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Preface

The subject of this year's conference, as of the six previous meetings, was again elementary particle physics. One of the main topics of current interest here is the understanding of strong interactions; some of the attempts to explain them are discussed at length in these proceedings, including current algebra, Regge pole theory and effective Lagrangians. On the other hand, the processes of weak interactions, too, are not yet fully understood, especially when strong interactions intervene, as in the radiative corrections to weak decays. Several other aspects of elementary particle physics are also treated in this volume, every one of them another step towards a thorough understanding of the sub-atomic world.

I want to take this opportunity to thank all the members of my staff for their assistance in organizing the meeting and for their help in preparing the manuscripts.

Graz, June 1968 **P. Urban**

Contents

Dear Colleagues, Ladies and Gentlemen:

Once again in the history of our annual International Meeting in Schladming we have started with the lectures before the official opening ceremony, on account of the carnival festivities going on here, which I hope you are enjoying. The word "history" I just used is not an overstatement since we are now meeting for the seventh time in Schladming and to many of you our symposium is quite familiar. These people especially, and all those who are participating for the first time, I would like to welcome most cordially and wish all of you two weeks of success in physics and perhaps also in skiing.

To our guests of honour, who show by their attendance the serious interest of the governmental and provincial authorities in our enterprise, I most respectfully extend my welcome. It is a great privilege for me to have the representative of the Ministry of Education with us; may I take the opportunity here to express our sincerest thanks for the generous support we have received over the years. Other grants which have contributed much to the successful organization of this meeting were given by the International Atomic Energy Agency and the Ministry of Trade and Industry. We are further indebted to the Provincial Government of Styria, whose representative I also welcome here, as well as the Chamber of Commerce. It gives us special pleasure to have among our guests of honour the Dean of the Philosophische Fakultät of the University of Graz. We are certain that through the scientific reputation of this meeting the name of our University is already well known to the scientific community. But also our host city, Schladming, and its beautiful surroundings become more familiar each year to a larger group of people, who meet with us here. May I take the opportunity now to thank especially the Mayor of the City, Director Laurich, both personally and as representative of the authorities of Schladming for their continued help throughout the years.

Science—and possibly also skiing—have again this year drawn to Schladming about 180 participants from 18 nations, a number which impressively shows the value of our meeting as a place for learning about new developments and discussing recent results. We are happy that we have succeeded in obtaining the cooperation of outstanding experts as lecturers and I want to thank them for coming here; I hope they will also benefit from their stay.

As expressed in the title of this year's meeting, "Particles, Currents, Symmetries", we shall be dealing with the world of elementary particle physics. Here especially, in the absence of a theory as powerful as quantum-electrodynamics, the strong and weak interaction processes have successively forced us to change our point of view in order to restore the agreement between experiment and theory. The latest such event, caused by the detection of CP violation processes, has again destroyed another supposed conservation principle. These

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ON HIGH ENERGY COLLISIONS OF HADRONS AT VERY LARGE MOMENTUM TRANSFERS†

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Content

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I. Introduction

Under collisions with large momentum transfers we understand in the following elastic and inelastic scattering reactions where the absolute values of the quantities $t = (p_1-p_1')^2 < 0$ and $u = (p_1-p_2')^2 < 0$ are larger than the square of the nucleon mass M. Here p_1, p_2, p_1' and p_2' are the 4-momenta [1] of the two primary particles before and after the collision (we shall consider only those processes in which the two initial particles occur in the final state, too).

In other words, we shall not discuss forward and backward scattering here, the structure of which seems to be dominated by diffraction-like mechanisms and resonances in the direct and crossed channels [2].

Whereas a large amount of very interesting theoretical and experimental work has been done in order to analyze the features of forward and backward scattering, the large momentum transfer collisions - in particular the inelastic ones - are rather poorly investigated and understood in comparison [3]. There are several reasons for this:

On the theoretical side we are no longer near the region of known low energy resonances in the crossed t- and u-channels of the (hopefully!) analytic scattering amplitudes. Thus we can no longer assume that these resonances dominate the physical region we are interested in. Furthermore, large momentum transfer reactions are usually accompanied by the emission of many secondaries, because very large momentum transfers tend to break up the primary particles. Our theoretical knowledge of many-particle inelastic scattering amplitudes - which in turn strongly influence the elastic amplitudes because of the unitarity relations - is, however, extremely poor.

On the experimental side the cross sections for the individual channels, for instance the elastic one, become extremely small for large momentum transfers (down to $10^{-34}cm^2/ster$ in elastic p-p scattering [4]), implying very small counting rates.

As for the inelastic channels it is difficult to obtain a detailed kinematical analysis of all the different particles in the final state.

Although our present theoretical and experimental knowledge of large momentum transfer collisions is very unsatisfactory, these reactions are of principal importance for at least three main reasons:

1. They are supposed to yield information about the short-range properties of the various interactions. The usual "uncertainty" argument for this goes as follows: If R is the interaction range to be tested, then we have approximately $R|t|^{1/2} \simeq 1$, where t is the momentum transfer squared. In other words, the more "central" the collision, the larger is the probability that the particles are being scattered with large momentum transfers.

A slightly different argument characterizing large momentum transfer reactions, or equivalently, high energy collisions with scattering angles $\Theta \neq 0,\pi$, goes as follows: If ℓ is the relative orbital angular momentum between the two incoming particles in the c.m. system and b their impact parameter, then short range collisions will have a small b. Because of $\ell = bp$, where p is the c.m. momentum of the incoming particles, this means that short-range interactions are mainly characterized by the lower partial waves of the scattering amplitude, if p is kept fixed. However, this argument fails in the extreme relativistic limit $p \to \infty$. In this case not only low partial waves might become important even for $b \to 0$.

2. Because the rest masses of particles become negligible in comparison to the variables $s=(p_1+p_2)^2$, t and u etc., we hope to learn something about the realm of validity of approximate symmetries like SU(3), SU(2) $\otimes$ SU(2) etc. which are assumed to be the better the more mass differences or even rest masses themselves become negligible. Thus one suspects large momentum transfer reactions to provide a suitable testing ground for unitary and chiral symmetries.

3. In the framework of axiomatic field theories it is possible to derive lower and upper bounds for the high energy large momentum transfer behaviour of scattering amplitudes from some very general assumptions [5]. An experimental violation of these bounds would be very interesting! No such violations have been observed so far. We shall not deal with this aspect of large momentum transfer collisions.

I have described the first two motivations for analyzing large momentum transfer collisions rather uncritically. In what follows I shall try to give a critical analysis in the framework of a rather elementary and simplified model.

There are other interesting models for large momentum transfer reactions [6], which we shall not discuss here.

The model to be considered is more or less a phenomenological one. Qualitatively it employs the rather old bremsstrahlung picture for meson production, keeping an eye or two on our basic interests listed above and adding several new features [7] which provide interesting experimental predictions.

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Acknowledgments

I benefited a great deal from conversations with Dr. G. Mack. A part of the final version of these lectures was written during a visit at DESY, Hamburg. I am very much indebted to Professor H. Joos and several other physicists for their very kind hospitality and for stimulating discussions.

II. A Statistical Soft Meson Model for Elastic and Inelastic Nucleon-Nucleon Scattering

1. Description of the model

We start this section II by giving a critical discussion of the four basic assumptions concerning the model to be employed. The assumptions are as follows:

a) We consider only laboratory energies E_{lab} larger than 10 GeV. At such energies of the incoming nucleons many collision channels are open and <u>we assume the features of a given channel to be dominated by what is going on in all the other channels.</u> This assumption is, of course, applicable to all high energy collisions, not merely to those with large momentum transfers. A typical relation expressing this overall property is the Optical Theorem which says that the imaginary part of the elastic forward scattering amplitude is proportional to the total cross section at the same energy.

