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A BREMSSTRAHLUNG MODEL FOR PROTON-PROTON ELASTIC SCATTERING AT HIGH ENERGIES‡

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Abstract: We calculate the elastic proton-proton cross section in the framework of a bremsstrahlung model. Adjusting only one free parameter we obtain good overall agreement with experiments for lab energies up to at least 20 GeV.

In the following we shall analyse elastic proton-proton scattering at high energies in the framework of a model the details of which have been discussed previously [1]. The main features of the model are these: In addition to ignoring spin and isospin of the protons as well as isospin and parity of the secondary mesons (mostly pions) in *inelastic* proton-proton collisions, we approximate the source $j(x)$ for the mesons in the equation

$$(\Box + \mu^2)A(x) = j(x) \tag{1}$$

by a c-number (as to the incorporation of unitary symmetries see ref. [1]). These approximations lead to a factorization of the S-matrix for proton-proton collisions where the second factor can be calculated explicitly in terms of the source $j(x)$. For the elastic cross section we obtain

$$\frac{d\sigma^{el}}{dt} = \bar{\sigma}(s,t)\, e^{-b(s,t)} , \tag{2}$$

where s is the c.m. energy squared and t the invariant momentum transfer squared. Further

$$b = \int \frac{d^3k}{2k_0} \,|\tilde{j}(k;s,t)|^2 , \qquad k_0 = +(\boldsymbol{k}^2 + \mu^2)^{\frac{1}{2}} ,$$

$$j(x;s,t) = \int_0^\infty \frac{dk_0}{2\pi} \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \,[\tilde{j}(k;s,t)\, e^{-ikx} + \tilde{j}^*(k;s,t)\, e^{ikx}] . \tag{2a}$$

The 'potential' cross section $\bar{\sigma}(s,t)$ has the following double meaning [1]:

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On the one hand it describes the scattering of the 'bare' nucleons if we ignore their long-range meson cloud (which is taken care of by the second factor $\exp(-b)$). On the other hand: Approximating the source in eq. (1) by a c-number means that we ignore the recoil suffered by the protons as a consequence of the emission of mesons. From this approximation we get the sum rule

$$\bar{\sigma}(s,t) = \frac{\mathrm{d}\sigma^{\mathrm{tot}}}{\mathrm{d}t}(s,t) = \sum_{n=0}^{\infty} \frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}t}(s,t) , \tag{3}$$

where $\mathrm{d}\sigma^{(n)}/\mathrm{d}t$ is the cross section for the quasi-elastic scattering of two protons accompanied by the emission of n soft mesons. By identifying the potential cross section in the *elastic* cross section (2) with the *inelastic differential total* cross section (3) we have assumed that the inelastic kinematics are not too distorted in comparison to the elastic ones. The problems associated with this assumption are discussed in ref. [1].

The main object of the present paper is to calculate the quantity $b(s,t)$. In order to do this we make the following assumptions:

(i) Each of the two protons is assumed to provide the following [2] meson source $j^{\alpha}(x)$, $\alpha = 1,2$, $j = j^1 + j^2$:

$$j^{\alpha}(x) = \theta(-x_0)\rho_{\mathrm{in}}(x - v^{\alpha}_{\mathrm{in}}\, x_0) + \theta(x_0)\rho_{\mathrm{out}}(x - v^{\alpha}_{\mathrm{out}}\, x_0) . \tag{4}$$

This ansatz gives

$$\tilde{j}^{\alpha}(k) = -i\left(\frac{\tilde{\rho}_{\mathrm{in}}(k)}{k_0 - k\cdot v^{\alpha}_{\mathrm{in}} - i\epsilon} - \frac{\tilde{\rho}_{\mathrm{out}}(k)}{k_0 - k\cdot v^{\alpha}_{\mathrm{out}} + i\epsilon}\right) , \tag{5}$$

$$\tilde{\rho}(k) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}^3x\ \mathrm{e}^{-ik\cdot x}\, \rho(x) .$$

