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SYMMETRIC STATES IN QUANTUM GEOMETRY

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In the kinematical sector of loop quantum gravity, states are represented as functions on the space $\overline{\mathcal{A}}$ of generalized connections with gauge group $G = SU(2)$ on a space manifold Σ. All these states can be decomposed in terms of the spin network basis which are special states associated with graphs in Σ. Given a symmetry group S acting on Σ, we can ask for states which are symmetric with respect to that action and therefore can be used to study the full theory in a simpler regime. However, as the decomposition into spin networks shows, no non-trivial ordinary state can be exactly symmetric: the discrete structure of space breaks any continuous symmetry. Nevertheless, we can look for symmetric *generalized* states which is automatic in our definition:[1]

Definition. *A symmetric state is a distribution on $\overline{\mathcal{A}}$ whose support contains only connections being invariant under the action of the symmetry group.*

In order to describe symmetric states more explicitly, we need more information about invariant connections. All we need here is the fact (which is well-known from the example of an $SU(2)$-connection which is invariant under the rotation group) that an invariant connection can be decomposed into a reduced connection and scalar fields which are functions on the reduced manifold $B := \Sigma/S$.[2,3] So we arrive at a convenient representation of symmetric states as functions on the space of generalized connections and scalar fields on the reduced manifold. A basis for these states is given by spin network states with Higgs field vertices in the reduced manifold with gauge group G. We can use these states either as reduced models by restricting all considerations to them, or as distributional states in the full theory. In the latter interpretation, our symmetric states are idealized states, but can be approximated by ordinary (weave) states.

Spherically symmetric states are described by spin networks with Higgs field vertices in a one-dimensional (radial) manifold B which immediately implies that a spherical surface (represented by a single point in B, the radius) intersects a given spin network state in only one point. Quantizing the area along the lines of the full theory[4] then leads to the area spectrum (γ is the Immirzi parameter and $l_{\rm P}$ the Planck length)

$$A_j = \gamma l_{\rm P}^2 \sqrt{j(j+1)} \quad , \quad j \in \tfrac{1}{2}N_0 \tag{1}$$

which for large values of j is *equidistant* and compatible with the Bekenstein spectrum[5] for the horizon area of spherically symmetric black holes. On the contrary, the full area spectrum in loop quantum gravity has an exponentially decreasing level distance.[4] We can interpret the large difference of the two spectra in the spherically symmetric and the non-symmetric regime as a *level splitting* familiar from the spectroscopy of atoms: breaking the spherical symmetry leads to a splitting of the levels resulting in an almost dense spectrum. A necessary requirement for this to happen is a huge degeneracy of the levels in the spherically symmetric sector which is also expected from thermodynamical considerations (black hole entropy). However, in quantum geometry spherically symmetric states are distributional and so their degeneracy is not well defined which prohibits a simple counting of states.

In specializing the framework to homogeneous or even isotropic states we have the basics of loop quantum cosmology.[6] In this case, the reduced manifold is a single point and so homogeneous states are defined purely in terms of scalar fields (point holonomies). For Bianchi models (anisotropic) we have three independent point holonomies, whereas for isotropic models there is only one point holonomy (however, in this case the concept of spin networks has to be generalized in order to describe all states[7]). Again, we can observe the phenomenon of *level splitting*, this time for the volume operator.[7] For isotropic states, the operator simplifies so much that the complete volume spectrum can be computed explicitly (V_0 is an arbitrary constant entering via a homogeneous auxiliary metric):

$$V_j = \gamma^{\frac{3}{2}} V_0^{-\frac{1}{2}} l_{\mathrm{P}}^3 \sqrt{j(j+\tfrac{1}{2})(j+1)} \quad , \quad j \in \tfrac{1}{2} N_0 \,. \tag{2}$$

The fact that homogeneous states are distributional also implies that there is a discrepancy between minisuperspace quantizations and approximations (weaves) in the full theory: even small inhomogeneous perturbations cause a transition to the full volume spectrum. This is in contrast to the treatment of inhomogeneities in more standard quantum cosmological models where symmetric and slightly perturbed geometries are smoothly connected.

In cosmological models there is a familiar procedure to study intrinsic dynamics by introducing the volume as internal time. The volume quantization in quantum geometry suggests that such a procedure in loop quantum cosmology leads to a discrete time. This can in fact be made more precise: using a quantization of the Hamiltonian constraint for cosmological models[8] and transforming to a dreibein representation leads to an interpretation of the Wheeler–DeWitt equation as a *discrete time* evolution equation[9] (rather than differential equation). In these models, one can also show that the *physical* volume spectrum is identical to the kinematical one; in particular, it is discrete.