More general: what we are concerned with in our first assumption are the consequences of the unitarity of the S-matrix, which relates the different channel amplitudes to each other. As it is impossible,

at least at the moment , to take into account and to exploit in detail all unitarity restrictions in the case of many-particle final states, one has to introduce some statistical hypotheses concerning the contribution of all open channels to an individual one. We shall specify these assumptions under c).

The general moral to be drawn from our first hypothesis is that we should try to get information pertaining to our basic interests in large momentum transfer collisions from the combined inspection of <u>all</u> open channels, not only of a single, for instance the elastic, one. We shall sharpen this assertion later on.

b) Our second assumption is typical for large momentum transfer scattering: At very high energies the hadronic fields - we neglect the electromagnetic and weak interactions in this section - of the two incoming particles are strongly Lorentz-contracted in the c.m. system. The same holds for the outgoing particles. For large momentum transfer reactions the contraction necessitates a strong rearrangement of the long-range parts of the hadronic fields. This strong rearrangement is very probably accompanied by the emission of secondaries.* <u>Our second assumption therefore is that the main bulk of the secondaries is a consequence of the strong rearrangement of the hadronic field tails in large momentum transfer collisions.</u>

A first qualitative conclusion from this hypothesis is that the secondaries should be mostly pions because they constitute the tail of the long-range parts of the hadronic fields. This is supported by cosmic ray and accelerator experiments which show about 80% of the secondaries to be pions [8] . The rest con-

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* By "secondaries" we always mean the particles produced in addition to the outgoing "primary" particles which initiated the reaction.

sists mainly of kaons. However, it is at present not well-known how many of the pions are decay products of, for instance, vector mesons which were produced in the first place. Furthermore, one does not know the variation of the ratio of the different kinds of secondaries as a function of the momentum transfer of the primary nucleons.

What is very much needed is an experiment which determines the identities and the kinematics of all or most particles - primaries and secondaries - in high momentum transfer collisions with multiparticle final states.

A main conclusion to be drawn from the second assumption is that we shall not get much information about the short-range properties of the hadronic fields if we are not able to subtract the long-range effects. This was first pointed out by Wu and Yang in a slightly different context [9].

c) The last pessimistic conclusion from our second assumption poses the question whether it is possible to separate long and short range effects, for instance by a factorization of the cross sections. In order to obtain such a factorization we assume the secondaries to be "soft", i.e. we neglect energy-momentum-, spin- and unitary spin recoils associated with the emission of the secondaries.

Before discussing the factorization implied by this assumption, let us look at the experimental situation in order to see how realistic this third hypothesis is : Many of the observed cosmic ray events seem to be quasielastic, i.e. only a relatively small fraction [8] (about 30 - 40 %) of the initial energy is converted into secondaries, and the primary particles emerge with a relatively small energy loss after

the scattering ("leading particles"). However, the analysis of accelerator experiments [10] suggests that large momentum transfers in inelastic nucleon-nucleon collisions are strongly suppressed and that most of the observed cosmic ray events are very likely associated with small momentum transfers of the primary particles. This conclusion is not completely convincing because present accelerator energies are quite small in comparison to the interesting cosmic ray energies ($E_{lab} > 10^2$GeV), and it may be that large momentum transfer reactions are more frequent at very high energies. A detailed analysis of this question in cosmic ray physics would be very interesting.

As it stands, the simple assumption of "soft" secondaries may be too unrealistic considering the present state of our knowledge although the soft pion hypothesis has met with considerable success at low energies in the framework of current algebra [11][12]. However, there may be at least two ways out of this dilemma: First, the collision cross sections may be "soft" or quasielastic in certain variables but not in others. Perhaps, if one is lucky, one will be able to pick the right variables in which the cross sections are insensitive to our "softness" approximation. It seems, for instance, that large momentum transfer elastic and inelastic nucleon-nucleon cross sections depend in a first approximation only on the transverse momentum transfer $p_\perp$ of the outgoing nucleons, not on their longitudinal momentum [4],[10],[13]. It is therefore tempting to assume these collisions to be quasielastic with respect to $p_\perp$. The assumption is supported by measurements of the pion c.m. $p_\perp$- and $p_\parallel$-spectra [14] in inelastic proton-proton collisions at 12.5 GeV. The $p_\perp$ - spectra have a Gaussian distribution around $p_\perp = 0$, but the maximum of the $p_{\shortparallel}$-spectra is around 0.5 GeV/c. Thus the pions seem indeed to be soft with respect to their transverse momentum transfer. Unfortunately, the result is inconclusive for us because the kinematics of the outgoing protons were not measured simultaneously, and so we do not know whether the soft mesons are associated with small or large momentum transfers of the primary nucleons. What is needed is an experiment which measures the kinematics of the primary and secondary particles at the same time.

We shall discuss this first refined possibility of a quasi-elastic approximation in detail in parts 2 and 3 of this section.

Secondly, we can weaken the "softness" - hypothesis by keeping it for the amplitudes but dropping it for the phase spaces in question. This is, of course, more realistic and the results are quite encouraging [15]-[17]. We shall deal with this approach in detail in part 5.

Let us briefly discuss now the factorization implied by the assumption of negligible energy-momentum and spin recoil. The unitary spin problem will be treated in part 4. Here we assume that there is only one type of primary particles and one type of secondaries. Furthermore we assume spin and parity effects to be negligible. There is practically no information as to whether this last assumption is justified for large momentum transfer collisions or not.

With these simplifications we get, as the cross section for the quasielastic scattering of two primary particles accompanied by the emission of n soft secondaries, the expression

$$\frac{d\sigma^{(n)}}{d\Omega} = \bar{\sigma}(E,\Theta)w_n(E,\Theta) \quad . \qquad (1)$$

Here E is the c.m. energy of one of the primary particles and Θ their c.m. scattering angle. (In the following, we always mean c. m. quantities if not stated otherwise.)

The factorization (1) is nothing other than the multiplication law of probability theory with $w_n(E,\Theta)$ as the conditional probability for the emission of n soft secondaries, provided the primary particles are being scattered at energy E with scattering angles Θ and $\pi-\Theta$ respectively.

Negligible recoil means that the secondaries are emitted statistically independently, and therefore we have for w_n the Poisson probability

$$w_n(E,\Theta) = \exp[-\bar{n}(E,\Theta)] \cdot \frac{1}{n!} \cdot [\bar{n}(E,\Theta)]^n \quad . \qquad (2)$$

As the second factor in eq. (1) describes the emission of secondaries, which according to our assumption b) is a consequence of the rearrangement of the long-range parts of the hadronic fields, this factor $w_n(E,\Theta)$ is characteristic for the long-range effects. Therefore the first factor, the so-called "potential" cross section $\bar{\sigma}(E,\Theta)$, should be characteristic for the short-range properties. Thus eq. (1) provides us with a separation of long- and short-range effects, valid, of course, at present only in the framework of our approximations.

The quantity $\bar{n}(E,\Theta)$ is the average number of secondaries, provided the primary particles have the energy E and scattering angles Θ and $\pi - \Theta$ respectively. This "differential" multiplicity is of considerable importance for our model.

We further notice that the factor $\exp[-\bar{n}]$ is contained in elastic and inelastic cross sections. It represents the overall consequence of all the inelastic

S-matrix unitarity restrictions for each individual channel, including the elastic one.

Summing over n in eq. (1) gives

$$\frac{d\sigma^{tot}}{d\Omega}(E,\theta) = \sum_{n=o}^{\infty} \frac{d\sigma^{(n)}}{d\Omega} = \bar{\sigma}(E,\theta) \quad . \tag{3}$$

From this we conclude that the differential total cross section gives the cleanest information about the short-range interactions, not the individual ones! This interesting feature is a consequence of our approximations, but its realm of validity may go beyond them.

d) Our last main assumption concerns the form of the potential cross section at large momentum transfers, or, equivalently, the structure of the short-range interactions: We assume the short - range interactions of the "bare" primary hadrons - i.e. without their long - range meson cloud - to be pointlike.

