Polarized Protons in HERA

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Abstract: This article outlines the implications of obtaining polarized protons at high energy in HERA and presents some results of calculations of the equilibrium spin distribution at high energy.

1 Introduction

In May 1994 longitudinal spin polarization was achieved at the HERA electron ring [1] and longitudinal polarization is now provided on a regular basis for the HERMES experiment.

Following this success one should investigate whether we can complement the polarized $e^{\pm}$ beam with polarized protons. For the high energy physics justification see the contributions by the Polarized Protons and Electrons Working Group in these Proceedings.

However, owing to their high mass in comparison to electrons, protons at HERA-p emit almost no synchrotron radiation so that there is no natural polarization process analogous to the Sokolov-Ternov effect, which is used to generate the $e^{\pm}$ polarization [2]. So the attainment of polarized protons in storage rings must proceed along lines very different from those for electrons.

The first ideas on how to approach this task emerged at DESY in 1994 and since then extensive studies of spin motion at high energy in HERA have been made by DESY [3, 4, 5, 6, 7] and by the Institute for Nuclear Research, Troitsk, Russia [8].

Early in 1996 we were joined by the SPIN Collaboration, a very large group dedicated to topics in the high energy physics of proton polarization and the machine physics of providing polarized protons and accelerating them [9]. The results of a study by the SPIN collaboration and the DESY team will appear soon [10]. In addition, polarization specialists from the Budker Institute of Nuclear Physics in Novosibirsk are participating.

As we will see in subsequent sections, the attainment of stored high energy protons in HERA will not be simple and will require extensive additions to the DESY accelerators. Thus at least four requirements must be fullfilled in order for this project to be realised. It must be demonstrated by preliminary tests and by simulations that the machine physics is feasible, the high energy physics community must present a strong case, there must be sufficient luminosity, and of course the money and infrastructure must be available.

This article will explore some aspects of the feasibilty and the luminosity. No attempt will be made to present a final solution —- we are still in the process of identifying problems. Instead, the minimum modifications to the HERA complex likely to be necessary to obtain polarized protons will be described together with some results of calculations for high energy spin motion.

2 Obtaining high energy polarized protons

Currently, the only promising way to obtain high energy polarized protons in HERA is to accelerate polarized protons provided by a source of polarized H^- ions. This is far from being straightforward, but polarized protons of a few GeV have been in regular use for many years and polarized protons have been accelerated to above 20 GeV at the Brookhaven AGS [11]. See Table 1.

Table 1: The quest for high energy polarized protons.

Accelerator	Energy
ZGS	12 GeV
KEK PS	12 GeV
AGS	22 GeV
IUCF	1 GeV
Saturn II	3 GeV
PSI Cyclotron	0.59 GeV
TRIUMF Cyclotron	0.5 MeV
LAMPF	0.8 MeV

So one might claim that acceleration of polarized protons up to a few tens of GeV is understood, at least in principle. But as we will see in the following sections, acceleration beyond the top energy of PETRA up to about 1 TeV is a much more demanding exercise. Fortunately, other studies of the problems have already been made and help to show the way. These are listed in Table 2.

Table 2: Other high energy accelerators whose polarized proton capability has been analyzed.

Accelerator	Energy
Fermilab Main Injector	120 GeV
Fermilab TEVATRON	900 GeV
LISS	20 GeV
RHIC	250 GeV

A key aspect of HERA is the fact that the protons must remain polarized at $\approx$ 820 GeV, without replenishment of the polarization, for the lifetime of the beam, namely about ten hours.

Furthermore, no extensive spin tracking studies of acceleration to the 1 TeV domain have been made previously. These are now underway at DESY. Therefore the strategy adopted at DESY is to separate the investigations conceptually into three parts:

- The polarized source and the acceleration process to $\approx 39GeV$.
- Acceleration in HERA to high energy.
- The phase space distribution of the polarization at high energy and its lifetime.

