ACTA PHYSICA AUSTRIACA / SUPPLEMENTUM V

PARTICLES, CURRENTS, SYMMETRIES

PROCEEDINGS OF THE
VII. INTERNATIONALE UNIVERSITÄTSWOCHEN
FÜR KERNPHYSIK 1968 DER KARL-FRANZENS-UNIVERSITÄT
GRAZ, AT SCHLADMING (STEIERMARK, AUSTRIA)
26th FEBRUARY—9th MARCH 1968

SPONSORED BY
BUNDESMINISTERIUM FÜR UNTERRICHT
BUNDESMINISTERIUM FÜR HANDEL, GEWERBE UND INDUSTRIE
THE INTERNATIONAL ATOMIC ENERGY AGENCY
STEIERMÄRKISCHE LANDESREGIERUNG AND
KAMMER DER GEWERBLICHEN WIRTSCHAFT FÜR STEIERMARK

EDITED BY
PAUL URBAN
GRAZ

WITH 36 FIGURES

1968

SPRINGER-VERLAG / WIEN · NEW YORK

Organizing Committee:
Chairman: Prof. Dr. PAUL URBAN
Vorstand des Institutes für Theoretische Physik, Universität Graz
Committee Members: Dr. H. KÜHNELT
Dr. H. LATAL
Proceedings: Dr. R. BAIER
H. J. FAUSTMANN
Secretary: A. SCHMALDIENST

Acta Physica Austriaca / Supplementum I
Weak Interactions and Higher Symmetries
published in 1964

Acta Physica Austriaca / Supplementum II
Quantum Electrodynamics
published in 1965

Acta Physica Austriaca / Supplementum III
Elementary Particle Theories
published in 1966

Acta Physica Austriaca / Supplementum IV
Special Problems in High Energy Physics
published in 1967

Library of Congress Catalog Card Number 68-55816

Printed in Austria

Titel-Nr. 9237

Preface

The subject of this year's conference, as of the six previous meetings, was again elementary particle physics. One of the main topics of current interest here is the understanding of strong interactions; some of the attempts to explain them are discussed at length in these proceedings, including current algebra, Regge pole theory and effective Lagrangians. On the other hand, the processes of weak interactions, too, are not yet fully understood, especially when strong interactions intervene, as in the radiative corrections to weak decays. Several other aspects of elementary particle physics are also treated in this volume, every one of them another step towards a thorough understanding of the subatomic world.

I want to take this opportunity to thank all the members of my staff for their assistance in organizing the meeting and for their help in preparing the manuscripts.

Graz, June 1968 **P. Urban**

Contents

Dear Colleagues, Ladies and Gentlemen:

Once again in the history of our annual International Meeting in Schladming we have started with the lectures before the official opening ceremony, on account of the carnival festivities going on here, which I hope you are enjoying. The word "history" I just used is not an overstatement since we are now meeting for the seventh time in Schladming and to many of you our symposium is quite familiar. These people especially, and all those who are participating for the first time, I would like to welcome most cordially and wish all of you two weeks of success in physics and perhaps also in skiing.

To our guests of honour, who show by their attendance the serious interest of the governmental and provincial authorities in our enterprise, I most respectfully extend my welcome. It is a great privilege for me to have the representative of the Ministry of Education with us; may I take the opportunity here to express our sincerest thanks for the generous support we have received over the years. Other grants which have contributed much to the successful organization of this meeting were given by the International Atomic Energy Agency and the Ministry of Trade and Industry. We are further indebted to the Provincial Government of Styria, whose representative I also welcome here, as well as the Chamber of Commerce. It gives us special pleasure to have among our guests of honour the Dean of the Philosophische Fakultät of the University of Graz. We are certain that through the scientific reputation of this meeting the name of our University is already well known to the scientific community. But also our host city, Schladming, and its beautiful surroundings become more familiar each year to a larger group of people, who meet with us here. May I take the opportunity now to thank especially the Mayor of the City, Director Laurich, both personally and as representative of the authorities of Schladming for their continued help throughout the years.