In other words, the short-range interactions do not have any fixed hard core, all rest masses are negligible, and all relevant coupling constants do not contain a fixed length.

From this it follows that

$$\bar{\sigma}(E,\theta) = E^{-2} A(\theta); \quad \theta \neq 0,\pi \quad ; E>>M \quad , \tag{4}$$

because E is the only quantity left which can provide us with a dimension of length.

Another way of describing the hypothesis (4) is that we assume the short-range interactions to become invariant under dilatations, i.e. all fixed lengths like rest masses, etc., become negligible at very high energies and large momentum transfers as far as the potential cross section is concerned.

The assumption (4) certainly cannot be applied to

the second factor in eq. (1), because the range of the hadronic fields is determined by rest masses and the factor $w_n(E,\theta)$ depends very much on this range. Therefore, we cannot neglect all rest masses in $w_n(E,\theta)$. Thus, the naive assumption that all rest masses become negligible for large s, -t and -u obviously cannot be true in general!

2. Comparison of the model with experiments

In this part we shall compare some of the predictions of that version of the model described in part 1 which supposes the soft meson hypothesis to be a good approximation for some suitably chosen variables. The refined version which takes realistic phase spaces into account will be discussed in part 5.

2.1. Elastic scattering

For the elastic cross section $d\sigma^{el}/d\Omega \equiv d\sigma^{(o)}/d\Omega$ we get from eqs. (1), (2) and (4):

$$\frac{d\sigma^{el}}{d\Omega} = E^{-2} A(\theta)\, e^{-\bar{n}(E,\theta)} \quad ; \theta \neq 0,\pi \quad . \tag{5}$$

We see that - at a fixed θ - the only unknown energy dependence of the cross section is contained in the differential multiplicity $\bar{n}(E,\theta)$. From accelerator and cosmic ray experiments we know only something about the integral multiplicity

$$\bar{n}_{tot}(E) = \int \frac{d\sigma^{tot}}{d\Omega}\, \bar{n}(E,\theta)\, d\Omega/\sigma^{tot}(E) \quad . \tag{6}$$

Experimentally [8] we have $\bar{n}_{tot}(E) \sim E^{\alpha}$ in nucleon-nucleon collisions, where α varies roughly between 0.5

and 1. Under the assumption that the angle integration (6) does not change the energy dependence of $\bar{n}(E,\Theta)$ very much, we expect the elastic p-p cross section to drop exponentially with increasing energy E for fixed scattering angles $\Theta \neq 0,\pi$. This feature is borne out experimentally: Orear fitted [18] the earlier data [4] by the following expression :

$$\frac{d\sigma^{el}}{d\Omega} = \text{const. } E^{-2}\, e^{-ap_\perp} \quad , \qquad (7)$$

where $a \simeq 5(\text{GeV/c})^{-1}$ and $p_\perp = p \sin\Theta$. Later measurements [19] showed that the fit (7) is only a first approximation and that there is some fine structure. But as a first approximation the expression (7) is quite useful for us. It obviously has the same structure as the formula (5). It is therefore tempting to identify the two exponentials:

$$\bar{n}(E,\Theta) = a\, p_\perp \quad . \qquad (8)$$

However, the identification (8) may be wishful thinking, because it is not a priori clear which percentage of the empirical exponential $\exp(-ap_\perp)$ has to be attributed to the inelastic channels characterized by $\exp(-\bar{n})$ in our model. If the fraction is β then we have $\bar{n}(E,\Theta) = ap_\perp + \ln\beta$. If β is energy- and angle-independent, we can neglect the constant lnβ at large enough energies. But it is not clear whether β=1 is a realistic choice for the present accelerator energies. Nevertheless, let us stick to the relation (8) for the moment - β = 1 - and let us look at its implications:

Eq. (8) means that the differential multiplicity $\bar{n}(E,\Theta)$ in p-p scattering should depend in a first approximation only on the transverse momentum transfer of the nucleons, not on their longitudinal one. This will have to be tested by future experiments. Numerically we get $\bar{n} \simeq 15$ for $p_\perp$ = 3 GeV/c. This seems to be a rather large number, which will probably be cut down by realistic phase space limitations (see part 5 of this section).

Furthermore, eqs. (7) and (8) combined imply $d\sigma^{tot}/d\Omega = \text{const. } E^{-2}$, i.e. the differential total cross section becomes independent of Θ if we use the fit (7). This contradicts the results [10] of Anderson et al. which show clearly that $d\sigma^{tot}/d\Omega$ decreases with increasing Θ. (see also ref. [17]). A large part of this discrepancy is probably due to the fact that the fit (7) describes only the gross features of the data.

2.2. Inelastic collisions

By inserting the relation (8) into eqs. (1) and (2) we get the prediction that the inelastic cross sections should depend dominantly on the nucleon transverse momentum transfer. Unfortunately the quantities $d\sigma^{(n)}/d\Omega$ or $d^2\sigma^{(n)}/d\Omega dp$ have not yet been measured for large momentum transfer p-p collisions. What has been measured, however, is the sum

$$\frac{d^2\sigma}{d\Omega dp} = \sum_{n=0}^{\infty} \frac{d^2\sigma^{(n)}}{d\Omega\, dp} \quad .$$

Anderson et al. fit [10] their data for a given initial energy by the expression

$$\frac{d^2\sigma}{dp_\parallel dp_\perp} \sim p_\perp^2\, e^{-ap_\perp} \qquad (9)$$

We see that this cross section indeed depends dominantly on the transverse momentum transfer. Moreover, we have the same exponential $\exp(-ap_\perp)$ with the same constant a as in Orear's fit (7) for the elastic data. The factor $p_\perp^2$ in front of the exponential in eq. (9) may be interpreted as the approximate replacement of the sum $\sum_n 1/n!\ (ap_\perp)^n$ by one of its terms.

We have to stress, however, that the comparison of the fit (9) with eq. (1) cannot be done without raising serious questions: In deriving eq. (1) we assumed the scattering to be quasielastic. On the other hand the protons suffer a considerable energy loss in the experiments of Anderson et al. As already pointed out earlier, one way out of this dilemma seems to be the feature [10] that the dynamics does not seem to care much about losses of longitudinal momentum. If, therefore, the reactions are somewhat quasielastic with respect to the transverse momentum transfer, our first version of the bremsstrahlung model may still be reasonable.

2.3. Differential total cross section

Eq. (4) predicts the differential total cross section

$$\frac{d\sigma^{tot}}{d\Omega} = \int dp\, \frac{d^2\sigma}{d\Omega dp}$$

to go down like $E_{c.m}^{-2}$ for fixed $\theta_{c.m.}$. The date of ref. [10] allow for a certain comparison of this prediction at one single angle $\theta_{c.m.} = 29^o$. A numerical graphical integration by hand yields

$$d\sigma^{tot}/d\Omega(E_{lab} = 10\ \text{GeV}) \simeq 3.0 \pm 0.5\ \text{mb/ sr} .$$

$$d\sigma^{tot}/d\Omega(E_{lab} = 30\ \text{GeV}) \simeq 2.2 \pm 0.5\ \text{mb/ sr}.$$

The theoretical prediction is 3:1 instead of 3:2.2. However, this comparison cannot claim to be a serious one for several reasons: At $E_{lab} = 10$ GeV we certainly cannot neglect all hadron rest masses in the c.m. system as has been supposed in eq. (4). In addition the momentum transfers are not large compared to the nucleon rest mass at $\theta = 29^o$ and $E_{lab} = 10$ GeV. Finally the values of $d^2\sigma/dpd\Omega$ are not known experimentally for $p \leq 1$ GeV/c, although the maxima of the curves lie in this region. Qualitatively we can say that $d\sigma^{tot}/d\Omega$ seems to drop slowly with increasing energy, in agreement with the hypothesis (4).

