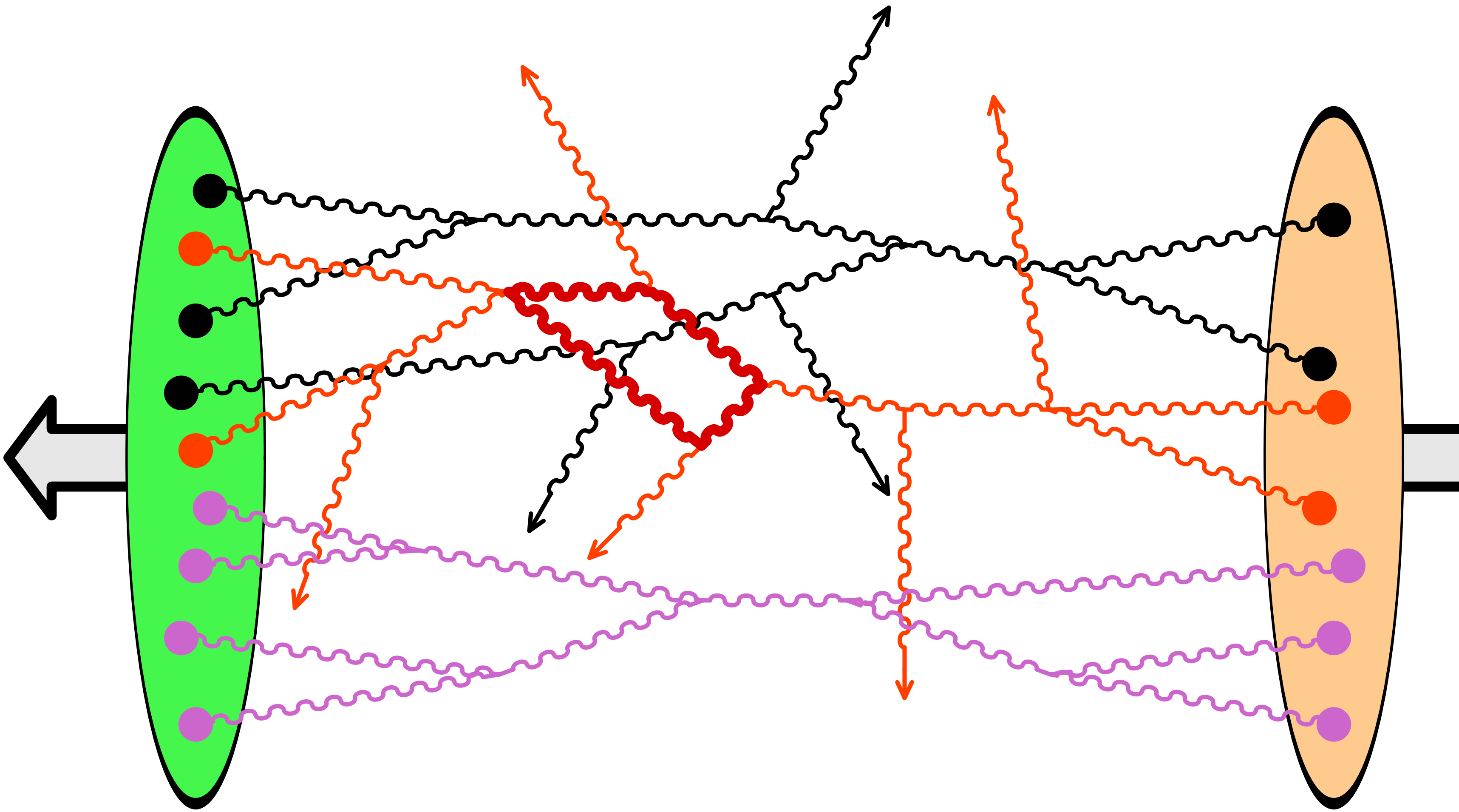
- Four Wilson lines (q, $\bar{q}$) $\Longrightarrow$ "2 dipoles"

$$\mathrm{tr}\big\langle V_{\boldsymbol{x}}^{\dagger}\, t^{a}\, V_{\boldsymbol{y}}\, V_{\boldsymbol{x}'}^{\dagger}\, t^{b}\, V_{\boldsymbol{y}'}\big\rangle, \qquad \mathrm{tr}\big\langle V_{\boldsymbol{x}}^{\dagger}\, t^{a}\, V_{\boldsymbol{y}}\, t^{c}\, \tilde{V}_{\boldsymbol{x}'}^{cb}\big\rangle$$

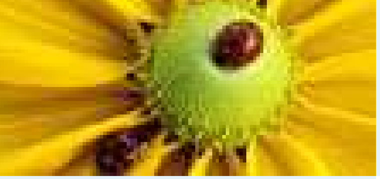
- A new set of 'generalized pdf's' :
  the ensemble of gauge–invariant correlations of Wilson lines



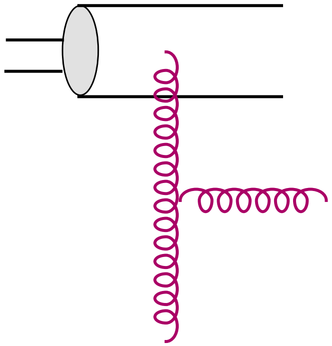
AA collisions



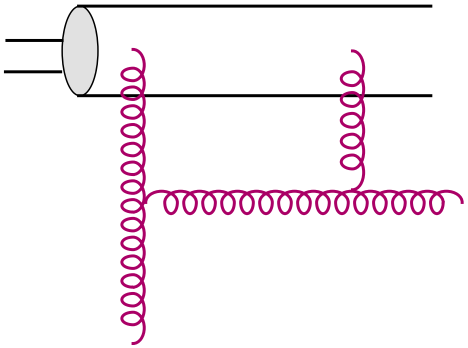
- Solve the classical Yang–Mills eqs. with two sources



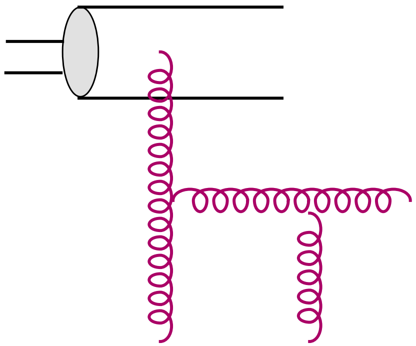
Non–linear evolution at small x



BFKL



saturation



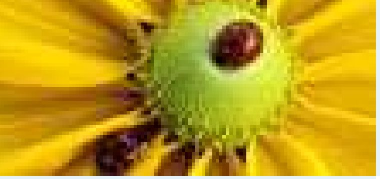
bremsstrahlung

- Infinite hierarchy for the N–body gluon densities :

$$\frac{\partial\langle\varphi\rangle}{\partial Y} \simeq \alpha_s\langle\varphi\rangle - \alpha_s^2\,\langle\varphi\varphi\rangle$$

$$\frac{\partial\langle\varphi\varphi\rangle}{\partial Y} \simeq 2\alpha_s\langle\varphi\varphi\rangle - 2\alpha_s^2\,\langle\varphi\varphi\varphi\rangle + \alpha_s^2\langle\varphi\rangle \; \ldots$$

  - Early stages: Correlations are generated via fluctuations
  - Intermediate stages: ... then amplified by BFKL evolution
  - High density: ... and eventually lead to saturation !