Science—and possibly also skiing—have again this year drawn to Schladming about 180 participants from 18 nations, a number which impressively shows the value of our meeting as a place for learning about new developments and discussing recent results. We are happy that we have succeeded in obtaining the cooperation of outstanding experts as lecturers and I want to thank them for coming here; I hope they will also benefit from their stay.

As expressed in the title of this year's meeting, "Particles, Currents, Symmetries", we shall be dealing with the world of elementary particle physics. Here especially, in the absence of a theory as powerful as quantum-electrodynamics, the strong and weak interaction processes have successively forced us to change our point of view in order to restore the agreement between experiment and theory. The latest such event, caused by the detection of CP violation processes, has again destroyed another supposed conservation principle. These

GAUGE PROPERTIES OF THE MINKOWSKI SPACE†

By

H. A. KASTRUP

Universität München, Sektion Physik, München,
Germany

I. Introduction

Gauge groups have been playing an important role in particle physics. Some examples are:

1. Phase transformations of states ($\phi \to e^{i\alpha}\phi$) or of non-observable fields ($\psi(x) \to e^{i\alpha}\psi(x)$). Such symmetry transformations are associated with the conservation of charge-like quantities such as, for instance, the baryon number.

2. Local gauge transformations $A_\mu(x) \to A_\mu(x) + \partial_\mu f(x)$, $\psi(x) \to e^{if(x)}\,\psi(x)$ in quantum electrodynamics which imply the conservation of the electric charge and which are considered to be intimately connected with the vanishing rest mass of the photon. These position dependent transformations further have the important consequence that all charged particles are coupled in a universal way [1].

3. Generalizations of these local gauge transformations to strong interactions (Yang-Mills-groups [2]) played an important role in the creation of the Eightfold Way [3]. There is, however, the problem that the associated vector particles, which take the place of the photon, have finite rest masses. Perhaps for this reason, and certainly because there are mass differences between members of the same multiplets, these symmetries are only approximately valid.

4. As a last example we mention the chiral group $\psi(x) \to e^{i\alpha\gamma_5}\,\psi(x)$, acting on spinor fields and being an exact symmetry only in the limit of (at least) vanishing pion masses [4].

I mention these different well-known examples of gauge transformations for the following reasons: They show that even approximate gauge groups which are broken by rest masses can be very useful for particle physics. We further notice that all the groups listed above transform only the state vectors or fields but not the coordinates x^i. One may, therefore, ask whether there are useful gauge groups, perhaps only approximate ones, which transform the coordinates $x = (x^0, \vec{x})$ of the Minkowski space, too.

By a gauge group of the Minkowski space we mean the following: We assume distances ds to be defined by the differential quadratic form

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \; . \qquad (1)$$

We take the differential form (1) because we want to allow for nonlinear transformations, in particular local ones. The structure of the Minkowski space is pre-

† Seminar given at the VII. Internationale Universitätswochen f.Kernphysik, Schladming,February 26-March 9,1968.

served by requiring the form (1) for all space-time points.

As a gauge transformation of the Minkowski space we denote any mapping $x^i \to x^{i'} = f^i(x)$, $i=0,1,2,3$, which multiplies the form (1) by a - in general position dependent - factor.

