Exercises 1 (13. August 2012)

Monte Carlo technique

1. construct a uniform random number generator from the congruential method:

$$I_{i+1} = mod(a \cdot I_i + c, m)$$

$$R_{i+1} = \frac{I_{i+1}}{m}$$

   with $I_0 = 4711$, $a = 205$, $c = 29573$ and $m = 139968$
   Compare the correlation of 2 random numbers. Compare this with RANLUX.

2. construct a Gaussian random number generator from a uniform random number generator

3. write a small program that integrates (with Monte Carlo method) the function $f(x) = 3x^2$ for $\int_0^1 f(x)dx$, and calculate the uncertainty.

4. write a small program that integrates (with Monte Carlo method) $\int_0^1 \int_0^x dxdy$ with $0 < x, y < 1$.

5. write a small program to integrate a simple function in one dimension:
   $\int_{x_{min}}^1 g(x)dx = \int_{x_{min}}^1 (1-x)^5 \frac{dx}{x}$, using Monte Carlo integration, with $x_{min} = 0.0001$
   Improve the above integration by using importance sampling.

If you have time, you can do some more exercises:

- use the LHAPDF library to calculate the flavor sum rules:

$$\int_0^1 dx \, u_V(x, Q^2) = 2$$

$$\int_0^1 dx \, d_V(x, Q^2) = 1$$

  use the LHAPDF library and calculate the momentum sum rule:

$$\int_0^1 dx \, \sum_{i=-6}^6 xp_i(x, Q^2)$$

  use the MRST(MRST2004nlo) set and the LO* (MRST2007lomod) set. How much is the momentum sum rule violated in the LO* set ? Is the momentum sum rule satisfied (or violated in the same way) for different Q^2 values (use $Q^2 = 5, 10, 100, 1000$ GeV^2).

- use TMDlib to calculate the integral over k_t of the TMD parton density for a gluon for $0.0001 < x < 0.9$ at $q = 10$ GeV. Integrate in the range $0.01 < k_t^2 < 100$ GeV^2. Use TMD set: `PB-NLO-HERAI+II-2018-set2`.
  Plot the integrate TMD in a histogram for $0.0001 < x < 0.9$.
  Compare your results with the ones you can obtain from TMDplotter: tmdplotter.desy.de.
  The integral is:

$$\int_{k_{t\,min}^2}^{k_{t\,max}^2} dk_t^2 \mathcal{A}(x, k_t, q) = x f(x, q)$$