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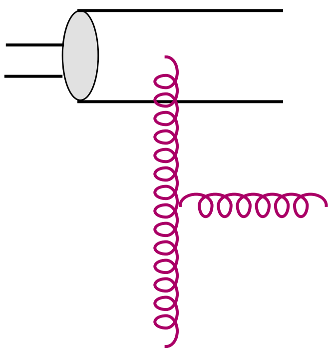
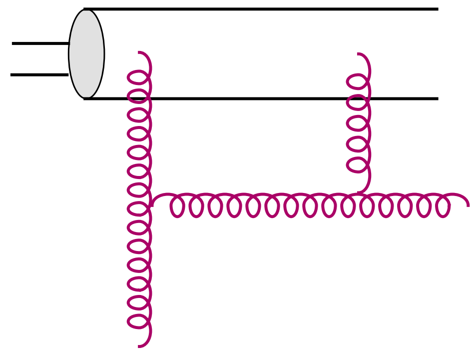
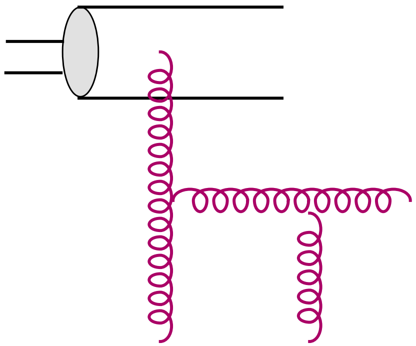


BFKL



saturation



bremsstrahlung

- Equivalent to a single Langevin (stochastic) equation:

$$\frac{\partial \varphi}{\partial Y} = \alpha_s \partial_\rho^2 \varphi + \alpha_s \varphi - \alpha_s^2 \varphi^2 + \sqrt{\alpha_s^2 \varphi}\, \nu\,, \quad \langle \nu(Y_1) \nu(Y_2) \rangle = \delta(Y_1 - Y_2)$$

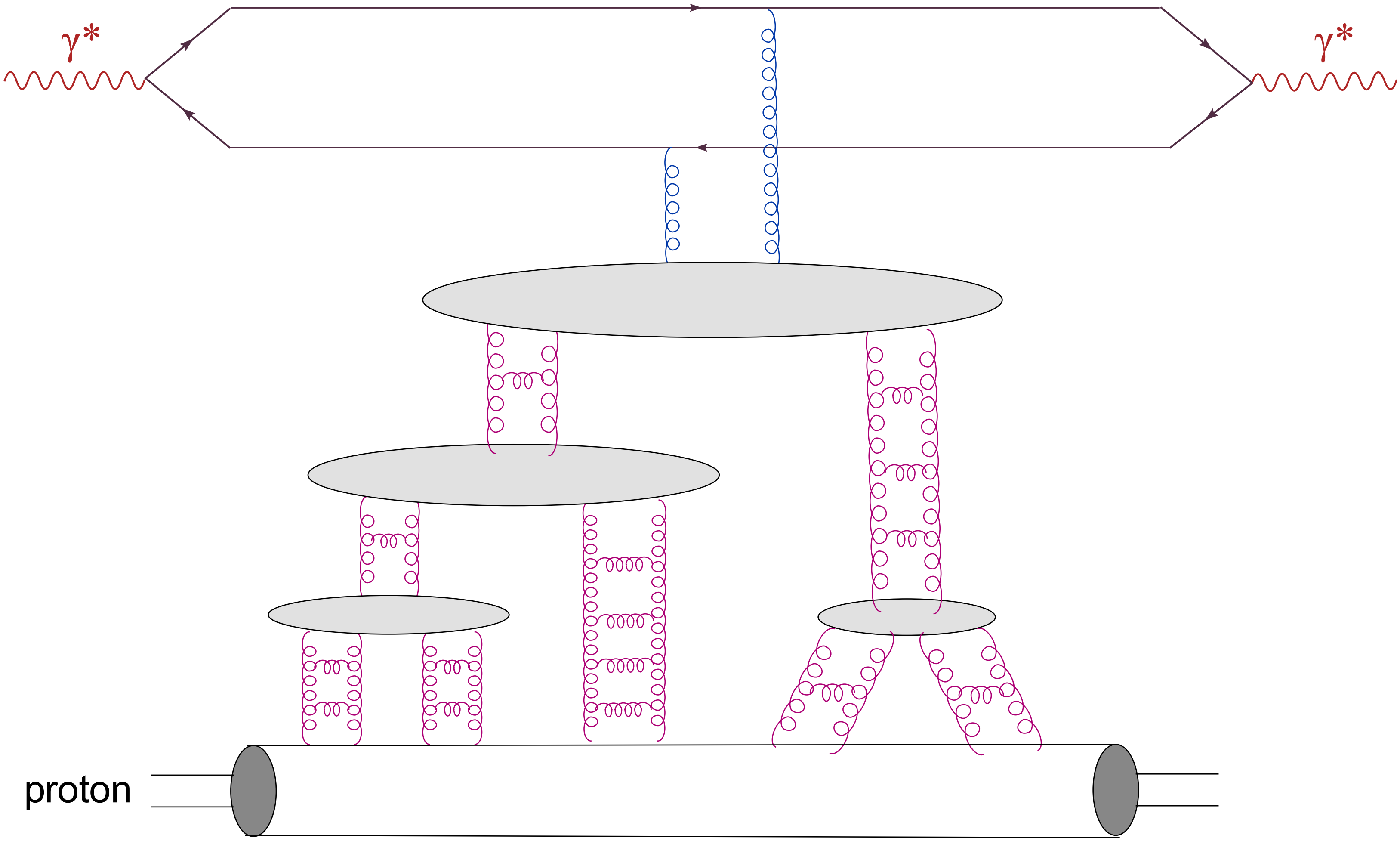
- Link to statistical physics : 'Reaction–diffusion' $A \rightleftharpoons 2A$

  High–energy evolution in QCD $\approx$ A classical stochastic process

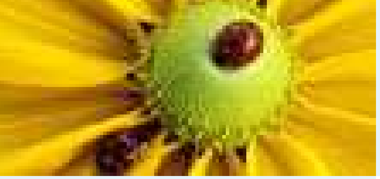
- Mean field approximation: ignore the noise $\Longrightarrow$ BK equation

  ... almost never right !