Naturally, much effort could be saved if there were a way to polarize the high energy proton beam *in situ*. This would also have the potential advantage of continually 'topping up' the polarization if there were significant depolarization (e.g. by the beam–beam interaction). Indeed, two alternative concepts based on polarizing the beam at high energy have been suggested [3]. But since the alternatives do not appear to be promising at present we will limit the discussion to exploration of the conventional route, namely acceleration of a polarized beam from low energy.

2.1 The source and acceleration from low energy

The acceleration chain for HERA consists of:
$SOURCE \rightarrow RFQ(750KeV) \rightarrow LINAC\ III(50MeV) \rightarrow DESY\ III(7.5GeV/c) \rightarrow PETRA\ II(39GeV/c) \rightarrow HERA(820GeV/c)$.

Now we discuss some of these systems. For polarized protons one needs a new H^- source. There are two basic types: the 'atomic beam' and the 'optically pumped' sources [9]. They supply up to 80 percent polarization. So far currents of 1.6 mA, DC, have been reached with an optically pumped source [9, 10, 12]. But it is expected that a pulsed current of 20 mA could be reached after some further development [10]. If the transmission efficiency of the low energy beam transport line and the LINAC could be brought close to 100%, there would probably be no loss of luminosity with respect to the luminosity currently available. Thus the existing radio frequency quadrupole (RFQ) might have to be replaced with a new version with much greater acceptance.

Spin precession in electric and magnetic fields is described by the T–BMT equation [13, 14]:

$$\frac{d\vec{S}}{ds} = \vec{\Omega} \times \vec{S} \tag{1}$$

where $\vec{\Omega}$ depends on the electric and magnetic fields, the velocity and the energy. In magnetic fields equation (1) can be written as:

$$\frac{d\vec{S}}{ds} = \frac{e\vec{S}}{mc\gamma} \times ((1+a)\vec{B}_{||} + (1+a\gamma)\vec{B}_{\perp}) \ , \quad a = \left(\frac{g-2}{2}\right) \tag{2}$$

where $\vec{B}_{||}$ and $\vec{B}_{\perp}$ are the magnetic fields parallel and perpendicular to the trajectory. The arc length is denoted by s and the other symbols have the usual meanings. The gyromagnetic anomaly, $\frac{g-2}{2}$, for protons is about 1.79. The full T–BMT equation contains terms depending on the electric field but these vanish if the electric field is parallel to the velocity as in a linac.

The T–BMT equation shows that for motion transverse to the magnetic field, the spin precesses around the field at a rate which is $a\gamma$ faster than the rate of rotation of the orbit direction. Thus

$$\delta\theta_{spin} = a\gamma\delta\theta_{orbit} \tag{3}$$

in an obvious notation. So in one turn around the design orbit of a flat storage ring a non–vertical spin makes $a\gamma$ full precessions. We call this latter quantity the 'naive spin tune' and denote it by ν_0. It gives the natural spin precession frequency in the vertical dipole fields of the ring. At 820 GeV, ν_0 is about 1557. Therefore a 1 mrad orbit deflection will cause about 90 degrees of spin precession. Thus spin is extremely sensitive to perturbing fields and in particular to the radial fields in the quadrupoles.

To reach DESYIII the spins pass through the off–axis horizontal and vertical fields of the various quadrupoles in the LINAC and the transfer line only once. But acceleration in DESY, PETRA, and HERA requires that the spin ensemble passes through the complex structure of quadrupole fields many millions of times. This is the point at which a detailed understanding of spin dynamics in storage rings becomes necessary.

If a circular accelerator only had vertical (dipole) fields, vertical spins would not be affected. But the spins 'see' radial fields in the quadrupoles or combined function magnets since the beam has a non–zero height and even in a nominally flat ring the periodic (closed) orbit is offset vertically owing to unavoidable magnet misalignments. According to the T–BMT equation these fields tend to tilt the spins away from the vertical. The disturbance is different for each spin since it depends on the position of the particle. For particles circulating for many turns the total disturbance can grow to become very big if there is coherence between the natural spin motion and the oscillatory motion in the beam characterized by the spin–orbit resonance condition:

