SOME ASTROPHYSICAL SPECULATIONS ABOUT SCALING IN VERY HIGH ENERGY COLLISIONS

H. A. KASTRUP
Universität München, Sektion Physik, Munich, Germany

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The paper consists of two parts: First, equilibrium distribution functions are discussed under the assumption that binary collisions are dilation on conformally invariant. Second, supposing that 'scaling' in several high energy collisions is associated with dilation or (and) conformal invariance at short distances, some speculative astrophysical implications for a hot expanding universe are presented.

When asymptotic scaling of collision amplitudes, as a consequence of dilatation and conformal invariance at short distances, was suggested [1] some years ago, no clearcut experimental information was available in order to test this hypothesis and there was also the problem of selecting the right scaling variables. In the meantime, due to the theoretical work by Bjorken, Feynman, Mack, Wilson and many others [2, 6], much more is known about the dynamical background of scaling and about the right choice of the scaling variables. The main experimental support comes from deep inelastic electron-nucleon scattering [3] and from 'inclusive' hadronic reactions [4]. Though there are still many open problems, it seems that scaling and dilatation and conformal invariance at short distances will play an important part in high-energy physics [5].

Suppose now that scaling of high energy collision amplitudes is associated with asymptotic dilatation and conformal invariance at short distances - that this is so in deep inelastic e-N scattering was shown [6] by Mack - and suppose further that this implies, in a sense to be specified below, the approximate conservation of dilatation and (or) conformal momenta [7] in the reactions in question. Then, according to our knowledge about relativistic Boltzmann equations [8], there should be equilibrium solutions of this equation in flat space such that the exponent of the distribution function is linearly related to these momenta.

Now, if the very early universe was a very hot one, with temperatures in the high energy region (10^{13} degree Kelvin and higher) and if approximate scaling for different processes was present (which is likely) and *if* it indeed led to distribution functions of the kind just mentioned (which is not so obvious!), then this scaling would have and might still have an tremendous influence on the expansion of the universe which could have gone much beyond its gravitational one!

Obviously there are many 'ifs' and accordingly our paper is separated into two parts: We first discuss some statistical properties of systems with binary dilatation and conformally invariant collisions. We shall see that conservation of dilatation momentum leads to Milne's universe [9] and conservation of the conformal momenta to a new isotropic but not homogeneous model. We then discuss several reactions with scaling, which, after the 'big bang', *might* have caused at least a part of the distribution and expansion of matter in the universe. The first part is not speculative, but the second one is!

1. In the first part we can use the general results [10] of Ehlers, Geren and Sachs, applying them to the flat Minkowski space. According to those results, a locally isotropic distribution function $f(x, p)$ for particles with vanishing or negligible rest masses has the form *

$$f(x, p) = g[\xi^{\mu}(x) p_{\mu}] \quad ,$$

where $\xi^{\mu}(x)$ is a conformal Killing vector defining a velocity field $u^{\mu}(x)$, $u^{\mu}u_{\mu} = c^2$, in terms of $\xi^{\mu} = (\exp U(x))u^{\mu}$, where $U(x)$ is some function. In order to simplify our discussion technically, we shall talk explicitly only about Boltzmann distributions, the generalizations to Einstein-Bose and Fermi-Dirac statistics are straightforward, using the results of ref. [8].

1.1. Dilatations. The transformations

$$D(\alpha): x^{\mu} \rightarrow \hat{x}^{\mu} = e^{\alpha} x^{\mu}, \quad \alpha \text{ real},$$

give the conformal Killing vector

* We use the metric $x^2 = x^{\mu}x_{\mu} = (x^0)^2 - (\boldsymbol{x})^2$, $x^0 = ct$.

$$\xi^{\mu}_{D}(x) = (\partial\hat{x}^{\mu}/\partial\alpha)_{\alpha=0} = x^{\mu} \qquad (1)$$

and we obtain the equilibrium distribution function

$$f(x, p) = F_D \exp\left[-\frac{p^{\mu} u_{\mu}(x)}{kT(x)}\right] , \qquad (2)$$

$$u^{\mu}(x) = \frac{cx^{\mu}}{(x^2)^{1/2}} , \qquad x^2 > 0 ,$$

$$kT(x) = \frac{c}{\hat{\beta}_D (x^2)^{1/2}} , \qquad F_D, \hat{\beta}_D = \text{const.}$$