The continuous and discrete groups which induce such transformations in x-space were already determined in the last century. Liouville did this analysis for three [5] and S. Lie for arbitrary [6] dimensions [2]. We shall call the largest group of the Minkowski space, which has the property in question, the "full Liouville group of the Minkowski space". Its orthochronous proper part, i.e. that part of the group which is continuously connected with the identity transformation, consists of the following subgroups:

The orthochronous proper Poincaré group $D^{\uparrow}_{10}(a;\Lambda)_+$:

$$x^j \to \Lambda^j_{\ i}\, x^i + a^j, \quad j = 0,1,2,3,$$

$$ds^2 \to ds^2 \quad ; \qquad (2)$$

the dilatations $D_1(\alpha)$:

$$x^j \to e^\alpha x^j, \quad j = 0,1,2,3,$$

$$ds^2 \to e^{2\alpha} ds^2 \quad , \qquad (3)$$

and the special Liouville group $SL_4(c)$:

$$x^j \to RT(c)\, Rx^j = \frac{1}{\sigma(x)} (x^j - c^j x^2), \quad j=0,1,2,3,$$

$$ds^2 \to \frac{1}{\sigma^2(x)}\, ds^2 \quad , \qquad (4)$$

where

$$Rx^j = -\frac{x^j}{x^2} \; ; \; T(c)\, x^j = x^j + c^j \; ;$$

$$\sigma(x) = 1-2c.x + c^2x^2 \quad .$$

The full 15-parameter Liouville group L_{15} may be obtained from the proper transformations (2) - (4) by adjoining the usual parity transformation P and the time inversion T or, equivalently, the "length inversion" R and the discrete group $\tilde{R}\, x^i = x^i/x^2$.

The group L_{15} is isomorphic to the group O(2,4). We notice that the special Liouville group SL_4 and the length inversion R are not well-defined in x-space, because some light cones $((x^j-c^j/c^2)(x_j-c_j/c^2)c^2 = \sigma(x)=0$ or $x^2 = 0$ respectively) are mapped into infinity. We shall deal with this problem later on, when we discuss the geometrical meaning of the transformations (3) and (4) in detail.

II. The Classical Free Relativistic Particle.

In order to illustrate the "conserved" quantities associated with the dilatations and the special Liouville group, we consider the very simple case of a classical relativistic free particle characterized by the relations

$$p^o = + (\vec{p}^2 + m^2)^{1/2} \; , \quad \vec{r} = \frac{\vec{p}}{p^o}\, x_o + \vec{a} \; . \qquad (5)$$

The constants of motion associated with the homogeneous Lorentz group are

$$\vec{m} \equiv \vec{r} \times \vec{p} = \vec{a} \times \vec{p} \; , \; \vec{n} \equiv \vec{r}\, p_o - x_o\, \vec{p} = p_o\, \vec{a} \; . \qquad (6)$$

The "dilatation momentum" is [7]

$$s \equiv p^o x^o - \vec{r}.\vec{p} = \frac{m^2}{p_o} x_o - \vec{a}.\vec{p} \quad , \tag{7}$$

and the corresponding "Bessel-Hagen momenta" of the special Liouville group are [7]

$$h_o \equiv 2\, x_o\, x^j p_j - x^2 p_o = \frac{m^2}{p_o} x_o^2 + \vec{a}^2\, p_o \ ,$$

$$\vec{h} \equiv 2\vec{x}\, x^j\, p_j - x^2 \vec{p} =$$

$$= \frac{m^2}{p_o} \left(\frac{\vec{p}}{p_o} x_o^2 + 2\vec{a}\, x_o\right) + \vec{a}^2\vec{p} - 2(\vec{a}.\vec{p})\, \vec{a} \quad . \tag{8}$$

We see that the gauge groups $D_1(\alpha)$ and $SL_4(c)$ lead to conserved quantities only in the limits $m \to 0$ or $p_o \to \infty$. Therefore the situation here is very similar to those of the last two examples discussed in the Introduction.

It is interesting to notice that in the limits $m \to 0$ or $p \to \infty$ all of the 15 conserved quantities listed above may be expressed by the two independent vectors $\vec{p}$ and $\vec{h}$:

$$p_o = + (\vec{p}^2)^{1/2} \ , \ h_o = + (\vec{h}^2)^{1/2} \ , \quad 2s^2 = p.h \ ,$$

$$\vec{h} \times \vec{p} = 2s\, \vec{m} \ , \quad p_o \vec{h} - h_o \vec{p} = 2\, s\, \vec{n} \ . \tag{9}$$

Thus the quantities s, $\vec{m}$ and $\vec{n}$ are determined by $\vec{h}$ and $\vec{p}$ up to a relative sign.