$$\nu_{spin} = m + m_x \cdot Q_x + m_z \cdot Q_z + m_s \cdot Q_s \tag{4}$$

where the m's are integers and the Q's are respectively the horizontal, vertical and longitudinal tunes of the orbital oscillations. Here, ν_{spin} is the real (i.e. not naive) spin tune. In the presence of misalignments, rotators and snakes (see below), the unit vector describing the periodic spin direction on the closed orbit, $\vec{n}_0(s)$, is not vertical so that the concept of spin tune must be generalised to be the effective number of spin precessions around $\vec{n}_0$ per turn. In general $\nu_{spin} \neq \nu_0$ but in a nominally flat ring ν_{spin} will not deviate very far from $a\gamma$. The vector $\vec{n}_0(s)$ also describes the direction of the average spin vector of a bunch i.e. of the polarization direction of a bunch.

Now we can see the difficulty: during acceleration, ν_{spin} passes through the resonance condition many thousands of times since ν_0 increases by one unit for every 523 MeV increase in energy. There is therefore a good chance that the beam loses polarization at each resonance so that by full energy the polarization has vanished. Even at fixed nominal beam energy, there are longitudinal (energy) oscillations due to the presence of the RF cavities installed to keep the beam bunched, and the consequent oscillations in the spin tune can, if they have large enough amplitude, also cause resonance crossing.

Thankfully, the situation is not as bad as it might appear: in 1975 Derbenev and Kondratenko proposed an ingenious class of devices which are now called 'Siberian Snakes' [15, 16]. They consist of magnet systems designed to rotate spins (and $\vec{n}_0$) by 180 degrees around an axis in the horizontal plane independently of the particle energy. With such devices it can be arranged that ν_{spin} is *independent* of the particle energy and that at least the first order resonance condition, ($|m_x| + |m_z| + |m_s| = 1$ in eqn.4) is never satisfied. Thus at one stroke depolarization during acceleration or due to energy oscillations is eliminated — at least if the effect of higher order resonances can be ignored. Typically one would arrange to set $\nu_{spin} = 1/2$. For detailed sketches of spin motion in the presence of snakes the reader is referred to [17]. From those sketches one can obtain a helpful view of the resonance suppression mechanism. The spin perturbations accumulated during one turn tend to be cancelled on the next turn after the snake has flipped the spins by 180 degrees. This is very reminiscent of the spin echo effect used in spin–magnetic resonance investigations in materials.

From the T–BMT equation it is clear that at high energy, transverse magnetic fields are much more efficient than solenoid fields for rotating spins. For transverse fields the T–BMT equation takes the form

$$\frac{d\vec{S}}{ds} = \frac{e\vec{S}}{mc} \times (1/\gamma + a)\vec{B}_{\perp} \tag{5}$$

from which it is clear that for protons at high energy the spin precession rate is essentially independent of energy and proportional to the field. We have seen that at 820 GeV with $\nu_0 \approx 1557$ the field needed to give a 1 mrad orbit deflection will cause about 90 degrees of spin precession. Equivalently, a simple rotation of 90 degrees requires a field integral of 2.74 Tesla metres. So at high energy, snakes should be built from strings of interleaved horizontal and vertical dipoles which generate a small local closed bump in the orbit but produce, via the noncommutation of large spin precessions, the required overall 180 degree spin rotation. Since the HERA electron spin rotators operate on this principle, expertise in building such devices already exists at DESY [18]. Since the snake fields are independent of energy, a key problem in snake design is to find solutions for which the orbit excursion at low energy is small, i.e. one or two centimetres. The ultimate in 'interleaving' is a superconducting helical dipole, as for example, in the design proposed for RHIC [19, 20]. It is not yet clear how many snakes will be needed for suppression of depolarization during ramping to 820 GeV in HERA.