The invariant density $n(x)$, defined by

$$n^{\mu}(x) = \int \frac{d^3p}{p_0} p^{\mu} f(x, p) = n(x)\ u^{\mu}(x) ,$$

is calculated to be

$$n_D(x) = 8\pi F_D (kT(x))^3; \qquad n_D(x=0, t) = \frac{8\pi F_D}{c^2(\hat{\beta}_D t)^3}. \qquad (3)$$

The energy-momentum tensor $T^{\mu\nu}(x)$, defined by

$$T^{\mu\nu}(x) = \int \frac{d^3p}{p_0} p^{\mu} p^{\nu} f(x, p) =$$

$$= (\hat{\mu} + \hat{p}/c^2) u^{\mu} u^{\nu} - g^{\mu\nu}\hat{p} ,$$

where $\hat{\mu}(x)$: proper energy density, $\hat{p}(x)$: proper pressure and $3\hat{p} = \hat{\mu}c^2$ for vanishing rest masses, is determined by

$$\hat{p}_D(x) = 8\pi c^2 F_D (kT(x))^4 . \qquad (4)$$

Because $u^{\mu}(x) = c\,\partial^{\mu}(x^2)^{1/2}$, the flow associated with the velocity field is hypersurface orthogonal. We further have

$$u_{\mu;\nu} \equiv \partial_{\nu} u_{\mu}(x) = \frac{c}{(x^2)^{1/2}} (g_{\mu\nu} - \frac{1}{c^2} u_{\mu} u_{\nu}) \qquad (5)$$

and the expansion velocity θ has the value $3c/(x^2)^{1/2}$. The acceleration $u_{\mu} = u_{\mu;\nu}u^{\nu}$ vanishes.

Transformation to comoving coordinates is obtained by the substitution

$$x^0 = \tau \cosh w , \qquad r = \tau \sinh w , \qquad (6)$$

$$\theta \to \theta , \qquad \varphi \to \varphi , \qquad \tau = (x^2)^{1/2} ,$$

$$ds^2 = d\tau^2 - \tau^2 [dw^2 + \sinh^2 w (d\theta^2 + \sin^2\theta\ d\varphi^2)] .$$

In the comoving system we have $\bar{u}^{\mu} = cg^{\mu}_{0}$.

We see that the system so obtained is just Milne's universe [9], extensively discussed in the literature [11]. In our case we obtain it from the requirement that the dynamics of binary collisions is invariant under dilatations.

1.2. Special conformal transformations:

$$SC(c):\ x^{\mu} \to \hat{x}^{\mu} = (x^{\mu} - c^{\mu}x^2)/\sigma(x;c) ,$$

$$\sigma(x;c) = 1 - 2c\cdot x + c^2 x^2 . \qquad (7)$$

Here we get the four conformal Killing vectors

$$\xi_{\alpha} = (\xi^{\mu}_{\alpha}) , \qquad \xi^{\mu}_{\alpha}(x) = 2x^{\mu} x_{\alpha} - g^{\mu}_{\alpha} x^2 ,$$

$$\alpha, \mu = 0, 1, 2, 3 .$$

Let b be a time-like constant unit vector: $b^2 = 1$, then we can form (for $x^2 \neq 0$) the time-like vector $\xi^{\mu}_{c} = \xi^{\mu}_{\alpha} b^{\alpha}$, $\xi^2_c = (x^2)^2$. In this case equilibrium distribution function, velocity field and temperature are given by

$$f(x, p) = F_c \exp[-u^{\mu}(x) p_{\mu}/kT(x)] ,$$

$$u^{\mu}(x) = -cx^2\,\partial^{\mu}(b\cdot x/x^2) ,$$

$$kT(x) = c/\hat{\beta}_c x^2 , \qquad F_c,\ \hat{\beta}_c = \text{const.} \qquad (8)$$

Proper density $n_c(x)$ and proper pressure $\hat{p}_c(x)$ are calculated to be

$$n_c(x) = 8\pi F_c (kT)^3 , \quad \hat{p}_c(x) = 8\pi c^2 F_c (kT)^4 . \qquad (9)$$