The relations (9) bear a close resemblance to the following commutator of the generators P_j and K_ℓ of the groups $T_4(a)$ and $SL_4(c)$:

$$[P_j, K_\ell] = 2i(g_{j\ell} D - M_{j\ell}) \ , \tag{10}$$

where D and $M_{j\ell}$ are the generators of the groups $D_1(\alpha)$ and $L_+^\uparrow$. Because of $K_j = RP_jR$ one can generate the full Liouville group by the translations T_4, the length inversion R, and the time inversion T.

Under the discrete groups P, T and R the above constants of motion transform as follows:

$$P : s \to s \ , \quad \vec{h} \to -\vec{h} \ , \ h_o \to h_o \ ;$$

$$T : s \to -s \ , \quad \vec{h} \to -\vec{h} \ , \ h_o \to h_o \ ;$$

$$R : \vec{m} \to \vec{m} \ , \ \vec{n} \to \vec{n} \ , \ s \to -s \ , \ h_j \to p_j, \quad p_j \to h_j \ ,$$

$$j = 0,1,2,3,$$

In addition we have $\tilde{R} = PTR$.

We see that the transformation R leaves the energy p_o positive because $h_o \geq 0$. The same is true for the group SL_4: the quantity h_o stays positive under translations (see eq. (8)). Because of eq. (4) this means that p_o stays positive under special Liouville transformations.

Remark:

For any free elementary excitation with a dispersion law

$$E = A\, p^\alpha \ , \tag{11}$$

where A and α are constants, the Bessel-Hagen momentum $\vec{h}$, and because of the eqs. (9) the quantities h_o, s, $\vec{m}$ and $\vec{n}$, too, are constants of the free motion [8]. This can be seen by replacing $(x^o, \vec{x})$ in the above formulae by $(vt, \vec{x})$, where $\vec{v} = \partial E/\partial\vec{p}$. In particular we have all these constants of motion for the nonrelativistic particle with $E^2 = p^2/2m$. Thus, all these free motions are invariant under the group [9] O(2,4).

However, the situation is different, if there is an energy gap in the dispersion law: $E = A(p^\alpha + B)^\beta$. The "gap" parameter B introduces a fixed length into the theory, which breaks the dilatation and Liouville symmetries.

III. Geometrical Interpretation

The geometrical interpretation of the dilatations is relatively easy: they map a given length ds onto another length $e^\alpha ds$. Therefore, physical systems which contain a fixed length can only be approximately invariant under the group [10].

I think that the interpretation of the special Liouville group is very similar: it induces local geometrical gauge transformations instead of the global dilatations (the gauge factor $\sigma(x)$ in eq. (4) is position dependent but e^α is not!). There are, however, some rather complicated problems associated with the special Liouville group in x-space. I shall discuss three of them:

1. It was already pointed out in the Introduction that the Liouville group is not well-defined in x-space because the mapping (4) is not one-to-one. This feature can be taken care of quite easily by separating a Poincaré invariant unit of length κ^{-1} (scale factor) from the position coordinates x^j. One defines

$$x^j = \frac{\eta^j}{\kappa}\ , \quad j = 0,1,2,3, \qquad (12)$$

and the spurious coordinate λ by

$$\kappa\,\lambda = \eta^j\,\eta_j \quad . \qquad (13)$$

For each point in space-time the four numbers η^j characterize its position and κ the unit of length employed at this point [7].