An aspect of the HERA proton ring that has nontrivial implications for spin motion is that there are sets of interleaved vertical and horizontal bends on each side of the North, South, and East interaction points. These serve to adjust the height of the proton ring to that of the electron ring at the collision points. Coming from the arc, a proton is first bent 5.74 milliradians downwards, then 60.4 milliradians towards the ring center and finally the orbit is made horizontal by a 5.74 milliradians upward bend. The structure is repeated with interchanged vertical bends on the far side of the interaction point at the entrance to the arc. It is easy to see that with $a\gamma \approx 1557$ the interleaved vertical and horizontal bends cause a large disturbance to the spin motion. In a perfectly aligned ring without vertical bends the equilibrium spin direction on the design orbit is vertical. With vertical bends $\vec{n}_0$ is no longer vertical and strongly energy dependent. Thus the vertical bends must be compensated. One way to do this, proposed by V. Anferov [10], would be to insert a radial Siberian Snake at the midpoint of the 60.4 mrad horizontal bends. More details can be found in [10, 7].

Naturally, spin rotators are also needed to provide a choice of spin orientations at the interaction points.

A suggestion for a snake layout in PETRA can be found in [10].

DESYIII poses a special problem. Down at a few GeV, dipole fields of the magnitude needed for snakes would cause unacceptable orbit distortions. A detailed discussion of spin preservation in DESYIII is beyond the intended scope of this article but more details can be found in [3, 4]. Suffice it to say that devices called 'Partial Snakes' and techniques like orbit and spin tune jumping are under investigation [10, 19, 21]. We have also not discussed the transfer lines.

But one topic deserves special mention, namely proton polarimetry. One clearly needs a polarimeter at the final energy in order to make sense of the measurements of the experimenters [22]. But polarimeters, each chosen for its suitability to handle its energy range, are needed on the low energy part of the chain. Otherwise one will be accelerating 'blindly'.

So far we have dealt only with protons but there is some interest in the particle physics community in obtaining polarized high energy deuterons [23]. For deuterons, $\frac{g-2}{2}$ is only about –0.14 and the deuteron mass is about twice that of the proton. So there are far fewer spin–orbit resonances to cross. But equivalently, the influence of dipole fields on deuteron spins is very small and the design of spin rotators would be far from simple. Nevertheless the possiblity should be kept in mind. In summary, the following modifications, marked by the underlining, to the acceleration chain will be needed.

- The source: <u>atomic beam or optically pumped source of polarized H^- ions.</u>
- Radio frequency quadrupole (RFQ) to 750 KeV: <u>better optimization.</u>
- Proton LINAC to 50 MeV.
- Transfer line to DESY III.
- DESY III: stripping foil injection, $50MeV \rightarrow 7.5GeV$: <u>partial snake and pulsed quads.</u>
- Transfer line DESY III to PETRA II: <u>snakelike spin direction tuner</u>.
- PETRA II: $7.5GeV/c \rightarrow 39GeV/c$: <u>two Siberian Snakes.</u>
- Transfer line PETRA II to HERA: <u>snakelike spin direction tuner</u>.
- HERA: $39GeV/c \rightarrow 820GeV/c$: <u>Siberian Snakes and spin rotators.</u>
- <u>Polarimeters.</u>

This completes the outline of the modifications needed for acceleration from low energy. More information can be found in [3, 4, 10].

Clearly, before embarking on the kinds of upgrades described above one should make computer simulations of the acceleration from low energy and storage at high energy by integrating the equations of combined spin–orbit motion for an ensemble of particles. This is currently the main thrust of the work at DESY. Some results from such calculations are presented in the next section.

3 The equilibrium polarization distribution

The polarization at a point $\vec{z} = (x, p_x, y, p_y, \delta t, \delta E)$ in phase space and azimuth θ in a spin 1/2 beam in an accelerator is defined by the phase space density distribution $\rho(\vec{z};\theta)$, the *value* of the polarization at each point in phase space $\mathcal{P}(\vec{z};\theta)$, and the *direction* of the polarization at each point $\hat{\mathcal{P}}(\vec{z};\theta)$. Once we know these three functions we have a complete specification of the polarization state of the beam.

Since the T–BMT equation is linear in the spin, $\vec{\mathcal{P}}(\vec{z};\theta) = \mathcal{P} \cdot \hat{\mathcal{P}}$ obeys the T–BMT equation [24] which we now write in the form

$$\frac{d\vec{\mathcal{P}}}{d\theta} = \vec{\Omega}(\vec{z},\theta) \times \vec{\mathcal{P}} \; . \tag{6}$$

Because equation (6) describes precession, $\mathcal{P}(\vec{z},\theta)$ is constant along a phase space trajectory.