Differentiation with respect to x^{ν} gives

$$u_{\mu;\nu}(x) = \frac{1}{c^2}\dot{u}_{\mu} u_{\nu} + \frac{2x\cdot b}{x^2}(g_{\mu\nu} - \frac{1}{c^2} u_{\mu} u_{\nu}) ,$$

$$\dot{u}_{\mu} = u_{\mu;\nu}u^{\nu} = \frac{2}{x^2}(x\cdot b\ u_{\mu} - cx_{\mu}) , \qquad (10)$$

i.e. in this case the acceleration does *not* vanish! Choosing $b = (1,0,0,0)$, we have $u^{\mu} = -cx^2\,\partial^{\mu}(x^0/x^2)$ and the inversion

$$\tau = x^0/x^2, \quad \rho = r/x^2, \quad \theta \to \theta, \quad \varphi \to \varphi, \qquad (11)$$

leads to comoving coordinate frames:

$$u^{\mu} \to \bar{u}^{\mu} = c(\tau^2 - \rho^2) g^{\mu}_{0} , \qquad (12)$$

$$ds^2 = \frac{1}{(\tau^2 - \rho^2)^2} [d\tau^2 - d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta\ d\varphi^2)] .$$

We see from eq. (12) that the system obtained is isotropic, but not homogeneous, i.e. it is not a Friedman universe with a Robertson-Walker metric!

Let us discuss a simple but important consequence: Suppose there is an observer at ($t = t_0$, $x = 0$) $\leftrightarrow$ ($\tau_0 = 1/t_0$, $\rho = 0$). Suppose further that light was emitted at $(t, x) \leftrightarrow (\tau, \rho)$ with wavelength λ, arriving at $x = 0$ at time $t_0 = t + r/c$.

Then the red shift $z = (\lambda_0 - \lambda)/\lambda$ is calculated to be

$$\rho = \tfrac{1}{2}\tau_0 z \quad , \qquad \tau = \tfrac{1}{2}\tau_0(z+2) \quad , \qquad (13)$$

or, in terms of the Minkowski coordinates r, t and t_0:

$$r = \tfrac{1}{2}c\,\frac{z}{1+z}\,t_0 \quad , \qquad t = \tfrac{1}{2}\,\frac{2+z}{1+z}\,t_0 \quad . \qquad (14)$$

For $z \ll 1$ the first one of the eqs. (14) reduces to Hubble's law $r = (c/H_0)z$, $H_0 = 2/t_0$. For $z \approx 1$, however, Hubble's law would give distances larger than the exact relation (14) does!
Because of

$$1/x^2 = (1+z)/c^2 t_0^2 \quad , \qquad (15)$$

which follows from eq. (14), we can, in principle, determine the temperature $T(x)$, the density $n_c(x)$, the pressure $\hat{p}_c(x)$ and the energy density $\hat{\mu}(x)$ by redshift measurements *alone*, if we know t_0 and the constants of F_c and $\hat{\beta}_c$ (which may be gotten by local measurements!)! For a black body gas with two internal degrees of freedom we have $F_c = 2/(2\pi\hbar)^3$.

2. Astrophysical speculations. The two expanding systems of ultrarelativistic particles described above suggest very much to compare them with our expanding universe. As the features of the first one (associated with dilatations) are well-known, let us confine the discussion to the second one:

2.1. Locally, i.e. for $r^2 \ll c^2 t^2$, the system is isotropic *and* homogeneous and Hubble's law is valid as mentioned above. The local Robertson-Walker metric has an expansion function $R = R_1 t^2$, R_1 = const. This function is *not* a solution of Friedman's equation [11], but can be shown to be a solution of the corresponding Brans-Dick equation [12] with $\omega = -\frac{3}{2}$ (see ref. [13]), vanishing spatial curvature and the Brans-Dicke scalar

$$1/\varphi(t) = (1/\varphi_1)t^6 \quad , \qquad \varphi_1 = 2^8\pi^2 F_c/9c^9\hat{\beta}_c^4 \quad .$$

2.2. Whereas the Einstein-DeSitter model [11] of the universe (which takes gravitation into account) has a time decrease of the density $n(x)$ given by $n(x)t^2$ = const., according to eqs. (8) and (9) the densities in our model decrease much faster, overtaking the gravitational decrease. This may be one reason for the model being of some relevance!