The coordinates η^j transform like the x^j under the Poincaré group ($\eta^j \to \eta^j + a^j\kappa$, $\eta^j \to \Lambda^j_i\,\eta^i$). Under dilatations we have

$$\eta^j \to \eta^j\ , \ \kappa \to e^{-\alpha}\kappa,\ \lambda \to e^\alpha\lambda \quad , \qquad (14)$$

and the length inversion R gives

$$\eta^j \to \eta^j\ , \quad \kappa \to -\lambda\ , \quad \lambda \to -\kappa \quad . \qquad (15)$$

Since the group R generates the special Liouville group, the relations (15), which leave the position unchanged, make it even more evident that we are dealing with gauge transformations.

The full 15-parameter Liouville group leaves the quadratic form $\eta_j\,\eta^j - \kappa\lambda$ invariant, which shows its isomorphy to the group O(2,4).

The length ds can be expressed in terms of the new coordinates by

$$ds^2 = \quad dx^j\,dx_j \quad = \frac{1}{\kappa^2}\,(d\eta^j\,d\eta_j - d\kappa\,d\lambda)\ . \qquad (16)$$

with the subsidiary condition (13).

2. In the framework of the Poincaré group plus dilatations we can define space-like and time-like separations by the sign of the form $(x_1 - x_2)^2$. This is no longer true if we include the special Liouville group. The reasons are the following [7]:

a) The transformations (4) are nonlinear in x-space. From all that we know from differential geometry and general relativity we then have to use differential forms in order to define distances, not global ones like $(x_1-x_2)^2$.

b) Because the transformations (4) are not well-defined in x-space, it does not make much sense to use the x-coordinates themselves in order to define space- and time-like separations.

c) The sign of the global form $(x_1-x_2)^2$ is not invariant [7] under the transformations (4).

For all these reasons we have to define space- and time-like separations in the space of the coordinates η^j, κ and λ. Eq. (14) suggests how to do this: Locally we still can define space- and time-like separations in a Liouville invariant way by the sign of ds^2 - see eq. (4) - , or because $\kappa^2 \geq 0$ by the sign of $d\eta_j\, d\eta^j - d\kappa\, d\lambda$. Since all the transformations induced by the Liouville group are linear in η^j, κ and λ, we can define space- and time-like separations by the sign of the invariant form

$$(\eta_1-\eta_2)^j\,(\eta_1-\eta_2)_j - (\kappa_1-\kappa_2)\,(\lambda_1-\lambda_2)\ . \tag{17}$$

This definition seems to be the natural generalization of the usual one by means of the sign of $(x_1-x_2)^2$.

Remark: Because of the property c) the transformations (4) seem to violate a theorem by Zeeman [11] on the causal automorphisms of the Minkowski space. However, this is not the case, because Zeeman assumes the mappings of the Minkowski space he is dealing with to be one-to-one. The transformations (4) do not fulfill this assumption.

3. The conventional interpretation of the special Liouville group in x-space is that it represents a transformation from a system at rest to a relativistic uniformly accelerated one [12] (hyperbolic motion). I do not consider this interpretation to be a fruitful or even a right one for the following reasons:

a) The interpretation applies only to the spatial parameters c^j, j=1,2,3, not to c^o.

b) The group is not well-defined in x-space.

c) Invariance under translations and the special Liouville group implies [13] invariance under dilatations (eq. (10)). This means that uniformly accelerated systems would have continuous rest masses and continuous energy spectra. However, there is no evidence that the discrete energy spectra of atoms or the rest masses of elementary particles become continuous under uniform accelerations.

d) The group velocity of the wave packets formed by the eigenfunctions of the generators K_j of the special Liouville group describes linear motions [14] , whereas the phase velocity describes hyperbolic motions. From all that we know from quantum mechanics, the group velocity describes the motions of particles, not the phase velocity.

e) We have seen that the Liouville symmetry is not confined to the relativistic case, but that it occurs in any situation where there are elementary excitations of the form (11). Whereas the "gauge"-interpretation can be applied to all these situations, the "acceleration"-interpretation cannot.