BK equation

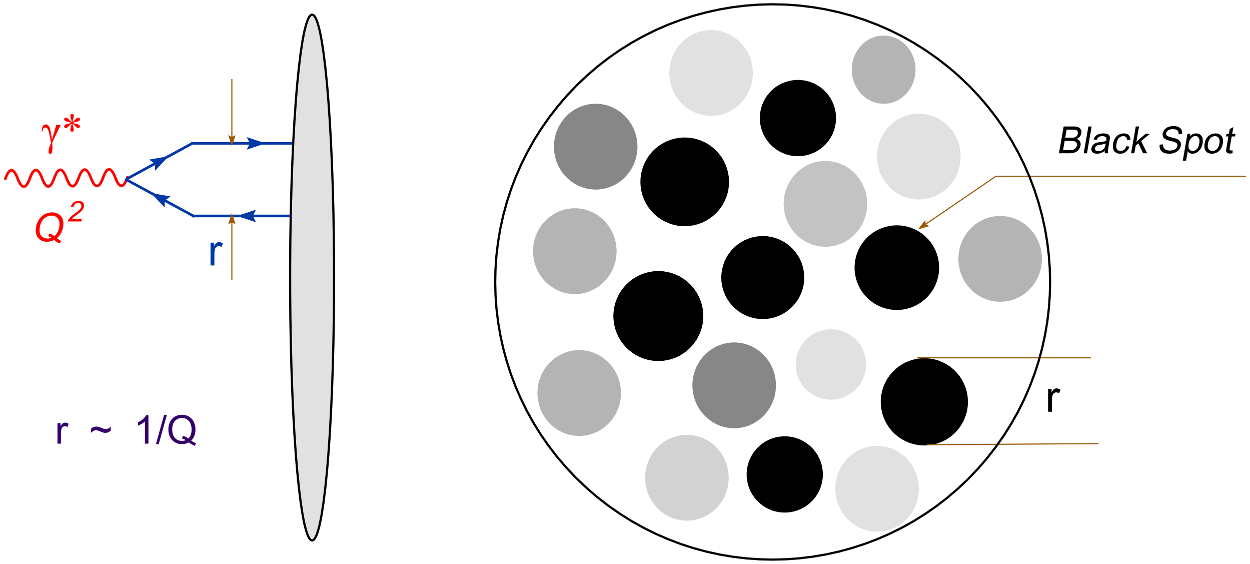


- 'Fan diagrams' : gluon recombination alone ('high density')



Black spots

- A physical picture of the proton as seen DIS at very high energy ($DY \gg 1$) and relatively large Q^2 : $Q^2 \gg \langle Q_s^2(Y) \rangle$
- Strong inhomogeneities due to fluctuations in the evolution



- ‘Grey spots’ : $T \ll 1$
- ‘Black spots’ : $T \sim 1$

- The the hadron looks dilute on the average : $\langle T \rangle \ll 1$

Black spots

- A physical picture of the proton as seen DIS at very high energy ($DY \gg 1$) and relatively large Q^2 : $Q^2 \gg \langle Q_s^2(Y) \rangle$
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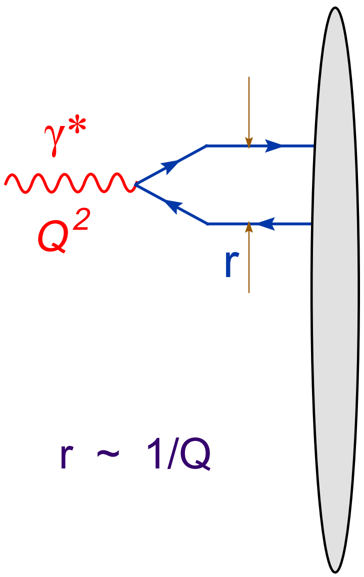
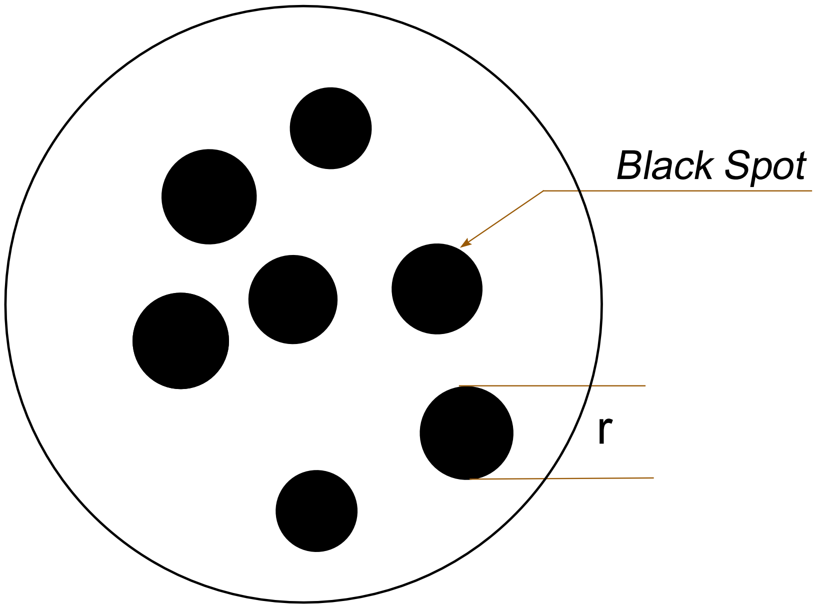
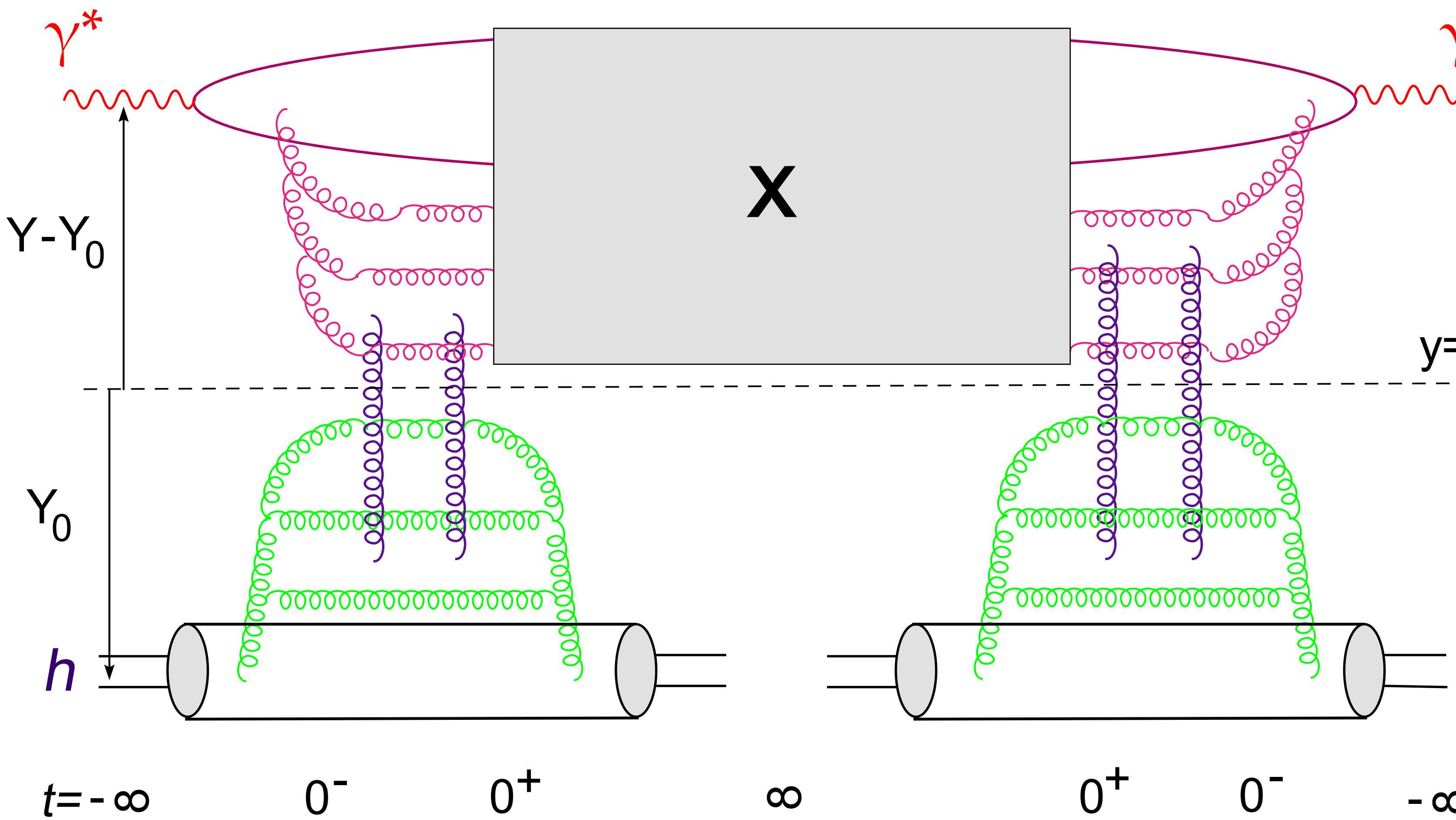