2.3. For redshifts of the order 1, the model is no longer homogeneous and Hubble's law is modified. If the modified version (14) has anything to do with, e.g. quasars, they would not be so far away as Hubble's law suggests and, on the other hand, because we have $\hat{\mu}(r,t) \propto (1+z)^4$ for the invariant energy density, the enormous energies and the deviations from uniform distribution [14] associated with quasars *may* have something to do with our model and the physical mechanisms which produce it! We are not suggesting that the properties of quasars can be understood completely in the framework of our model, nevertheless this field seems to be an interesting candidate for further investigations.

2.4. The most important physical problem in the context of our considerations is the question: which specific dynamical mechanisms could be able to give rise, in a reasonable approximation, to the type of distribution functions we have been discussing. In other words, are there high energy processes which are approximately dilatation or (and) conformally invariant and lead to the above systems? Presently we are not able to give a clearcut answer. However, there are several theoretical and experimental hints which suggests that one should take the above statistical systems seriously. Our procedure in this context is the following: we list several important reactions with cross sections of a form which can be related to an effective dilatation and (or) conformally invariant 'potential' like, e.g. the Coulomb potential, or a Born approximation in kinematical regions where all rest masses are negligible and where we can have approximate conservation of dilatation and (or) conformal momenta:

2.4.1. Purely electromagnetic interactions in lowest order:
a) Compton scattering off electrons for $s, |t| \gg m_e^2$:

$$\frac{d\sigma}{d|t|} = \frac{2\pi\alpha^2}{s^2}\left(\frac{s}{|t|} + \frac{|t|}{s}\right) \quad ,$$

$$s = (p_e + p_\gamma)^2 \quad , \qquad t = (p_e - p_e')^2 \quad .$$

b) Electron-electron scattering for s, $|t| \gg m_e^2$:

$$\frac{d\sigma}{d|t|} = \frac{2\pi\alpha^2}{t^2 s^2}(t^2 + 2st + 2s^2) \quad .$$

Higher order will introduce 'non-scaling' corrections, but we have at least an approximate scaling.

2.4.2. Inclusive semihadronic processes. The process e + N → N plus anything does not only show scaling of the structure functions [2, 3] but the total differential cross section is Coulomb-like [15, 16]:

$$d\sigma/d|t| \approx 4\pi\alpha^2/t^2 \quad .$$

This process - at very large initial energies -

may be looked at as an effective binary electron scattering off a Coulomb potential. However, in such a case we have conservation of the dilatation momentum [1] and the time-component of the conformal momenta *. Thus we do have in this case a kind of 'quasi'-binary collision we need for establishing the distribution function in question. In weak interactions we can apply the same reasoning only [17] if there is an intermediate boson W and if $|t| \gg m_W^2$.

2.4.3. Purely hadronic interactions. In inclusive hadronic reactions a + b → c + anything 'Feynman' scaling [4] is observed for $s \to \infty$: The production cross section for the particle c with longitudinal c.m. momentum $p_\parallel$ and transvers momentum $p_\perp$ is supposed to be of the form

$$d^2\sigma/p_\perp dp_\perp dy = f(p_\perp, y), \qquad y = 2p_\parallel/\sqrt{s},$$

which is supported by experiments [4]. Either if $f(p_\perp, y)$ factorizes [4]: $f(p_\perp, y) = f_1(p_\perp)f_2(y)$ or if we integrate over $p_\perp$, we get

$$d\sigma/dy = g(y).$$

This production cross section may be interpreted in the following way: A cross section in one space dimension, defined by the decrease $\Delta j = -\sigma j n(x^3)\Delta x^3$ of the current density j along Δx^3 is dimensionless. This means: if the dynamics do not contain any fixed lengths like rest masses or coupling constants with non-vanishing dimensions etc., then the production cross section for one particle in the final state, after summing over all other secondaries, can only be a function of the ratio y. In this sense purely hadronic interactions exhibit one-dimensional (longitudinal) dilatation invariance [1,18].

2.5. It is not clear, whether the high energy reactions just listed actually did lead to distribution functions in astrophysics as suggested above, because the physical situation is so complex. They or others *might* have, but obviously further studies and the inclusion of gravitation are necessary.

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* The conformal momentum here has the form $h^0 = 2x^0(xp) - p^0x^2 - 2gr$, where gr^{-1} is the Coulomb potential.

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