IV. The Nonrelativistic Case

It has already been mentioned that the nonrelativistic free motion is invariant under the full Liouville group O(2,4). The crucial difference between the relativistic and nonrelativistic case is in the way the symmetry is broken: in the relativistic case many interesting interaction Lagrangians are dilatation and Liouville invariant [15], but almost all nonrelativistic potentials are not [9]. Relativistically, these gauge symmetries are in general broken by the kinetic mass terms, not by the interaction Lagrangians, but nonrelativistically, the symmetries are broken by the potentials, not by the kinetic terms.

Several applications of the broken dilatation and Liouville symmetries for relativistic systems have been discussed elsewhere [16]. In the following we shall discuss some features of the nonrelativistic case [9],[17]:

1. The conserved quantities associated with the O(2,4)-invariance of the free particle may be obtained by the replacement $x^o \to y^o = vt$, for instance

$$s \equiv 2Et - \vec{r}\cdot\vec{p} \quad , \quad \vec{h} = 2\,\vec{x}\,s - (v^2t^2 - \vec{x}^2)\,\vec{p} \; ,$$

where $E = 1/2\, \vec{v}\vec{p}$. The energy E itself is not among the 15 constants of motion associated with the group O(2,4). It is a function of $p^o = p$. The same is true for the "Galilei"-momentum $\vec{g} = v^{-1}\,\vec{n}$, where $\vec{n}$ is the "Lorentz" - momentum $\vec{n} = \vec{x}\,p - vt\,\vec{p}$. More details are contained in ref. 9 .

2. It is very illustrative to analyze the additional constraints imposed on elastic scattering by the assumption of conserved dilatation- and Bessel-Hagen momenta. We consider the scattering of two particles with equal masses in the c.m. system with the c.m.coordinate $\vec{R} = \vec{r}_1 + \vec{r}_2 = 0$.
At large negative times t we have

$$\vec{r}_i = \pm \frac{\vec{p}}{m}\, t \pm \vec{a} \; , \qquad i=1,2 \; .$$

For large positive times we get - as a consequence of energy, momentum, and Galilei-momentum conservation- :

$$\vec{r}\,'_i = \pm \frac{\vec{p}\,'}{m}\, t \pm \vec{a}\,' \; , \qquad i=1,2; \quad |\vec{p}\,'| = |\vec{p}| \; .$$

The conservation of angular momentum implies that the perpendicular projection of $\vec{a}\,'$ on $\vec{p}\,'$ is the same as that of $\vec{a}$ on $\vec{p}$:

$$\vec{a}\,' \times \vec{p}\,' = \vec{a} \times \vec{p} \quad .$$

There are no constraints on the parallel projection $\vec{a}\,' \cdot \vec{p}\,'$ so far, and this freedom accounts for the possibility of positive and negative time delays during the interaction. However, if we require the conservation of the total dilatation momentum $s_1 + s_2$ we get

$$\vec{a}\,' \cdot \vec{p}\,' = \vec{a} \cdot \vec{p} \quad . \qquad (18)$$

Eq. (18) means that there can be no time delays if the total dilatation momentum is conserved. This is, of course, a severe restriction which is not valid for most of the nonrelativistic interactions. In quantum mechanical language the vanishing time delay means [18] that $d\delta_\ell / dp = 0$, where δ_ℓ is the usual phase shift. Thus only constant phase shifts are allowed by the conservation of the total dilatation momentum. The phase shifts do not have to vanish, however.

One can check that the conservation of the total Bessel-Hagen momenta is fulfilled automatically in our example of elastic scattering if all the other total momenta are conserved.

3. It is interesting to see how the dilatation- and Liouville symmetries can be broken in the nonrelativi-

stic case in order to describe more realistic interacting systems. Because of the qualitative difference between relativistic and nonrelativistic systems the following procedure may not be very useful for the relativistic case [19].