(ii) In the rest system of a proton its hadronic charge distribution is static and radially symmetric:

$$j^{\alpha}_{\mathrm{rest}}(x) = j^{\alpha}_{\mathrm{rest}}(x) = f(|x|) . \tag{6}$$

If we assume the source $\rho_{\mathrm{in,\,out}}(x)$ to transform like a scalar under Lorents transformations (see eq. (1)), we get for the charge distribution ρ_{p} of a proton moving with the velocity $v = p/E$:

$$\rho_{\mathrm{p}}(x) = f\left(\left|x^{\perp} + \frac{E}{m}x^{\parallel}\right|\right) , \qquad \tilde{\rho}_{\mathrm{p}}(k) = \frac{m}{E}\, g(\hat{k}) ,$$

$$g(\hat{k}) = \frac{(2/\pi)^{\frac{1}{2}}}{\hat{k}} \int_0^{\infty} \mathrm{d}y\ \sin(\hat{k}y)\, y f(y) , \qquad \hat{k} = k^{\perp} + \frac{m}{E}\, k^{\parallel} . \tag{7}$$

Here $x^{\perp}$, $k^{\perp}$ and $x^{\parallel}$, $k^{\parallel}$ are the components of the vectors x and k perpendicular and parallel to the vector p.

From the above two assumptions we get the following general results which can be proved by analytical calculations in the c.m. system (the details of which will be given elsewhere ‡) and which can be seen also from the numerical results given below:

a) For small momentum transfers t and $s \to \infty$ the function $b(s,t)$ has the form

$$b(s,t) \sim \tfrac{1}{3} 4\pi \frac{|t|}{m^2} \int_0^\infty d\kappa \frac{\kappa^7}{(1+\kappa^2)^4} [g(\mu^2\kappa^2) - \mu^2(1+\kappa^2) g'(\mu^2\kappa^2)]^2 , \qquad (8)$$

i.e. the function $\exp(-b(s,t))$ has an energy-independent asymptotic diffraction peak, the numerical value of which depends on the special choice for the function $g(u)$. The constant m in eq. (8) is the nucleon mass and $g'(u)$ the derivative of $g(u)$.

b) Because $|a_1 + a_2| \leq |a_1| + |a_2|$, an upper bound for $b(s,t)$ can be calculated which turns out to be energy- and momentum transfer independent:

$$b(s,t) \leq 8\pi \int_0^\infty d\kappa \frac{\kappa^3}{(1+\kappa^2)^2} |g(\mu^2\kappa^2)|^2 . \qquad (9)$$

The last property may spell a limitation of the model because it means that above some energy the factor $\exp(-b(s,t))$ in eq. (1) becomes a constant and the energy dependence of the elastic cross section has to be taken care of by the potential cross section $\bar{\sigma}(s,t)$. Where this limitation sets is well seen below. It is probably related to the fact that a current like eq. (5) corresponds to soft-meson emissions from external lines [1, 2]. At very high momentum transfers this mechanism may no longer be the relevant one [1].

(iii) Our third and final assumption is the following: We assume the hadronic charge distribution $f(y)$ of the proton to be proportional to that seen in elastic electron-proton scattering. Analytically we shall use the dipole-fit. This amounts to putting

$$g(\hat{k}) = \Gamma\left(1 + \frac{\hat{k}^2}{a^2}\right)^{-2} , \qquad (10)$$

where $a^2 = 0.71\ (\mathrm{GeV}/c)^2$ and Γ is an effective coupling constant. The numerical results (with μ = pion mass) for b/Γ^2 are shown in fig. 1. It clearly shows the almost energy independent peak for large energies and the upper bound of b/Γ^2 is $b^{\max}/\Gamma^2 = 16.25$. This value depends on the numerical value of the constant a^2 in the dipole fit. For $a^2 = 2.34$ or 0.18 we have $b^{\max}/\Gamma^2 = 29.52$ or 6.60. Fig. 1 shows that at least for the energy- and momentum transfer range which has been measured in detail [3], the function b/Γ^2 has the same qualitative features as the experimental elastic cross section.