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QUANTUM THEORY AND THERMODYNAMICS OF SCHWARZSCHILD BLACK HOLES

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Starting from a symplectic reduction of the pure Schwarzschild gravitational system in $D \geq 4$ space-time dimensions and an ensuing quantization in terms of the group $SU(1,1)$ yields a universal spectrum $A_{D-2} \propto n+k, n=0,1,\ldots,k \in (0,1]$ fixed, of the horizon area, and, because $A_{D-2}(M)$ is a function of the mass M, a corresponding spectrum of M. Attributing the $Z(2)$-valued degree of freedom "orientation" to each basic area quantum yields a degeneracy of the levels which, combined with the mass spectrum, implies the Bekenstein entropy and the Hawking temperature, up to a constant.

1 Mass spectrum in D space-time dimensions

As early as 1974 Bekenstein [1] – using Bohr-Sommerfeld-type arguments – suggested that the quantum mass spectrum of a Schwarzschild black hole in 4 space-time dimensions may be obtained by interpreting the area A of the horizon as an action variable which gets quantized like the angular momentum, namely proportional to a positive integer $n = 0, 1, \ldots$. As $A \propto M^2$ this implies a mass spectrum $M \propto \sqrt{n}$. In the meantime many authors have suggested different ways of obtaining such a spectrum[a].

A Dirac-type symplectic reduction of spherically symmetric (Schwarzschild) classical pure Einstein gravity [4,5,7] in D space-time dimensions leads to two canonically conjugate observables: mass M and the proper time τ of an observer at asymptotically flat spatial infinity. Quantizing the reduced phase space in the usual way - the mass operator $\hat{M}$ being the Hamilton operator - and imposing appropriate boundary conditions on the wave functions [6,3] with respect to τ yields the mass levels

$$M_n = \alpha_D\,(n+k)^{(D-3)/(D-2)}\,m_{P,D}\ ,\ n=0,1,\ldots,\ \alpha_D = O(1)\ , \tag{1}$$

where $k \in (0,1]$ is a fixed constant (see below) and $m_{P,D}$ Planck's constant in D-dimensional space-time. For $D = 4$ the old Bekenstein result is obtained. The mass spectrum (1) is equivalent to the horizon area spectrum

$$A_{D-2} = (n+k)\,a_{D-2}\ , \tag{2}$$

where a_{D-2} is the basic "Planck-sized" spherical horizon area element.

The ***Hilbert spaces*** associated with such systems may be obtained from a group theoretical quantization of the reduced phase space by using the area A_{D-2} and a canonically conjugate angle coordinate as variables [7]. The resulting Hilbert spaces are those of the positive discrete series of the unitary representations of the group $SO^{\uparrow}(1,2)$ or one of its (infinitely many) covering groups. The number k characterizes the representation.

[a] A list of those papers is provided in Refs. [2,3]

2 Level degeneracy due to the degree of freedom "orientation"

In order to obtain the well-discussed thermodynamical properties of Schwarzschild black holes (Bekenstein entropy, Hawking temperature etc.) the levels (1) or (2) must have the appropriate degeneracy d_n. A relation like $d_n = g^n, g > 1$, serves that purpose. The main question then is:

Which degrees of freedom are responsible for such a degeneracy ?

In Ref. [3] I suggested that the geometrical $Z(2)$-valued property ***orientation*** is the degree of freedom responsible for the required degeneracy! As spheres have two possible orientations it appears reasonable to attribute two Ising-type degrees of freedom to each basic area quantum a_{D-2}. This means that the n-th level (1) - or (2) - has the degeneracy

$$d_n = 2^n \; . \tag{3}$$

The well-known properties of the Ising model suggest to expect [3] a phase transition at very low (Hawking) temperatures, i.e. in the classical limit $\hbar \to 0$ or/and at very large masses, associated with a "spontaneous orientation" which essentially reduces the originally huge number of possible microscopic orientations to just two macroscopic classical ones!

3 Quantum statistics

The spectrum (1) combined with the degeneracy (3) leads - up to an overall normalization factor - to the Bekenstein entropy and to the Hawking temperature. The *canonical* partition function of the system has the very interesting property [8, 3] that it is the same as the *grand canonical* partition function of the primitive droplet nucleation model for 1st order phase transitions in $D-2$ space dimensions. This constitutes another close relationship to condensed matter physics.

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