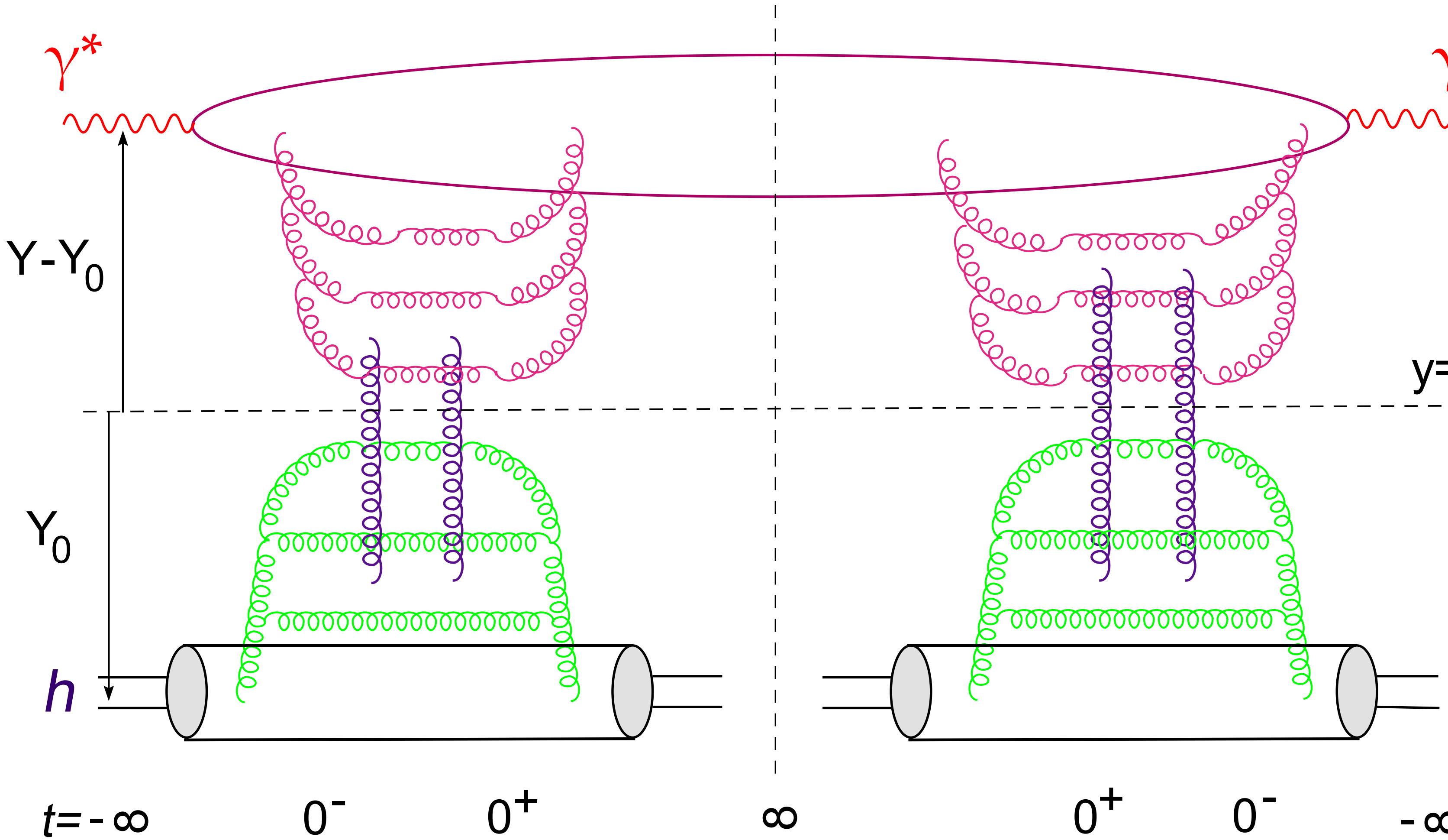
- Yet, this average (hence, the cross–section) is dominated by the rare–but–dense fluctuations (the 'black spots')