High energy physics experiments require that the particle bunches look the same from turn to turn, i.e. that they are in equilibrium. Thus in this section we will examine the equilibrium polarization at high energy.

At equilibrium $\rho(\vec{z};\theta)$ is periodic in θ. Furthermore, at equilibrium $\vec{\mathcal{P}}(\vec{z},\theta)$ not only obeys the T–BMT equation, but it is also periodic in θ. We write the equilibrium $\vec{\mathcal{P}}$ as $\vec{\mathcal{P}}_{eq}$ and we denote the unit vector along $\vec{\mathcal{P}}_{eq}(\vec{z},\theta)$ by $\vec{n}(\vec{z},\theta)$. $\vec{n}$ also obeys equation (6) and thus behaves like a spin. The vector $\vec{n}$ was first introduced by Derbenev and Kondratenko [25] in the theory of radiative electron polarization.

The maximizing of the equilibrium polarization of the ensemble implies two conditions:

1. The polarization at each point in phase space should be high.

2. The equilibrium polarization vector at each point in phase space should be almost parallel to the average polarization vector of the beam.

We have seen that at the very high proton energy of HERA the spin motion is extremely sensitive to disturbances and that the quadrupole fields can have a strong influence. As we will see later it can then happen that on average the equilibrium polarization vector deviates by tens of degrees from the average polarization vector of the beam [3]. So even if each point in phase space were 100% polarized the average polarization could be limited to a value much smaller than 100%. The first condition requires that the source delivers high polarization and that the polarization is maintained during acceleration. That was the topic of the last section. The second condition is an intrinsic property of the arrangement of the magnets in the ring, the energy, and the optic. At the interaction points the average direction should, of course, be oriented according to the requirements of the experiments. If it were to be demonstrated that the ring and the optic do not permit a high parallelism, there would be little point in striving to fulfill condition 1.

Clearly, it is very important to have accurate and efficient methods for calculating $\vec{n}(\vec{z},\theta)$.

3.1 Computational techniques

Much of the background to the following material has been covered in several other reports [3, 4, 5, 6] and they should be consulted for mathematical details. Furthermore, an extensive study based mainly on long term spin–orbit tracking can be found in [8]. This also gives examples of spin–orbit motion for various Siberian Snake schemes.

3.1.1 Straightforward polarization tracking

One can try to get information about the spread of the equilibrium spin directions over phase space by tracking a completely polarized beam for many turns. This is illustrated in figure 1. There are no geometrical distortions and the vertical bends are turned off so that the simulation represents a perfectly aligned flat ring. Particles at 100 different phases on a 1 sigma vertical phase space ellipse and zero longitudinal and horizontal emittance have been tracked through HERA for 600 turns while the beam was initially 100% polarized parallel to $\vec{n}_0$ [5, 7]. Similar kinds of tracking results have been presented in [8]. The polarization of the ensemble is defined as the average of the unit spin vectors across phase space. We see that the polarization oscillates wildly. This polarization distribution is obviously not an equilibrium distribution and this implies that the $\vec{n}$-axes are widely spread out away from $\vec{n}_0$. The bold horizontal line is the turn by turn polarization obtained when each spin is initially set parallel to $\vec{n}(\vec{z}, \theta)$ (calculated using the code SPRINT–see below) for a particle at its phase space position $\vec{z}$. Now the averaged polarization stays constant at 0.765. This latter value is consistent with the fact that the average deviation of $\vec{n}$ from $\vec{n}_0$ is about 40 degrees. Therefore, by starting simulations with spins parallel to the $\vec{n}$–axis one can perform a much cleaner analysis of beam polarization in accelerators. In effect one is now able to study the effect of spin perturbations (for example due to field noise, beam-beam effect etc) by searching for small deviations away from the 'stationary state' of the beam instead of having to detect long term drifts in a strongly fluctuating average polarization.