2.4. Ericson fluctuations

If the high energy large momentum transfer scattering of nucleons is dominated by some compound scattering mechanism like the collisions in the region of overlapping resonances in nuclear physics, one would expec Ericson fluctuactions [20] in the elastic p-p channel. Such fluctuations have not been found [21]. This is nice for us, because in our model we do not assume the primary particles to participate in any thermodynamical equilibrium, and therefore we do not expect such Ericson fluctuations to occur.

2.5. Difference between proton- and pion-spectra

There is a striking qualitative difference between the proton- and pion-spectra $d^2\sigma/dp_\perp\, dp_\parallel$ in the work [10] of Anderson et al.: Whereas the proton spectra do not depend on $p_\parallel$ (proton) the pion spectra drop approximately exponentially with increasing $p_\parallel$ (pion). Such a feature is hard to understand on the basis of a thermodynamical model which assumes the outgoing protons and pions to be produced by the same mechanism. In our model, however, the emission of pions and protons during the scattering is considered to be qualitatively different, although we are not able at the moment to account quantitatively for the observed difference. In order to do this, more dynamical assumptions than those discussed here will be necessary.

2.6. Forward-backward asymmetry

Intuitively we expect $\bar{n}(E,\pi-\Theta) > \bar{n}(E,\Theta)$, $\Theta < \pi/2$, because the rearrangement of the long-range parts of the hadronic fields is stronger for larger angles, implying the emission of more secondaries. Thus we expect a forward-backward asymmetry in those reactions where we do not have to symmetrize (or antisymmetrize) the scattering amplitudes or cross sections with respect to Θ and $\pi-\Theta$. Such a forward - backward asymmetry is indicated by the present p-$\bar{p}$- and p-n-scattering data [22]. A more quantitative analysis will be possible in the near future when more precise data become available.

3. Predictions for future experiments

We shall list a number of predictions implied by the first version of our model. These predictions can be tested by future experiments. Here we assume only one type of primary and one type of secondary (= produced) particles. Predictions which result from the inclusion of unitary spins and more realistic phase spaces for the secondaries will be discussed in part 4 and 5.

3.1. Correlations between elastic and inelastic kinematics

One of the basic features of the model under consideration is the assumption of correlations between the kinematics of the primary particles in elastic and inelastic collisions, i.e. we assume the final state momenta of the two primary particles in inelastic reactions not to be completely distorted compared to their elastic scattering. The best we can hope for is a relation like

$$\vec{p}_1{}' \simeq - \vec{p}_2{}' \quad , \tag{10}$$

even in multiparticle final states, where $\vec{p}_1{}'$ and $\vec{p}_2{}'$ are the c. m. momenta of the primary nucleons in the final state. Even if the relation (10) would not hold, the model might still work if, for instance,

$$\vec{p}_{1\perp}' \simeq \vec{p}_{2\perp}' \quad , \tag{11}$$

because most of the known nucleon-nucleon cross sections seem to depend - as discussed above - in a first approximation on the nucleon transverse momentum transfer only.

3.2. Differential multiplicities

A very important quantity in the framework of our model is the differential multiplicity

$$\bar{n} = \bar{n}\,(E;\, \vec{p}_1{}',\, \vec{p}_2{}') \quad , \tag{12}$$

where E is the initial c.m. energy. The expression (12) is written in a more general form than previously.

Of course, if the assumptions (10) or (11) hold or even the prediction (8), the number of independent variables is reduced drastically. However, the quantity (12) may be of considerable interest beyond the scope of our specific model, because it expresses rather simple correlations between primary and secondary particles. The decomposition of the differential multiplicity (12) with respect to different types of secondaries will be discussed in part 4.

3.3. Fluctuations

A specific test of the assumed Poisson distribution (2) consists in measuring the r.m.s. of $\bar{n}$. In the case of the Poisson distribution these fluctuations are given by $\bar{n}^{1/2}$.

3.4. Differential total cross sections

It was already pointed out before that the differential total cross sections $d\sigma^{tot}/d\Omega(E,\Theta)$ seem to be particularly appropriate in order to test short-range properties. Again we assume that the primary particles go approximately in opposite directions in the c.m. system after the collision, even in inelastic events. If only the relation (11) holds, then

$$\frac{d\sigma^{tot}}{dp_\perp}(E,p_\perp) = \int dp \frac{d^2\sigma}{dp_\perp dp}$$

may be the better quantity to consider.

3.5. Isobars

In many cases it will probably happen that one - or both - of the primary nucleons leaves the collision region in an excited state (isobar) and then goes back to its ground state (nucleon). If this happens one will not describe the pion from the isobar decay by a factor w_n in eq. (1), because this factor is supposed to describe secondaries produced by a nonresonant mechanism (they may be meson resonances, however). In such a case the isobar would have to be dealt with in the potential cross section $\bar{\sigma}$. If the momentum transfers and energies are much larger than the rest masses of the produced isobars, not much should change in $\bar{\sigma}$ compared to the nucleon case. If not, one has to incorporate at least a Breit-Wigner formula in $\bar{\sigma}$ in order to account for the final state interaction between the nucleon and the pion. In that case the assumption of negligible rest masses is, of course, no longer applicable to $\bar{\sigma}$.

4. Incorporation of unitary symmetries

Up to now we have considered only one type of primary and one type of secondary particles. In this part we shall deal with the case where the primary and secondary particles separately belong to a "primary" and "secondary" multiplet of a certain unitary symmetry group SU_n.

In order to include some features of a unitary symmetry, we shall make several crude approximations: We ignore spin and parity of all particles. Consequently, we ignore the usually necessary symmetrizations or antisymmetrizations due to Bose- or Fermi-statistics. This means, for instance, that we neglect the interference terms between amplitudes at Θ and $\pi-\Theta$ in elastic p-p scattering. We merely add the cross sections.

We further assume the total unitary spins of the primary and secondary particles to be decoupled, i.e. we ignore any unitary spin exchange between primaries and secondaries for the time being. Thus the total unitary spin of all the secondaries will be assumed to be zero. Such an assumption is evidently wrong if, for instance, one additional π^+ is produced in p-p collisions, but it might not be too bad an approximation in the case of multi- particle final states.

The hypotheses just stated again imply a factorization of S-matrix elements into a factor $\langle out|in\rangle_I$ describing only the primary particles and a factor

$$\langle out|0;\, in\rangle_{II} = \langle in'|S_{II}|0;\, in\rangle_{II}$$

for the secondaries which depends on the kinematics of $\langle out|in\rangle_I$ (conditional probability amplitude!). The state $|0;in\rangle$ does not contain any secondary particles. The unitary spin dependence can be exhibited in detail by writing down the field equations

$$(\Box + m^2)A_\beta(x) = j_\beta(x), \qquad \beta=1,\dots,r\,, \tag{13}$$

for the secondaries. The index β is a unitary spin index. Our assumptions from above are equivalent to approximating the sources $j_\beta(x)$ by c-numbers. In that case, we can solve the eqs. (13) for the S-matrix explicitly in a standard manner [23]:

$$S_{II} = \exp(-\tfrac{1}{2}b) \times \exp\Big[i\sum_{\beta=1}^{r}\int\frac{d^3\vec{k}}{\sqrt{2\omega}}\, j_\beta(k)\, a^+_{in}(k;\beta)\Big] \times$$
$$\times \exp\Big[i\sum_{\beta=1}^{r}\int\frac{d^3\vec{k}}{\sqrt{2\omega}}\, j^*_\beta(k)\, a_{in}(k;\beta)\Big], \tag{14}$$

where

$$\omega = +(\vec{k}^2+m^2)^{1/2};$$

$$b = \sum_{\beta=1}^{r} g_{\beta'} g_\beta(E,\theta) = \int\frac{d^3\vec{k}}{2\omega}|j_\beta(k)|^2 \quad ,$$

$$j_\beta(x) = \int_0^\infty \frac{d\omega}{2\pi}\int\frac{d^3\vec{k}}{(2\pi)^{3/2}}\,[j_\beta(k)\, e^{-ikx} + j^*_\beta(k)\, e^{ikx}] \tag{15}$$

The $a^+_{in}(k;\beta)$ and $a_{in}(k;\beta)$ are the creation and annihilation operators of the "in"-fields associated with the fields $A_\beta(x)$. The index II is to indicate that the S-matrix in eq. (14) describes only the secondary particles.