DIS Diffraction at high energy



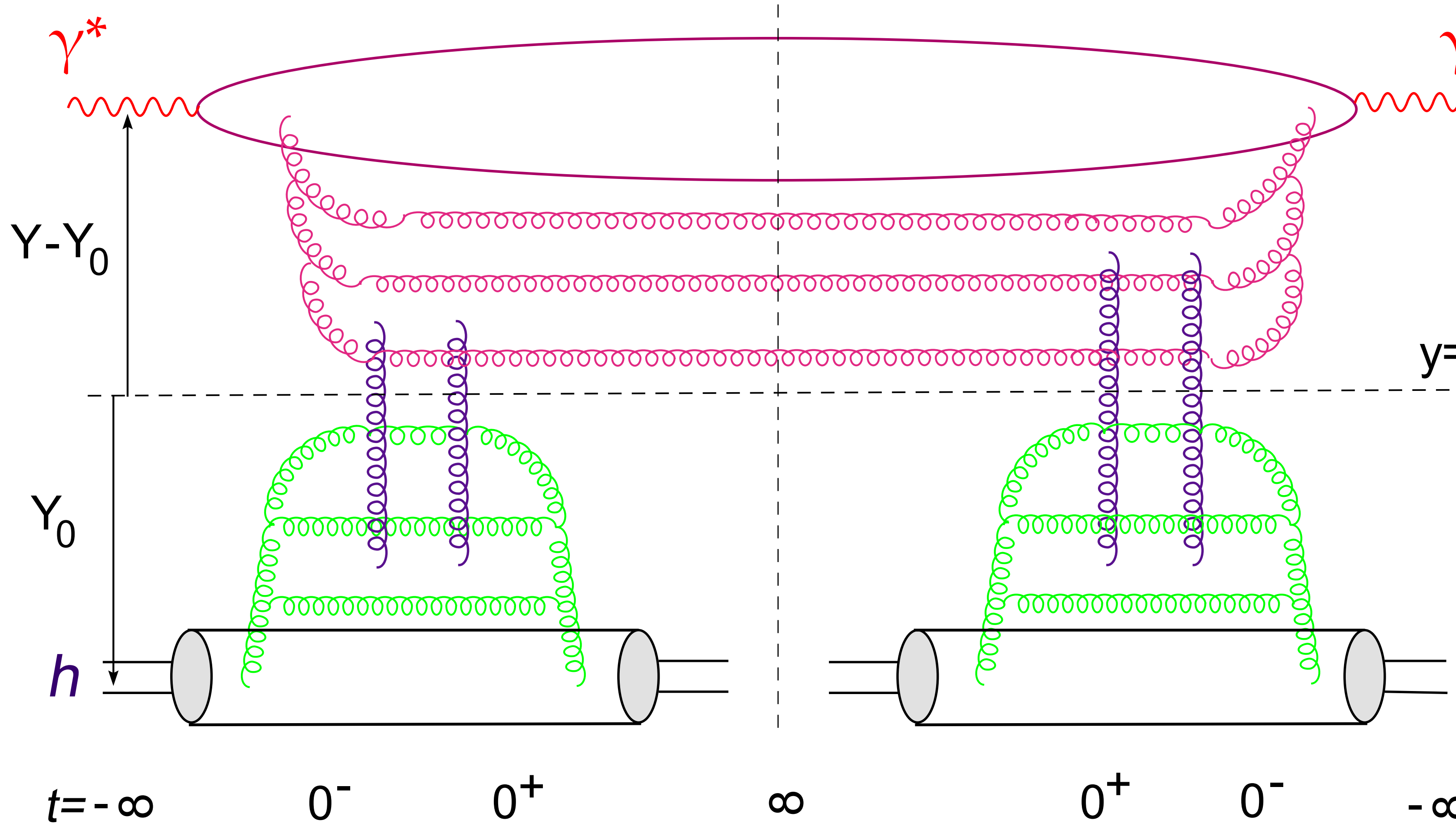
$$Y \geq Y_{\rm gap} \geq Y_0$$

DIS Diffraction at high energy



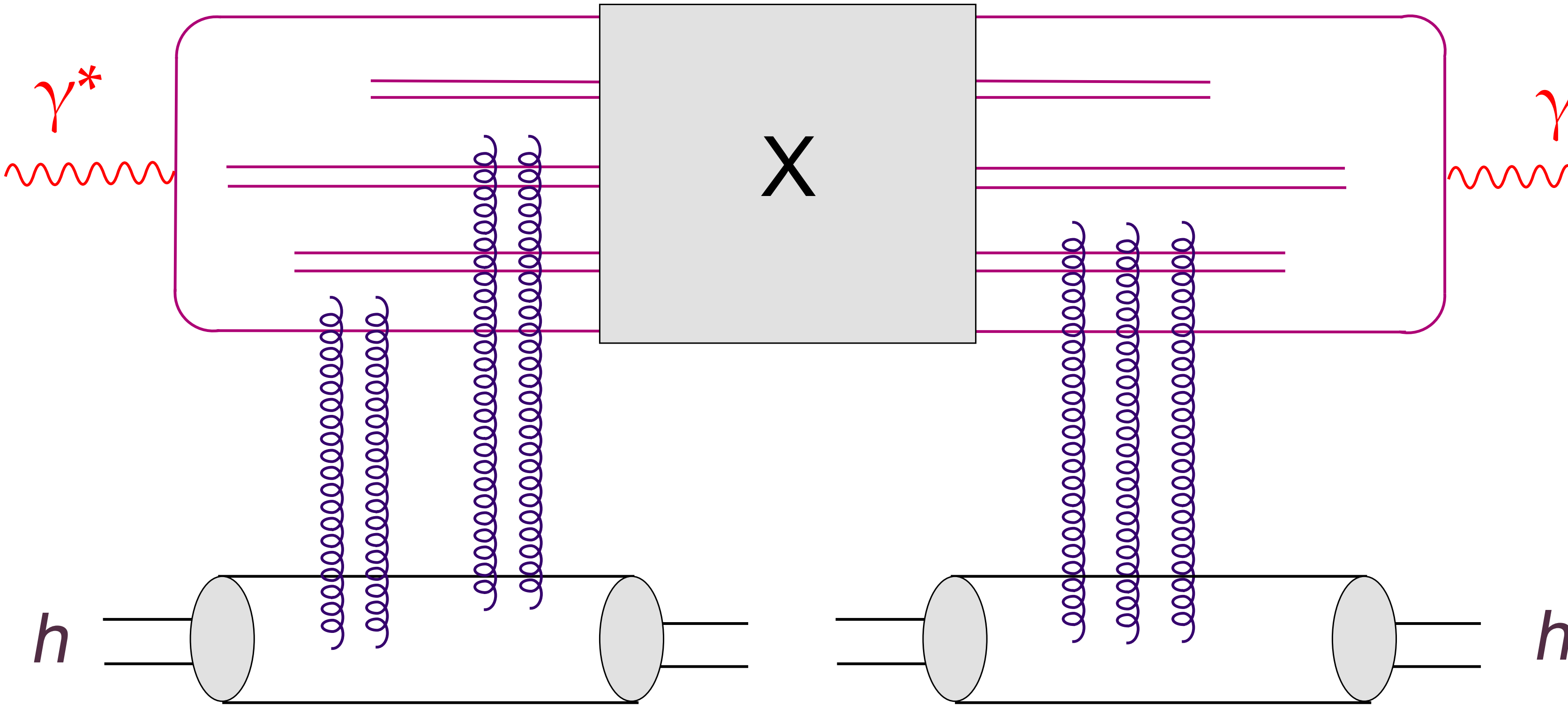
$Y_{\rm gap} = Y$ (elastic)