We consider the simplified situation of a stationary system (t=0). Then we have only the Euklidian group, the dilatations and a 3-dimensional Liouville group. A free nonrelativistic spinless quantum mechanical system is described by states $\phi(\vec{p})$ which form a Hilbert space with the scalar product

$$(\phi_1, \phi_2) = \int d^3\vec{p}\ \phi_1^*(\vec{p})\ \phi_2(\vec{p}) \ . \qquad (19)$$

In the space of these states we have a unitary representation of our groups. The Hermitian generators of this representation are given by

$$P^j = p^j, \quad M^{jk} = i^{-1}(p^j\partial_k - p^k\partial_j) \ ,$$

$$\partial_k = \frac{\partial}{\partial p^k} \ ; j,k = 1,2,3 \qquad (20a)$$

$$K^j = 3\partial_j + 2p^k\partial_k\partial_j - p^j\Delta \quad , D = i(\frac{3}{2} + p^j\partial_j) \ . \quad (20b)$$

It is important for later to mention that the first term in K^j and D is typical for the behaviour of the functions $\phi(\vec{p})$ themselves under finite transformations, for inst. $\phi'(p) = e^{3/2\ \alpha}\ \phi(e^\alpha\ \vec{p})$, - "intrinsic" parts of D and K^j -, whereas the rest is a consequence of the transformations of their argument $\vec{p}$ - "orbital" parts - or of the argument of their Fourier transform $\tilde{\phi}(\vec{x})$. We write down only one commutation relation:

$$[P^j \ , K^\ell] = 2i\ (\delta^{j\ell}\ D - M^{j\ell}) \ . \qquad (21)$$

The operator $H_o = 1/2m\ p^2$ for the free particle energy belongs to the enveloping algebra of the above Lie-algebra. Because of

$$e^{i\alpha D}\ H_o\ e^{-i\alpha D} = e^{-2\alpha}\ H_o$$

the spectrum of H_o is continuous.

Let us consider a system with interactions now. We assume the interaction to be described by a potential V in the energy operator $H = H_o + V$, where V is a function of the generators P^k and K^ℓ (because of eq. (21) all the other generators may be expressed by these). If we confine ourselves to velocity independent and rotational invariant potentials we have $V = V(\vec{K}^2)$. Now, if the dilatations are to be a symmetry group, then V has to transform in the same way as H_o. This implies $V \sim (\vec{K}^2)^{-1}$. Nature obviously is much richer than this single potential would allow her to be. More generally, invariance under finite dilatations implies a continuous energy spectrum which, however, is not observed in most of nonrelativistic atomic physics.

Therefore, we have to break the dilatation symmetry. We notice that the above constraints on the spectrum of H are no longer valid if D would commute with H. This happens if we drop the second (orbital) term of D in eq. (20b). In order to describe [20] this procedure mathematically we write $D_\varepsilon = i(3/2 + \varepsilon p^j\partial_j)$, where $0 \le \varepsilon \le 1$. $D_{\varepsilon=o}$ is a multiple of the unity operator and therefore commutes with H.

However, because of the closure relation (21) we cannot keep K^j as it is in eq. (20b) - we want to keep the momenta $\vec{p}$. We have to drop the orbital part of K^j, too, in order to get $D_{\varepsilon=o}$ on the right hand side of eq. (21). With

$$K_\varepsilon^j = 3\,\partial_j + \varepsilon(2p^k\partial_k\partial_j - p^j\,\Delta)$$

we get instead of eq. (21):

$$[P^j, K_\varepsilon^\ell] = 2i(\delta^{j\ell}\, D_\varepsilon - \varepsilon\, M^{j\ell}) \quad . \qquad (22)$$

In the limit $\varepsilon=0$ we have $Q^j = \frac{1}{3i} K^j_{\varepsilon=o}$ and $I = \frac{2}{3i} D_{\varepsilon=o}$, where Q^j is the usual quantum mechanical position operator and I the identity operator. Thus, the occurrence of the position operator is a consequence of our breaking of the dilatations,and quantum mechanics appears as a broken Liouville symmetry [17]. The fundamental relation (21) of the group goes over into the usual canonical commutation relation.