Before discussing some applications of the above formulae we notice that the S_{II}-matrix contains the overall SU_n - invariant factor exp(-1/2 b). According to our previous discussions, this factor dominates all cross sections for large momentum transfers of the primary particles. Thus, we expect a universal exponential decrease with increasing energy of all large momentum transfer cross sections for those particles which belong to unitary spin invariant couplings of "primary" and "secondary" multiplets. The universality may, of course, be distorted by symmetry breaking.

I would like to stress the following: Since the sources are c-numbers in eq. (13), the transformations $j_\beta \to V_{\beta\beta'}(SU_n)\, j_{\beta'}$ are not implementable by a unitary transformation. They are merely automorphisms in the vector spaces spanned by the fields $A_\beta(x)$ and the sources $j_\beta(x)$.

As a first application we discuss the isospin group SU(2). The primary particles, the nucleons, form an isodoublet, the secondaries, the pions, an isotriplet. We consider the following cases:

4.1. Nucleon-nucleon two-body collisions

Ignoring spins we have two independent amplitudes F_o and F_1 corresponding to the total isospins 0 and 1. Proton-proton and neutron-neutron cross sections are given by the same expression

$$\frac{d\sigma_{pp}}{d\Omega} = E^{-2}\left[|F_1(E,\theta)|^2 e^{-b(E,\theta)} + \right.$$

$$\left. + \; |F_1(E,\pi-\theta)|^2 e^{-b(E,\pi-\theta)}\right] = \frac{d\sigma_{nn}}{d\Omega} . \qquad (16)$$

We have already discussed the fact that $b(E,\pi-\theta) = \bar{n} = \bar{n}_+ + \bar{n}_- + \bar{n}_o$ is expected to be large compared to $b(E,\theta)$ for $\theta < \frac{\pi}{2}$, and therefore the second term in eq. (16) may be neglected for $\theta < \frac{\pi}{2}$. For $\theta > \frac{\pi}{2}$ the first term will become negligible.

In the case of proton-neutron collisions we have to differentiate between elastic and charge exchange scattering. Here we have

$$\frac{d\sigma}{d\Omega}\left(p + n \to \begin{matrix} p+n \\ n+p \end{matrix}\right) = \frac{1}{4E^2}|F_o \pm F_1|^2 e^{-b} . \qquad (17)$$

Experimentally, one cannot discriminate between elastic p-n scattering at the angle θ and charge exchange scattering at $\pi-\theta$. The observed cross section is therefore

$$\frac{d\sigma_{pn}}{d\Omega} = \frac{1}{4E^2}\left[|F_o+F_1|^2_\theta \; e^{-b(\theta)} + |F_o-F_1|^2_{\pi-\theta} \; e^{-b(\pi-\theta)}\right]. \qquad (18)$$

Here we did not write down the energy dependence explicitly. At the moment we do not know the relative phase between F_o and F_1 nor the ratio of their moduli.

Therefore, we cannot say whether one of the two terms in eq. (18) is negligible for certain values of θ.

4.2. Two - pion production

Let us consider the process [24]

$$p + p \to p + p + 2 \text{ pions},$$

where the protons are scattered with large momentum transfers, the total energy of the pions in the c.m. system is small compared to the nucleon energies, and the pions are "nonresonant" (no decay products of isobars or meson resonances). From eqs. (14) and (15) we get

$$\frac{d\sigma_{pp}^{(\pi^+\pi^-)}}{d\Omega} = \frac{d\sigma_{pp}^{el}}{d\Omega}\left(g_+^{(1)} \; g_-^{(2)} + g_+^{(2)} \; g_-^{(1)}\right) \qquad (19a)$$

and

$$\frac{d\sigma_{pp}^{(\pi^o,\pi^o)}}{d\Omega} = \frac{d\sigma_{pp}^{el}}{d\Omega} \, g_o^{(1)} \; g_o^{(2)} \quad , \qquad (19b)$$

where the $g_\beta^{(1)}$ etc. are the quantities defined in eq. (15). Now we make the plausible assumption that all the g_β, $\beta=+,-,o$, are equal:

$$g_+ = g_- = g_o = \frac{1}{3} b \quad . \qquad (20)$$

Taking our b from eq. (8) we can predict the absolute values of the cross sections (19a) and (19b) by inserting the known values of the elastic cross section. A comparison of these predicted values with experiments has not yet been done.

4.3. Ratios of multiplicities for different types of secondaries

We now discuss some estimates for the ratios of differential multiplicities $\bar{n}(E,\theta)$ for different types β of secondaries like pions, kaons, vector mesons etc. We can do this in the framework of SU(3) or probably even SU(6), because we assume the secondaries to "see" the primary particles in their nonrelativistic limit in the sense of negligible recoils. However, the notion of unitary symmetries is not necessary for the following.

We crudely approximate the integrand $|j_\beta(k)|^2$ in eq. (15) by the constant $g^2_{\beta BB}\ m^{-2}$, where $g_{\beta BB}$ is the (Yukawa) baryon coupling constant of the particle of type β . The arbitrary rest mass factor is introduced for dimensional reasons. With this approximation we have

$$\bar{n}_\beta(E,\theta) = g^2_{\beta BB}\ m^{-2}\ \Phi_\beta(E,\theta)\ , \tag{21}$$

where $\Phi_\beta(E,\theta) = \int \frac{d^3k}{2\omega}$ is the effective phase space of the particle of type β . This phase space is a function of the variables E and θ of the primary particles. From eq. (21) we get

$$\bar{n}_{\beta_1}(E,\theta) : \bar{n}_{\beta_2}(E,\theta) = g^2_{\beta_1}\ \Phi_{\beta_1} : g^2_{\beta_2}\ \Phi_{\beta_2}\ . \tag{22}$$

If the mass differences between the particles β_1 and β_2 are not too large one may be optimistic and assume $\Phi_{\beta_1} \simeq \Phi_{\beta_2}$.

Then we have

$$\bar{n}_{\beta_1} : \bar{n}_{\beta_2} = g^2_{\beta_1} : g^2_{\beta_2}\ . \tag{23}$$

Thus the ratio $\bar{n}_{\beta_1} : \bar{n}_{\beta_2}$ becomes energy independent and rather simple in the crude approximation considered here.

Example:

Using the empirical values [25] for the nucleon coupling constants g_π, g_ρ, g_ω and g_ϕ for the pions, ρ-, ω- and ϕ- mesons, eq. (23) predicts

$$\bar{n}_\pi : \bar{n}_\rho : \bar{n}_\omega : \bar{n}_\phi \simeq 15 : 1 : 5 : 0\ . \tag{24}$$

The ratios apply to the charge state 0. In the case of other charge states, (π,ρ) , one has to multiply the above values by the number of charge states.