DIS Diffraction at high energy

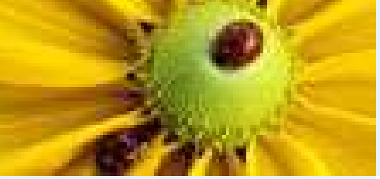


$$Y_{\text{gap}} = Y_0$$

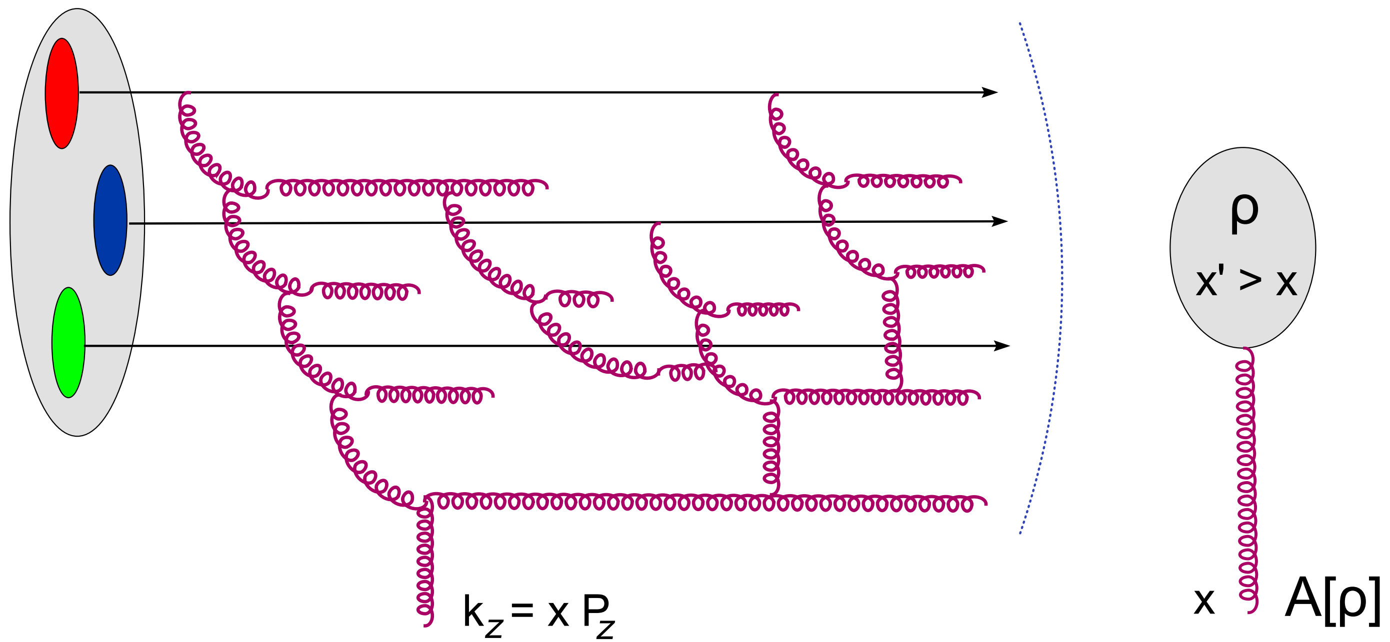
DIS Diffraction at high energy



$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b}(\boldsymbol{r}, Y, Y_0) = \sum_{N=1}^{\infty} \int \mathrm{d}\Gamma_N \, P_N(\{\boldsymbol{z}_i\}; Y - Y_0) \left| \langle 1 - S(1)S(2)\cdots S(N) \rangle \right.$$

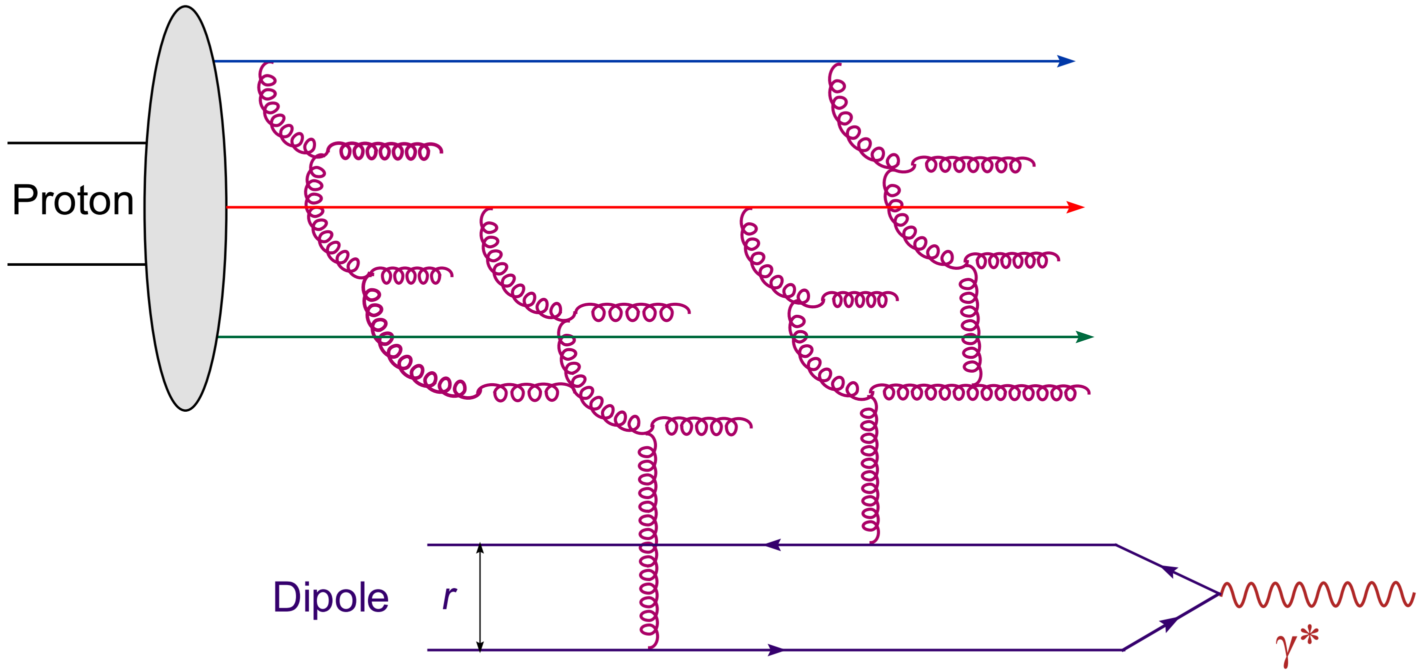


The Color Glass Condensate



- Small–x gluons :
  Classical color fields radiated by fast color sources (gluons with $x' \gg x$) 'frozen' in some random configuration
- $W_Y[\rho]$: Probability distribution for the color charge density (a 'generalized pdf' describing all the n–point correlations)
- Functional evolution equation for $W_Y[\rho]$: 'JIMWLK'