The main problem for comparing the model with experiments is the determination of the potential cross section $\bar{\sigma}$. Theoretically one can give arguments [1] for the smooth energy dependence of $\bar{\sigma}$ for fixed c.m. scattering

‡ See ref. [12].

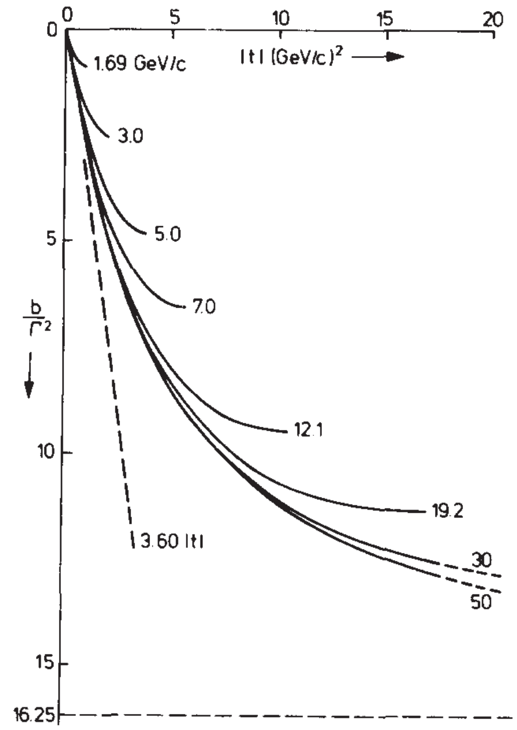

Fig. 1. The quantity b/Γ^2 as a function of $|t|$ at various lab momenta. The almost vertical dashed line gives the slope of the curves for $|t| = 0$, $s \to \infty$ and the horizontal dashed line at 16.25 shows the asymptotic value of b/Γ^2 for $|t| \to \infty$, $s \to \infty$.

angles $\theta^* \neq 0, \pi$ which is compatible with available data [4-6]. There is, however, no theory for small momentum transfers and the published experimental spectra $d^2\sigma/dt\, dW$, where W is the missing mass in inelastic proton-proton collisions, are incomplete, particularly for the region of large momentum transfers and large inelasticities. If these spectra were known, we would try to determine $\bar{\sigma}$ by means of the relation

$$\bar{\sigma}(s,t) = \int dW \frac{d^2\sigma}{dt\, dW}(s,t,W)$$

from experiments. The more copious data [7] for small inelasticities and momentum transfers indicate that $\bar{\sigma}$ is almost energy independent for fixed t in this region, contrary to the large momentum transfer region, where it seems to become very smooth in the energy for fixed θ^*.

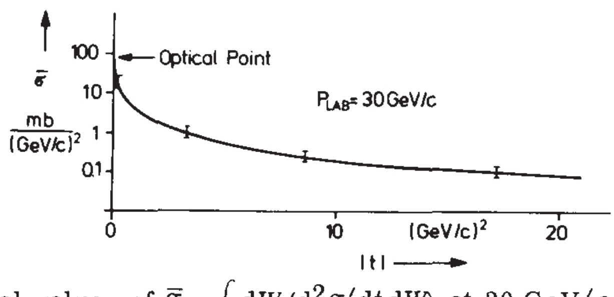

Fig. 2. Experimental values of $\bar{\sigma} = \int dW\, (d^2\sigma/dt\, dW)$ at 30 GeV/c as taken from refs. [4, 6, 8]. The errors are estimates.

Fig. 2 shows the quantity $\bar{\sigma}$ at 30 GeV as obtained according to eq. (11) from ref. [4]. The three points for large t are taken from Mack's analysis [6] of this work and the point at $t = 0$ was obtained by using the optical theorem for $\bar{\sigma}(t=0)$ (note that $b = 0$ for $t = 0$). The value of t in fig. 2 has been calculated by using the measured inelastic scattering angle and the elastic momentum.