The predicted rather small multiplicities of ρ-mesons by eq. (24) may be misleading, because we have ignored the rather large magnetic coupling $f_\rho \simeq 4g_\rho$ of the ρ-meson. This coupling, which seems to be rather small for the ω and ϕ mesons, may enhance the ρ-value in eq. (24).

5. Bremsstrahlung model with realistic phase spaces

Up to now we have optimistically more or less ignored the amount of energy and momentum carried away by the secondaries, assuming this amount to be small or at least ignorable for a clever choice of the variables. Such an attitude is in many cases too optimistic and one has to weaken the assumptions. One interesting possibility is to keep the "softness"- or "smoothness" - hypothesis for the collision amplitudes but to use realistic phase spaces for the secondaries.

Such a procedure was first proposed by Anderson and Collins for inelastic p-p collisions [15]. A similar analysis was applied by Sosnowski and coworkers [16] to inelastic pion-nucleon scattering. Here we shall discuss the method of Mack which is most closely related to the model described in the previous parts of this section [17]. We shall not outline all the details of the calculations which may be taken from Mack's paper, but rather mention the general features. Only pions are being considered as secondaries.

According to eq. (15) we have for the quantity $\bar{n}$ in eq. (2) the expression

$$\bar{n}(E,\theta) = \int \frac{d^3k}{2\omega} |j(k)|^2 .$$

We approximate this by

$$\bar{n}(E,\theta) \simeq J(E,\theta) \quad \Phi_1(E,\theta) ,$$

$$J(E,\theta) = |j(k \simeq o)|^2 ,$$

$$\Phi_1 = \int \frac{d^3k}{2\omega} .$$

Φ_1 is the effective phase space for one soft secondary. Eq. (2) then means that we have approximated the actual phase space of n pions by the n-th power of the one-pion phase space. In order to be more realistic one should use the actual phase space instead of the product Φ_1^n . If we keep the assumption that the two nucleons go into opposite directions after the scattering, even in inelastic collisions, and if only one proton is observed in the final state, we get for the reaction

$$p + p \rightarrow p + \text{nucleon} + n \text{ pions}$$

the cross section

$$\frac{d^2\sigma^{(n)}}{d\Omega dp}(E,p,\theta) = C_n B\sigma^{el}(E,\theta) \frac{1}{n!} J^n(E,\theta)\Phi_n(W) . \qquad (25)$$

Here $\Phi_n(W)$ is the invariant phase space for one nucleon and n pions with the invariant mass W(p), where p is the momentum of the observed proton after the scattering. C_n is a factor taking the different pion charge states into account and B is a kinematical factor. Both factors are defined in ref. 17 . The phase space can be calculated by the convenient method of Mazur and Lurçat [26] . The normalization in eq. (25) is such that

$$\int \frac{d^2\sigma^{(o)}}{dp\, d\Omega} dp = \sigma^{el} .$$

In order to exploit eq. (25) Mack's procedure is as follows: Experimentally, one knows [10] some values of

$$\frac{d^2\sigma}{dp\, d\Omega} = \sum_{n=o}^{\infty} \frac{d^2\sigma^{(n)}}{dp d\Omega}$$

for certain initial energies E and scattering angles θ as a function of p. By a least square fit one determines σ^{el} and J as a function of E and θ. As a result of this fitting one can say the following:

5.1. The experimental curves of $d^2\sigma/d\Omega dp$ can be accounted for very well by this 2-parameter fit (for fixed E and θ).

5.2. The values of $\sigma^{el}(E,\theta)$ so obtained are compatible with the experimentally measured ones. In particular one can predict the strong decrease of the elastic cross section with increasing energy E at fixed θ.

5.3. Predictions can be made for the following quantities at the energies, angles and momenta dealt with in ref. 10):

a) The differential multiplicity $\bar{n}(E,\theta,p)$,

b) the fluctuations $(\overline{n^2}-\bar{n}^2)(E,\theta,p)$,

c) the differential total cross section $\bar{\sigma}(E,\theta) = \int dp \frac{d^2\sigma}{dp d\Omega}$

d) and the multiplicity

$$\bar{n}_{av}(E,\theta) = \frac{1}{\bar{\sigma}} \int dp\ \bar{n}(E,p,\theta) \frac{d^2\sigma}{dp d\Omega} .$$

The numerical values of these predictions are contained in ref. 17 . One has to wait for future experiments in order to test them.

III. Electron-Nucleon Collisions

We shall now apply the general assumptions of section II to elastic and inelastic electron-nucleon scattering. Unfortunately, there is at the moment almost no experimental information about electroproduction cross sections where more than the electron and the recoil nucleon have been observed in multi-particle final states with large momentum transfers of the electron [27]. Thus one can only make predictions. In order to get something to compare with experiments we follow Mack [28],[29] and add some new dynamical assumptions in part 2 which allow us to calculate an explicit expression for the nucleon magnetic form factor at large momentum transfers.

1. A Model for the Electroproduction of Soft Mesons at Large Electron Momentum Transfer

We again start by assuming only one type of secondaries. Extensions to different types along the

lines discussed for p-p collisions in section II are obvious. Therefore, we shall have only a few remarks about the role of unitary symmetries in this section.

Furthermore, we shall ignore all electromagnetic radiative corrections.

1.1. Kinematics

We use the variables

$$s=(p_{el} + p_{nucl})^2 , \quad t = (p'_{el} - p_{el})^2, \quad q^o = (p_{el}-p'_{el})^o ,$$

where the dashed variables correspond to the final state.

For s, $-t \gg 4 M^2$ and $G_E(t) \le G_M(t) \equiv G(t)$, where M is the nucleon mass and G_E and G_M the usual electric and magnetic form factors, the Rosenbluth formula [30] has the simple form

$$\frac{d\sigma^{el}}{dt} = \sigma_B\ (s,t)\ G^2(t) \quad , \qquad (26)$$

where

$$\sigma_B = \frac{2\pi\alpha^2}{t^2 s^2} (t^2+2st+2s^2)$$

is the Born approximation which goes to

$$\sigma_B \xrightarrow{s\to\infty} \frac{4\pi\alpha^2}{t^2} \qquad (27)$$

for $s \to \infty$ and fixed t. For details of the inelastic kinematics - which we do not need here - see ref. 31).

1.2. Elastic scattering

Comparing the eqs. (1) and (26) gives

$$\sigma_B(s,t)\ G^2(t) = \bar{\sigma}(s,t)\ e^{-\bar{n}(s,t)},\ -t >> 4M^2 \quad . (28)$$

In order to extract more information from this equation we follow Wu and Yang [9] and assume the elastic form factor for large -t to be completely determined by the inelastic channels. Since the inelastic channels in our model are accounted for by the exponential on the right hand side of eq. (28), it is tempting to put

$$\bar{n}(s,t) = -\ln G^2(t) \ . \qquad (29)$$

As in the case of p-p scattering (II,2.1.) we have the normalization problem here, too. Small deviations of the normalization constant $\beta = G^2(t)/\exp(-\bar{n})$ from the value 1 do not affect the relation (29) for large momentum transfers. Despite some doubts, let us stick to the normalization $\beta = 1$ in the following. At the end we shall use the slightly different normalization $\beta = \mu_p^2$, where μ_p is the magnetic moment of the proton (or neutron). This latter normalization seems to be more appropriate if one wants to extrapolate the relation (29) to small values of -t. It then seems reasonable to have $\bar{n} = 0$ for $t = 0$.

Eq. (29) gives us an interesting relation between the elastic magnetic form factor G(t) and the differential multiplicity $\bar{n}(s,t)$ in quasielastic electron-nucleon scattering. Unfortunately, there are no data in order to test this relation. If we insert the experimental value [32] of G(t) at $-t = 10(\mathrm{GeV}/c)^2$, we

get $\bar{n} \simeq 8\text{-}9$. This seems to be a rather large number, but one has to wait for the experiments and see! One characteristic feature of the relation (29) is that it predicts that the differential multiplicity will become independent of the initial energy $\sqrt{s}$ for $s,-t >> >> 4M^2$.