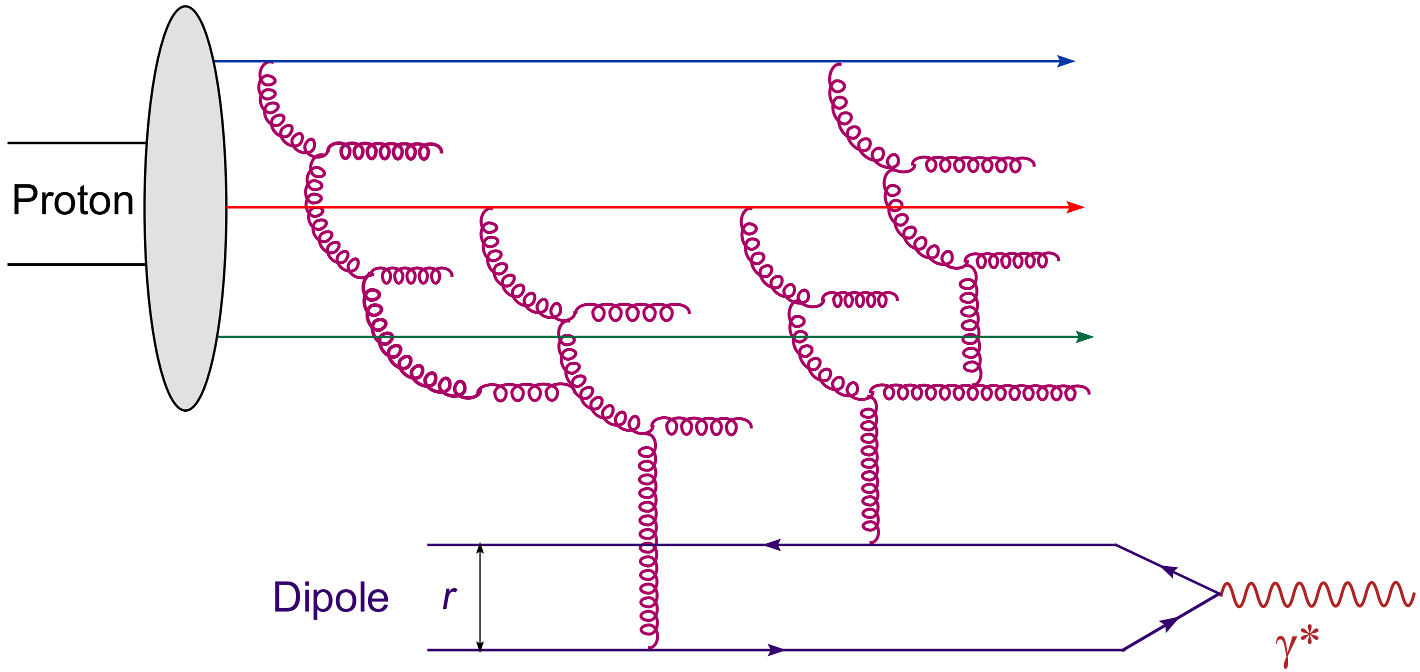
Deep Inelastic Scattering off the CGC



- Multiple scattering off a given configuration of (strong) color fields $A[\rho] \implies T(r)[\rho]$ (eikonal approximation)
- Average over ρ with weight function $W_Y[\rho]$ ('glass')

$$\langle T(r,Y)\rangle = \int \mathrm{D}[\rho]\; W_Y[\rho]\; T(r)[\rho]$$

Deep Inelastic Scattering off the CGC

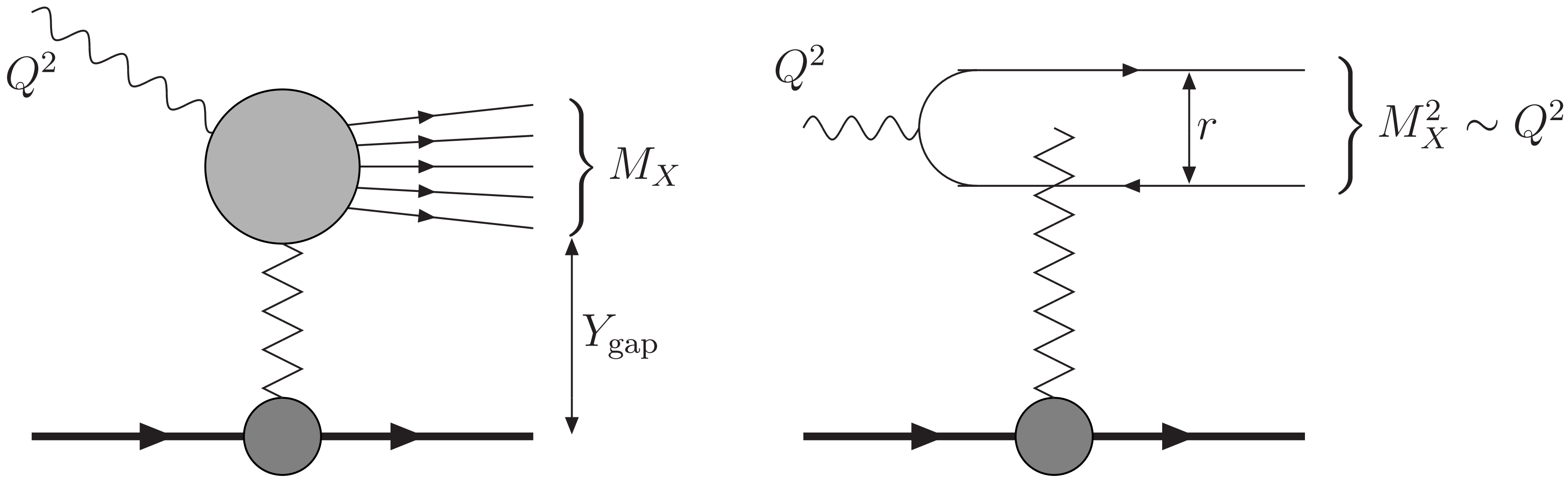


- Increase Y : Evolution equation for $W_Y[A]$ (JIMWLK)

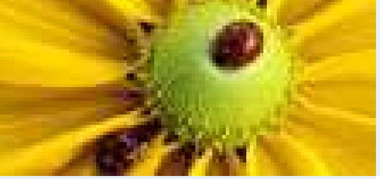
$$\partial_Y \langle T(r,Y)\rangle_Y = \int \mathrm{D}[A]\, \partial_Y W_Y[A]\; T(r)[A]$$

$\Longrightarrow$ Non–linear equation for $\langle T\rangle_Y$ (Balitsky-Kovchegov)

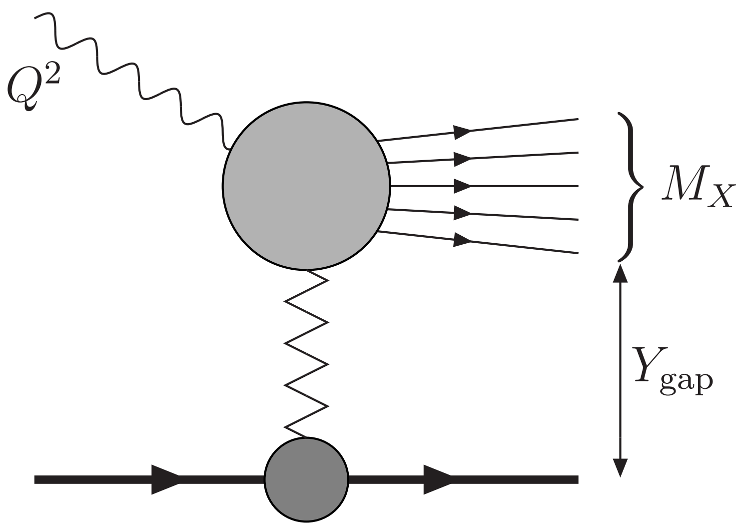
DIS Diffraction

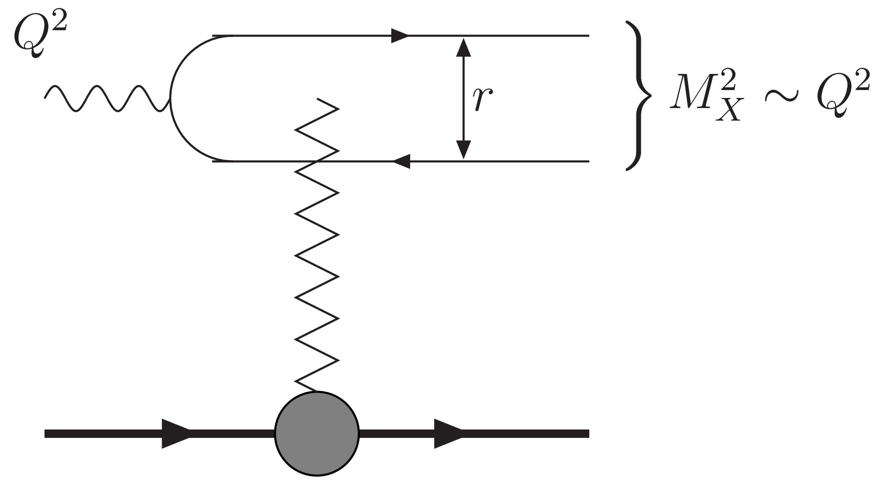


- An ideal laboratory to study saturation/unitarity effects
  - sensitive to relatively large dipole sizes
  - sensitive to theoretical models (or prejudices)