Because of our incomplete experimental and theoretical information concerning the potential cross section $\bar{\sigma}$ we have adopted the following preliminary point of view for comparing the curves of fig. 1 with the experimental ones: If we put

$$\bar{\sigma}(s,t) = \text{const} \times e^{-\tilde{b}(s,t)} ,$$

then the available experimental information mentioned above indicates a qualitative structure for $\tilde{b}$ which is similar to that of b/Γ^2 but on a smaller scale, i.e.

$$\tilde{b}(s,t) \approx \frac{\beta}{\Gamma^2}\, b(s,t) , \qquad \text{where} \qquad \beta \ll \Gamma^2 . \tag{12}$$

Defining $g_{\rm eff}^2 = \Gamma^2 + \beta$ we get for the elastic cross section

$$\frac{d\sigma^{\rm el}}{dt} = A\, e^{-(g_{\rm eff}^2/\Gamma^2) b(s,t)} . \tag{13}$$

The constant A is determined by the optical theorem to be $A = 77.7$ mb.

The effective coupling constant was adjusted in the following way: We calculate $g_{\rm eff}$ at the four points $\theta^* = 50^{\rm o}$ and $90^{\rm o}$ and 12.1 and 19.2 GeV/c by comparing the values of the function b at these points with the experimental results. The average is $g_{\rm eff}^2 = 1.82$. Using this value we calculate $d\sigma^{\rm el}/dt$ for various energies and momentum transfers. The result is shown

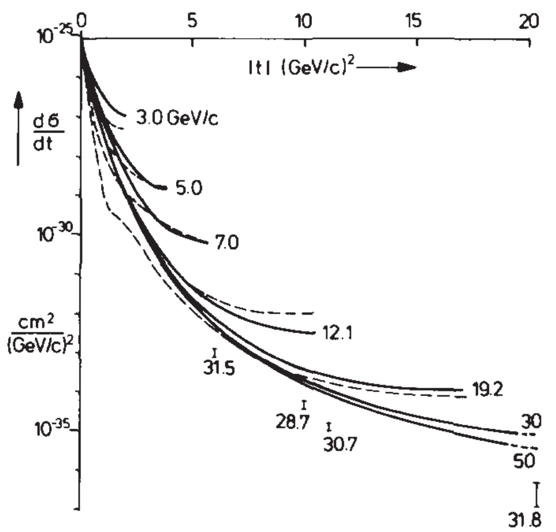

Fig. 3. Comparison of the predicted elastic cross section (solid lines) with the experimental results of ref. [3] (dashed lines) after adjusting one free effective coupling constant.

in fig. 3. In view of the simplicity of the model it is surprisingly good for energies up to 20 GeV. The experimental points at 30 GeV are systematically below the theoretical curve. This seems to be an indication for the rising influence of the constant asymptotic value of b which marks a limitation of the model. Part of this limitation is a consequence of the ansatz (12).

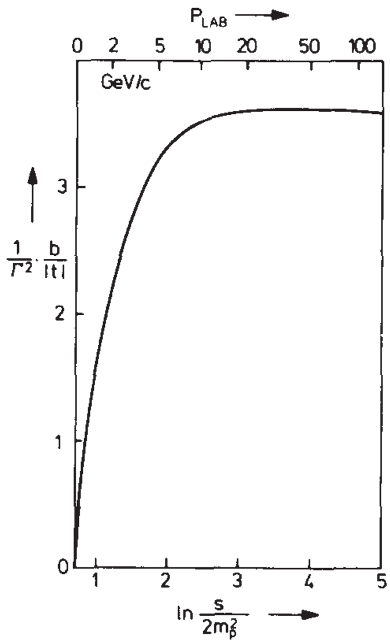

Fig. 4. Slope of the function b/Γ^2 as obtained from formula (8) by using the dipole density (10). The maximum value 3.62 is slightly above the asymptotic value 3.60 and lies at a lab momentum of about 40 GeV/c.