If eq. (29) is at least approximately reasonable, it may be useful for the determination of G(t) at very large momentum transfers, for the only quantities to be measured in order to determine G(t) are t and the average number of secondaries. Since the inelastic cross sections are expected to become larger than the elastic one for very large -t (see below), the application of eq. (29) may be helpful for the determination of G(t).

1.3. Inelastic collisions

From eqs. (1) and (29) we get

$$\frac{d\sigma^{(n)}}{dt} = \sigma_B(s,t)\ G^2(t)\ \frac{1}{n!}\ [-\ln G^2(t)]^n \ . \qquad (30)$$

Thus we have definite predictions for the cross sections of quasielastic electron-nucleon scattering with the additional emission of soft nonresonant secondaries. For small n one probably can neglect recoil effects associated with the emission of secondaries. For larger n one has to be more careful. There are several possibilities as to how one actually should compare eq. (30) with future data:

a) If $|t|^{1/2} >> q^o_{c.m.}$ it may be that

$$\frac{d^2\sigma^{(n)}}{dt\,dq^o_{c.m.}}(s,t;q^o_{c.m.}) \simeq \frac{1}{q^o_{c.m.}}\frac{d\sigma^{(n)}}{dt}(s,t), \qquad (31)$$

where $d\sigma^{(n)}/dt$ is given by eq. (30).

b) Another possibility is

$$\int dq^{o}_{c.m.} \frac{d^2\sigma^{(n)}}{dt\, dq^{o}_{c.m.}} (s,t;q^{o}_{c.m.}) \simeq \frac{d\sigma^{(n)}}{dt} (s,t) \ , \qquad (32)$$

where again the right hand side is given by eq. (30).

c) One makes similar phase space corrections as in the p-p case (see II,5.). So far this has not been done.

1.4. Short-range interactions and sum rule

Summing eq. (30) over n gives

$$\frac{d\sigma^{tot}}{dt} = \bar{\sigma}(s,t) = \sigma_B(s,t); \ s,-t >> 4M^2 \ . \qquad (33)$$

Since σ_B does not contain any rest masses (see eq. (26)), and since the fine structure constant α is dimensionless, we see that our fourth postulate from part 1 of section II, namely dilatation invariance for the short-range interactions, is fulfilled automatically here. In addition the sum rule (33) is compatible with a very similar inequality derived by Bjorken [33] for $d\sigma^{tot}/dt$ at fixed t in the limit $s \to \infty$ from chiral current algebra:

$$\frac{d\sigma_p^{tot}}{dt} + \frac{d\sigma_n^{tot}}{dt} \geq \frac{2\pi\alpha^2}{t^2} \ . \qquad (34)$$

The indices "p" and "n" refer to inelastic electron scattering off protons and neutrons respectively. If we assume the above proton and neutron cross sections to be approximately equal, we get from eq. (33) four times the value of the lower limit required by eq. (34). At the moment there are no data in order to test the

relations (33) and (34) for $-t > 4M^2$. At $t \simeq -1(\text{GeV/c})^2$ the inequality (34) seems to be roughly fulfilled [31].

We should not forget, however, that the numerical value of eq. (33) depends on the normalization β. With our second normalization $\beta = \mu_p^2$ the expression (33) would be larger by a factor μ_p^2.

1.5. Equality of proton and neutron magnetic form factors as a consequence of SU(2)

It was already pointed out in section II that the exponential $\exp(-\bar{n})$ is a unitary spin invariant. If we consider in particular the isospin group SU(2) and apply the above result to the proton and neutron magnetic form factors G_p and G_n by using the eq. (29), we obtain

$$\ln G_p(t) = \ln G_n(t) \ . \qquad (35)$$

This relation is fulfilled experimentally up to the logarithm of the ratio of the proton and neutron magnetic moments squared, a number which is negligible compared to $\ln G^2$ for large $-t$. If we chose our second normalization, the result (35) would be the same as the usual scaling law [34].

It is interesting that we already get the relation (35) from SU(2). It has previously been derived from higher symmetries [35].

2. A calculation of the nucleon magnetic form factor for large momentum transfers

A characteristic feature of the elastic cross section (26) and of our model in particular is a factorization into a dilatation invariant (better: covariant) factor $\bar{\sigma}$ and a second one which does not have this invariance, for otherwise it would be a constant because it is dimensionless. This second factor is therefore in some sense associated with the breaking of dilatation invariance at very large momentum transfers. Thus, it seems likely that more detailed information about the structure of this symmetry breaking will yield some information about the large momentum transfer behaviour of form factors.

In order to describe this symmetry breaking it is very useful to employ the methods developed in connection with the hypothesis of a partially conserved axial vector current. This has been done by Mack [29]. We shall not repeat the details of calculations here but shall briefly sketch the ideas and one result which is interesting for us.

2.1. Partially conserved dilatation current

The action integrals of the coupling terms in many interesting Lagrangian field theories are invariant under the transformations

$$x^\mu \to x^{\mu\prime} = \rho x^\mu, \qquad \mu = 0,1,2,3 \quad ;$$

$$\phi(x) \to \phi'(x') = \rho^\ell \phi(x) \;, \qquad (36)$$

where $\phi(x)$ is a certain spinor- or tensor-field with respect to the homogeneous Lorentz group. We have $\ell = -1$ for scalar, pseudoscalar and vector fields $A(x)$ and $A_\mu(x)$, $\mu=0,1,2,3$, and $\ell = -3/2$ for spin 1/2 fields $\psi(x)$. Invariant are all interactions with a dimensionless coupling constant like $\bar{\psi}\psi A$, $\bar{\psi}\gamma_5\psi A$, $\bar{\psi}\gamma_\mu\psi A^\mu$ etc. The kinetic terms are invariant, too, except for

the mass part: the rest masses destroy the dilatation invariance.

One can define a dilatation current $\mathcal{D}_\mu(x)$ in the usual way à la Noether and find its divergence $\partial^\mu \mathcal{D}_\mu$ proportional to the kinetic mass terms, for instance $m^2 A^+(x)A(x)$ for a free pseudoscalar field and $\alpha_o u_o + \alpha_8 u_8$ for the quark model, where $u_o = \bar{t}t$ breaks the chiral invariance and $u_8 = \bar{t}\,\lambda_8\, t$ breaks the SU(3) symmetry.

In the same way as in the algebra of currents one can define a time dependent "charge"

$$D(t) = \int d^3\vec{x}\; \mathcal{D}_o(x)$$

which generates the infinitesimal transformations (36):

$$[D(t)\;,\phi(x)]_{x^o=t} = i^{-1}(-\ell + x^\nu\partial_\nu)\phi(x) \;. \qquad (37)$$

The important point is now that the divergence $U(x) \equiv \partial^\mu\; \mathcal{D}_\mu(x)$ is a local field with quantum numbers $J^P = 0^+$, $I = 0$. This can be seen, for instance, from the examples given above. One now makes the "PCAC"-assumption that the matrix elements $\langle p'|U(o)|p\rangle$ are small for large momentum transfers $(p'-p)^2$, but relatively large for small $(p'-p)^2$. In addition one assumes $\langle 0^+, I=0|U(o)|0\rangle \neq 0$, i.e. $U(x)$ is an interpolating field for a 0^+, $I=0$ asymptotic free state. Such a state has the same quantum numbers as the more or less hypothetic σ-particle. But we do not require that such a particle really exists. The asymptotic state may be a 2-pion s-state, etc.