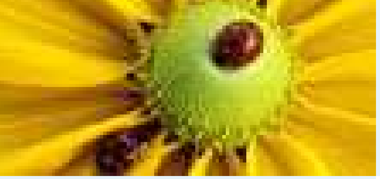
DIS Diffraction





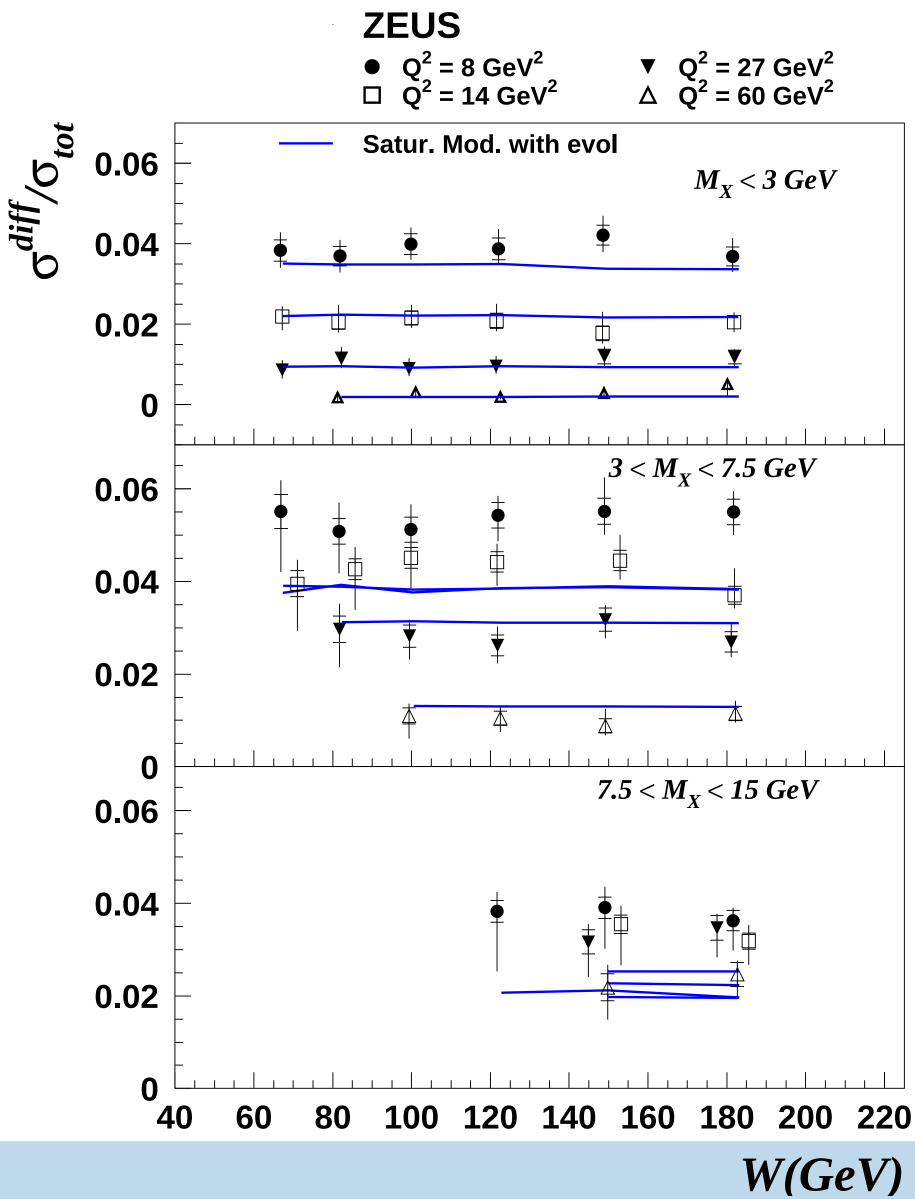
- An ideal laboratory to study saturation/unitarity effects
  - sensitive to relatively large dipole sizes
  - sensitive to theoretical models (or prejudices)
- Original prejudice: "Even for large Q^2, diffraction is soft"

$\sigma_{\rm diff} \propto x^{-2(\alpha_{\mathbb{P}}-1)}$ and hence $\dfrac{\sigma_{\rm diff}}{\sigma_{\rm tot}} \sim x^{-(\alpha_{\mathbb{P}}-1)}$ at small x ✓



Diffractive over inclusive ratio at HERA

Golec-Biernat, Wüsthoff (99) ; Bartels, Golec-Biernat & Kowalski (02)



Diffractive dissociation of the virtual photon

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} = \int \mathrm{d}z\, \mathrm{d}^2\boldsymbol{r}\, |\Psi_\gamma(z, r; Q)|^2\ \langle T(r, Y)\rangle^2$$

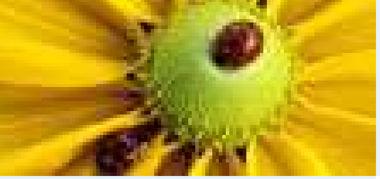
- The photon wavefunction favors small dipoles ($r \sim 1/Q$)

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} \sim \frac{1}{Q^2} \int_{1/Q^2}^{\infty} \frac{\mathrm{d}r^2}{r^4} \langle T(r, Y)\rangle^2$$

- The dipole amplitude favors relatively large dipoles :

$$T(r) \propto r^2 \qquad \text{(single scattering)}$$

- “The integral is dominated by large, non–perturbative, dipoles with size $r \sim 1/\Lambda_{\mathrm{QCD}}$, hence the soft pomeron ! ”



Hardening the diffraction (1)

- At sufficiently high energy, gluon saturation cuts off the large dipoles already on the ‘semi–hard’ scale $1/Q_s$!

$$\frac{\mathrm{d}\sigma_{\mathrm{diff}}}{\mathrm{d}^2 b} \sim \frac{1}{Q^2} \int_{1/Q^2}^{1/Q_s^2} \frac{\mathrm{d}r^2}{r^4} \left(r^2 Q_s^2(x) \right)^2 \sim \frac{Q_s^2(x)}{Q^2} \propto x^{-\lambda}$$

  - σ_{diff} is dominated by dipole sizes $r \sim 1/Q_s(x)$!
  - $\sigma_{\mathrm{diff}} \propto x^{-\lambda}$: single, hard pomeron increase with $1/x$ (instead of double soft !)
  - $\sigma_{\mathrm{diff}}/\sigma_{\mathrm{tot}} \approx$ constant ! ✓

- ‘Semi–hard diffraction’ ... at intermediate energies !

Hardening the diffraction (2)

- Very high energies: 'black spots' with $Q_s^2 \sim Q^2$
  - $\sigma_{\rm diff}$ is dominated by the hard scale $r \sim 1/Q$!
  - no 'pomeron' (power–like) increase, diffusive scaling, ...