3.1.2 Stroboscopic averaging

The first estimates of the distribution of the $\vec{n}$–axis were made using the SLIM formalism [3]. Further estimates were made using normal form theory [4, 8]. However, those calculations indicated that in a very high energy accelerator such as HERA, $\vec{n}$ can be widely spread out away from $\vec{n}_0$ in confirmation of the results presented in figure 1. Since the SLIM formalism and the normal form formalism are both based on perturbation theory and assume that the tilt of $\vec{n}$ from $\vec{n}_0$ is small it was clear that they were potentially unreliable and that a non–perturbative algorithm was needed.

Thus, a new method for obtaining $\vec{n}$ called stroboscopic averaging [5] has been developed. It is based on multi-turn tracking and the averaging of a special choice of spins viewed stroboscopically from turn to turn. Since this innovative approach only requires tracking data from *one particle*, it is fast and very easy to implement in existing tracking codes. Probably the main advantage over other methods is the fact that stroboscopic averaging does not have an inherent problem with either orbit or spin orbit resonances due to its non–perturbative nature. This allows the analysis of the periodic spin solution close to resonances. This algorithm has

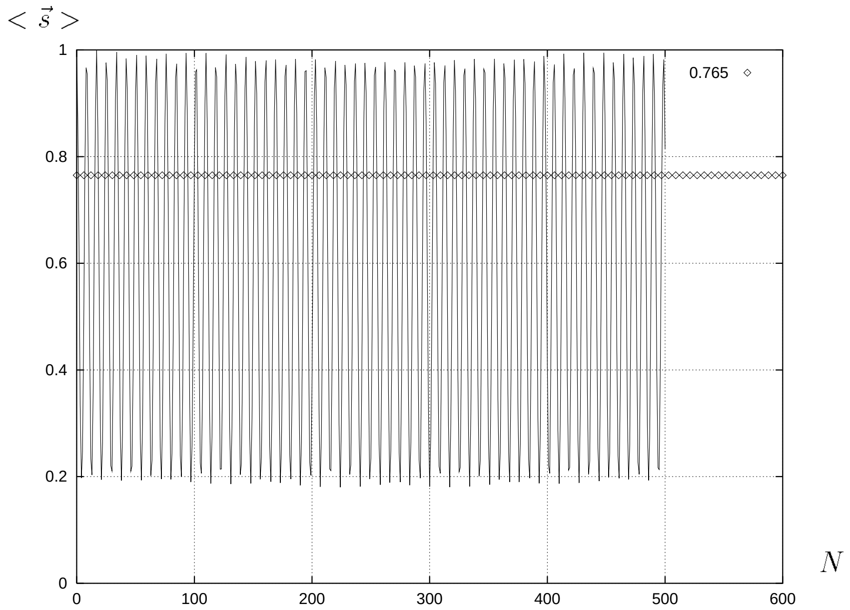

Figure 1: Propagation of a beam that is initially completely polarized parallel to $\vec{n}_0$ leads to a fluctuating average polarization. For another beam that is initially polarized parallel to the periodic spin solution $\vec{n}$ the average polarization stays constant, in this case equal to 0.765.

been implemented in the program SPRINT [5]. This is the code used to calculate the initial $\vec{n}$-axes before propagating the equilibrium distribution in figure 1.

By means of this new tool we are now able to make reliable calculations of $\vec{n}$–axes. We find confirmation of the predictions of SLIM and the normal form analysis, for the basic HERA layout, that the average tilt of $\vec{n}$ from $\vec{n}_0$ can be large, of the order of 60 degrees or more.

However, as mentioned before, Siberian Snakes are not only essential for suppressing depolarization during acceleration but also help at fixed beam energy. In particular they are essential for suppressing spin flip effects due to resonance crossing as the energies of individual particles execute (slow) synchrotron oscillations [8]. Snakes also stabilize the spin motion due to betatron oscillation since the orbital tunes can be chosen so as not to coincide with the fixed real spin tune generated by properly chosen snake layouts.