With the above assumptions one can play the usual soft - meson [36] game: Write down a generalized Ward-Takahashi identity, use the equal time commutation relations (37), take the Fourier transform etc. Having done all this one can express the τ-function for the process $A \to B + 0^+$(soft) in terms of the τ-function for the process $A \to B$:

$$\tau_{A\to B+0^+}[k(0^+) \simeq 0;\ p_2,\dots,p_n] =$$

$$= -i \sum_{j=2}^{n} (\ell_j + 4 + p_j^\nu \frac{\partial}{\partial p_j^\nu})\tau_{A\to B}\ (p_2,\dots,p_n)\ . \qquad (38)$$

Application of this relation to the electromagnetic form factor $F_+(t)$ of a scalar particle gives the amplitude $T^\mu(p,p',k\simeq 0)$ for the corresponding process where one additional soft 0^+-particle is emitted:

$$T^\mu(p,p',k) = -\ 2M^2 [\frac{1}{-2p.k+k^2} (p'+p-k)^\mu +$$

$$+ \frac{1}{2p'.k+k^2}(p'+p+k)^\mu]\ F_+(t) + \qquad (39)$$

$$+ 2(p'+p)^\mu\ t\frac{\partial}{\partial t}\ F_+(t) + O(k)\ .$$

In eq. (39) we have $p^2 = p'^2 = M^2$. The two first terms of the order k^{-1} describe those processes where the soft meson is emitted from the external lines, i.e. "before"and "after" the interaction of the primary hadron. The third term in eq. (39) describes the emission "during" the interaction.

2.2. Discussion of some approximations

In order to apply the relation (39) to the nucleon form factor G(t) we have to make some questionable approximations: First we shall ignore the spin of the

nucleon. This does not seem to be very critical. For in the region $-t \gg 4M^2$ and $s \to \infty$ which we consider here, the electron-nucleon elastic cross section in eqs. (26) and (27) has the same form as the cross section for the electron scattering off spin zero particles. Furthermore, in one approach we shall treat the pions which we expect to form the bulk of the secondaries in inelastic electron-nucleon collisions as scalar particles rather than pseudoscalar ones. Two heuristic arguments for such a crude-looking approximation may be given: First we have seen that we can neglect the spin terms in the Rosenbluth formula in the ultrarelativistic region. Thus spin, and therefore parity, too, do not look so important any more. Secondly, if we look at the first order $\bar{u}(1')\gamma_5 u(1)\ A(2)$ of the pseudoscalar coupling, we have $\gamma_5 u(1) \simeq \pm\ u(1)$ in the extreme relativistic limit for the nucleons. Thus the pseudoscalar vertex looks very similar to the scalar one in this limit, and it may be that we can ignore parity altogether. Obviously, all these arguments are not reliable.

If we keep the small momenta k fixed in eq. (39), then the third term contains one more power of the energy in the numerator than the two first terms. Thus we expect it to dominate in the high energy region, although it is of a higher order in k than the first two terms. The trouble here is that one has to deal with two limits at the same time: $p \to \infty$, $k \to 0$, a problem which does not arise in low energy soft meson theorems. Since the physical mesons do have a finite rest mass after all, it seems reasonable to keep $k \neq 0$ fixed and perform the limit $p \to \infty$ first. Intuitively this means that we assume the emission of secondary mesons at high energies and large momentum transfers to be more likely during the primary interaction than before and after.

It would be interesting to see how a "Low"-type formula [37] like eq. (39) looks for soft pseudoscalar pions. To the best of my knowledge, only those terms corresponding to the emission of soft pions from the external lines have been calculated [38]. Their application to high energy processes does not look very encouraging [12].

2.3. A differential equation for the form factor

Keeping only the third term in eq. (39) we get as the differential cross section for the electroproduction of one soft meson:

$$\frac{d^2\sigma^{(1)}}{dt\,d\Phi}\ (s,t;\ k\simeq 0) = \frac{4g^2}{M^2}\ [t\ \frac{\partial}{\partial t}\ G(t)]^2\sigma_B(s,t)\ . \quad (40)$$

Here Φ is the meson phase space and g the meson-nucleon coupling constant. Since we are not interested in the kinematics of the emitted meson, we rewrite eq. (40) in the following way

$$\frac{d\sigma^{(1)}}{dt}\ (s,t) = \frac{4g^2}{M^2}\ \sigma_B(s,t)\ [t\ \frac{\partial}{\partial t}\ G(t)]^2\ \Phi_{eff}\ , \quad (41)$$

where Φ_{eff} is some effective unknown phase space for the meson. Since we have assumed the mesons to be soft, their effective phase space should not depend too violently on s and t. In the following, we shall make the crude approximation Φ_{eff} = const.

The crucial step now is this: We assume that the cross section $d\sigma^{(1)}/dt$ in eq. (41) may also be expressed by the formula (30) with n = 1. This hypothesis

leads to the consistency condition

$$[t\ \frac{\partial}{\partial t}\ G(t)]^2\frac{4g^2}{M^2}\ \Phi_{eff} = -\ G^2(t)\ \ln\ G^2(t)\ . \quad (42)$$

The solution of this differential equation is

$$G(t) = \exp\ [-\ A\ \ln^2(-at)]\ , \quad (43)$$

with $A = M^2/(8g^2\Phi_{eff})$ and a as a constant of integration. The above procedure leading to the form factor (43) may be interpreted in different physical terms:

a) We ignore the intrinsic parity of pions and identify the scalar mesons with them. The problems associated with this interpretation have already been discussed. If we insert for g^2 the value of the pion-nucleon coupling constant squared and for Φ_{eff} the estimated value m_π^2, we obtain an A which is of the order of magnitude 10^{-1}. The value of the constant a is, of course, not known. A rough guess is that it may be of the order m_π^{-2} . These estimates of A and a give indeed a reasonable G(t) as can be seen by a comparison of the expression (43) with the latest Stanford data [39] up to $t = -25(GeV/c)^2$.

b) We interpret the scalar meson as a 2-pion s-state (nonresonant or resonant). The questionable assumption then is that we can describe this pair by the formula (30) with n=1, although we are essentially dealing with two secondaries. Furthermore, it seems doubtful that the main bulk of secondaries in multiparticle final states is produced in such pairs. However, nothing is known experimentally about this. In this second interpretation g is an effective 2-pion-nucleon coupling constant and Φ_{eff} a two-pion effective phase space. It is hard to give estimates for these quantities, although one may take for g the effective g_σ-coupling constant from dispersion theoretical analysis [25],[40].

c) A third possibility is to keep the 2-pion interpretation of the scalar meson but to use the formula (30) with n=2. The right hand side of eq. (42) is then slightly changed, and we get as a solution of this new differential equation

$$G(t) = \exp(-b|t|^B), \qquad B = \frac{M}{2g\sqrt{\Phi_{eff}}} . \qquad (44)$$

The quantity b is a constant of integration which should be positive if the expression (44) is to make any sense.

2.4. Ad hoc extension of the calculated G(t) to small t.

Despite the fact that the assumptions leading to the expression (43) are only valid for large -t, one may nevertheless ask how it behaves around t=0. It obviously has the wrong threshold properties at this point. The cut should start at $t = 4m_\pi^2$, not at t = 0. If one makes an ad hoc correction in order to have the cut started at the right t-value - even if the type of branch point remains wrong -, one gets

$$G(t) = \exp\{-A[\ln^2(-a(t-4m_\pi^2)) - \ln^2(a4m_\pi^2)]\}. \qquad (45)$$

In ref.[28],[39] the normalization on the left hand side of eq. (45) is $G(t)/\mu_p$ where μ_p is the total magnetic moment of the proton. As already mentioned before, a comparison with experiments [39] shows the expression (45) to be quite reasonable.

Footnotes and References

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