Fig. 4 finally shows the energy dependence of the peak $b(s,t)/|t|\Gamma^2$ for fixed small $|t|$ with the asymptotic limit (8). By using the above value $g_{\text{eff}}^2 = 1.82$ we have

$$\frac{g_{\text{eff}}^2}{\Gamma^2}\,\frac{b(s,t)}{|t|} \rightarrow 6.55\ (\text{GeV}/c)^{-2} \quad \text{for} \quad s \rightarrow \infty\,. \tag{14}$$

This is to be compared with the experimental value [8] $b_{\text{exp}}/|t| \approx$ $\approx 10\,(\text{GeV}/c)^{-2}$.

Since the function b is a scalar with respect to unitary symmetries [1], our model with its constant asymptotic diffraction peak may give an explanation for the very weak energy dependence of the Pomeranchuk trajectory.

We would like to point out that the model is also able to account for the similar shapes of the elastic proton-proton cross section and the corresponding ones with an isobar in the final state instead of a nucleon [7]. One reason is again that the function b is a unitary scalar, i.e. $b(s,t)$ is a universal function, independent of the particular collision channel. The potential cross section on the other side depends on the special channel. Thus one expects similar shapes (determined by the function b) but slightly changing magnitudes (determined by $\bar{\sigma}$). This is exactly what has been observed [7]. The second reason is independent of these unitary symmetry arguments: Experimentally the magnetic transition form factors $G_M^*(t)$ for the final state isobars in inelastic electron-nucleon scattering seem to be similar to the elastic ones [9]. In the framework of our model we therefore have to expect a corresponding similarity between elastic nucleon-nucleon scattering and those collisions where the final state nucleons are excited (isobars).

We would like to conclude by pointing out that the general spirit of our approach is similar to that of the work of Chou and Yang [10] and Ababarnel, Drell and Gilman [11]. There are, however, some important differences in detail, a comparison of which will be given elsewhere.

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REFERENCES

[1] H.A.Kastrup, in: Particles, currents and symmetries, ed. P.Urban (Springer-Verlag, Wien-New York, 1968), p. 407 and references therein;
A.D.Krisch, Lectures on high-energy proton-proton interactions. Argonne Nat. Lab. Preprint HEP 6808 (1968);
M.G.Gundzik, Argonne Nat. Lab. Preprint HEP 6911 (1969).

[2] W.Thirring, Principles of quantum electrodynamics (Academic Press, New York-London, 1958);
D.R.Yennie, 1963 Brandeis Summer Institute Lectures in Theoretical Physics, ed. K.W.Ford (Brandeis University, 1964), vol. 1, p. 165;
A.Białas and T.Ruijgrok, Nuovo Cimento 39 (1965) 1061.

[3] J.V.Allaby et al., Phys. Letters 28B (1968) 67 and references therein;
G.Cocconi et al., Phys. Rev. 138B (1965) 165.

[4] E.W.Anderson et al., Phys. Rev. Letters 16 (1966) 855; 19 (1967) 198.

[5] E.W.Anderson and G.B.Collins, Phys. Rev. Letters 19 (1967) 201.

[6] G.Mack, Phys. Letters 26B (1968) 515.

[7] J.V.Allaby et al., Phys. Letters 28B (1968) 229 and references therein.

[8] K.J.Foley et al., Phys. Rev. Letters 15 (1965) 45; 19 (1967) 857.

[9] W.K.H.Panofsky, in: Proc. 14th Conf. on high-energy physics, Vienna 1968 (CERN, Geneva, 1968) and references therein.

[10] T.T.Chou and C.N.Yang, Phys. Rev. 170 (1968) 1591.

[11] H.D.I.Arbarbanel, S.D.Drell and F.J.Gilman, Phys. Rev. Letters 20 (1968) 280.

[12] H.Gemmel and H.A.Kastrup, Z.Phys. (Germany) 229 (1969) 321.