We still have to deal with the following problem: the quantities $D_{\varepsilon=o}$ and $K^j_{\varepsilon=o}$ are both purely imaginary. This means that

$$u(\alpha) = e^{+i\alpha D_{\varepsilon=o}} \quad , \quad u(\vec{c}) = e^{ic_j K^j_{\varepsilon=o}}$$

are no longer unitary if α and c_j are kept real. We can avoid this by considering $\alpha = \alpha(\varepsilon)$ and $c_j = c_j(\varepsilon)$ as complex-valued functions of ε which are real for $\varepsilon=1$ and imaginary for $\varepsilon=0$. Then $u[\alpha(\varepsilon=o)]$ and $u[\vec{c}(\varepsilon=o)]$ are unitary. In this way the dilatations degenerate into a phase transformation.

Footnotes and References

1. E. P. Wigner, Proc. Nat. Acad. Sciences (USA) 38, 449 (1952).
2. C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1956).

3. M. Gell-Mann and Y. Ne'eman, The Eightfold Way (W. A. Benjamin, Inc., New York, 1966).
4. See, for instance, the lecture of F. Gürsey during the school.
5. J. Liouville, editorial note VI in G. Monge's book Application de l'Analyse à la Geométrie (Bachelier, Paris, 1850).
6. S. Lie and F. Engel, Theorie der Transformationsgruppen (B. G. Teubner, Leipzig, 1893) Vol.III, Chaps 17 and 18.
7. H. A. Kastrup, Phys. Rev. 150, 1183 (1966).
8. H. A. Kastrup, Nuovo Cim. 48, 271 (1967).
9. H. A. Kastrup, Gauge Properties of the Galilei Space, to be published.
10. H. A. Kastrup, Nucl. Phys. 58, 561 (1964).
11. E. C. Zeeman, Journ. Math. Phys. 5, 490 (1964).
12. See, for instance, T. Fulton, F. Rohrlich and L. Witten, Nuovo Cim. 26, 652 (1962).
13. We always assume the generators we are dealing with to be represented by self-adjoint operators which can be integrated to continuous finite unitary representations. See, however, M. Flato, and D. Sternheimer, Phys. Rev. Letters 16, 1185 (1966).
14. H. A. Kastrup, Phys. Rev. 143, 1021 (1966).
15. H. A. Kastrup, Phys. Letters 3, 78 (1962).
16. H. A. Kastrup, lectures given at this school.
17. H. A. Kastrup, Quantum Mechanics as a Broken Symmetry, to be published.
18. E. P. Wigner, Phys. Rev. 98, 145 (1955); see also ref. 10.
19. Contractions of the relativistic Liouville group were first introduced by I. E. Segal, Duke Math. J. 18, 221 (1951). Their physical significance was

20. Technically we are dealing with contractions of Lie-algebras: I. E. Segal, ref. 19; E. Inönü and E. P. Wigner, Proc. Nat. Acad. Sciences (USA) 39, 510 (1953) and 40, 119 (1954).

SOMETHING NEW IN COSMIC RAYS †

By

S. L. GLASHOW

Department of Physics, Harvard University, USA

The remarks I shall make are inspired by the beautiful recent cosmic ray experiment done in Utah by Keuffel et al. reported in the Christmas 1967 issue of Physical Review Letters. The "sec Θ law" which they observe to fail was deduced by Barrett et al. in Review of Modern Physics 1952. Whatever I say in the way of controvertial explanation of the experiment is due to Curtis Callan and myself.

The Experiment

Energetic cosmic radiation found in deep mines is usually assumed to consist of muons, for no other particle of sufficient penetrating power is known. The vertical intensity of this radiation as a function of rock depth has been very well studied. Muons of given energy have a more or less well defined range in

† Seminar given at the VII. Internationale Universitätswochen für Kernphysik, Schladming, February 26 - March 9, 1968.