The next task is then to find snake configurations which indeed counteract the effects of synchrotron and betatron motion. Towards this goal a 'filter' algorithm has been developed which allows the most effective snake layouts to be selected from a starting set [4]. This filtering method has enabled us to identify configurations which strongly reduce the deviation of $\vec{n}$ from $\vec{n}_0$. Figure 2 shows the energy dependence of the average opening angle of the $\vec{n}$–axis for a snake configuration selected by filtering and for two representative orbit amplitudes. The technical details for this example can be found in [7].

4 Further investigations

Apart from the obvious need to find adequate and practical snake schemes, to decide on the best way to realize the compensation of the vertical bends and to design and position the spin

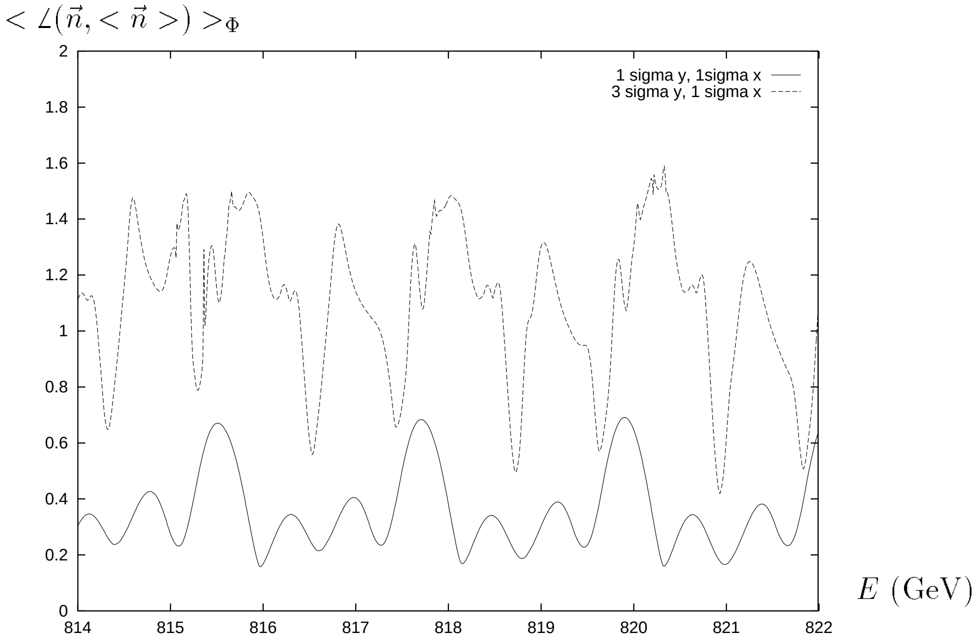

Figure 2: The average opening angle of the equilibrium spin distribution for energies between 814 and 822 GeV after the installation of the snake arrangement found by filtering. The two curves refer to particles with one sigma in the horizontal direction and one or three sigma in the vertical direction respectively.

rotators needed to provide various polarization directions for the experiments at the interaction points, the following topics must be addressed.

1. Understand the effects of misalignments. Find cures. (The most important additional limitation to the polarization is likely to be the spread in the $\vec{n}$-axes caused by misalignments.)
2. Include linear and nonlinear synchrotron oscillations.
3. Determine the maximum allowed emittances.
4. Study the efficacy of the chosen snake scheme for controlling the spin 'equilibrium' during the adiabatic acceleration.
5. Determine the influence of the beam-beam effect.
6. Understand the influence of noise in power supplies.
7. Calculate with optical nonlinearities and optical coupling.
8. Evaluate the effect of intra-beam scattering, if any.
9. Investigate the relevance and feasibility of spin matching [26] and in particular the 'strong spin matching' proposed by K. Steffen [27].
10. Evaluate the advantages of increasing the symmetry of the ring and the optic.

5 Conclusion

The HERA $e^{\pm}$-p colliding beam facility is unique. It provides a high energy, high quality and highly polarized electron (positron) beam for collisions with 820 GeV protons. As a next step one should attempt to polarize the proton beam.

Much work has already been invested in studying ways to accelerate polarized protons up to high energy and most of those ways could be adapted for HERA. It is clear that preparations must begin now.

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