$F_2(x, Q^2)$ and $F_L(x, Q^2)$ at Leading Order, in $\ln(1/x)$ as well as $\alpha_s(Q^2)$

R.S. Thorne

Rutherford Appleton Laboratory, Chilton, Didcot, OX11 0QX, UK

Abstract: We present a brief discussion of the full leading order, including $\ln(1/x)$ terms, renormalization group consistent calculation of structure functions. A fit to the available $F_2(x, Q^2)$ data is performed, and a prediction made for $F_L(x, Q^2)$. Comparison is made with two–loop results.

1 Introduction

Recent measurements of $F_2(x, Q^2)$ at HERA have provided data on a structure function at far lower values of x than any previous experiments, and show that there is a marked rise in $F_2(x, Q^2)$ at very small x down to rather low $Q^2 (\sim 1.5 GeV^2)$.[1, 2] These measurements have led to a great deal of interest in how one should best calculate structure functions, and in particular, whether one should include the leading $\ln(1/x)$ terms for a given power of α_s.

This presentation is a brief advertisement of the fact that the correct renormalization and factorization scheme independent expression for the structure functions naturally includes leading $\ln(1/x)$ terms, but leading in the expressions for the structure functions themselves, not in the unphysical (and hence factorization scheme dependent) anomalous dimensions and coefficient functions. Calculating the structure functions in this manner gives a vey good global fit to the data on $F_2(x, Q^2)$. Many more details of this are in a forthcoming paper.[3] As an example, let us consider an expression for a hypothetical physical quantity depending on the running coupling $\alpha_s(Q^2)$ and another variable N (N may be interpreted as the moment space variable), which can be expressed as

$$F(N, Q^2) = \sum_{m=1}^{\infty} \alpha_s^m(Q^2) \sum_{n=-m}^{\infty} a_{nm} N^n. \tag{1}$$

In the standard loop expansion one includes all terms in the expression for $F(N, Q^2)$ up to a given m, and uses the m–loop coupling. Uncertainties in the expression for $F(N, Q^2)$ due to uncertainty in the definition of the coupling are then of order $\alpha_s^{m+1}(Q^2)$. However, any change in the defintion of the coupling $\alpha_s(Q^2) \to \alpha_s(Q^2) + \mathcal{O}(\alpha_s^2(Q^2))$, does not alter any of the terms of the form $(\alpha_s(Q^2)/N)^m$. Hence, each of these terms is renormalization scheme independent, and should be included in the leading order expression, which uses the one–loop coupling. (Also, since $F(N, Q^2)$ is a physical quantity, factorization scheme independence is guaranteed.) Terms $\sim \alpha_s(Q^2)(\alpha_s(Q^2)/N)^m$ are next–to–leading order, etc. When working with structure functions the details are rather more complicated, but the principle is the same. In essense, since the structure functions may be factorized into inputs at some starting scale Q_0^2, and the evolution to other scales Q^2, the expressions for both the input and the evolution should be

Table 1: Comparison of quality of fits for full leading order (including $\ln(1/x)$) terms, and two–loop fits, MRSR1 and MRSR2. For the LO(x) fit the HERA data is corrected for the low value of $F_L(x,Q^2)$ obtained using this method.

Experiment	# data	χ^2		
		R_1	R_2	LO(x)
H1 F_2^{ep}	193	158	149	121
ZEUS F_2^{ep}	204	326	308	257
BCDMS $F_2^{\mu p}$	174	265	320	200
NMC $F_2^{\mu p}$	129	155	147	147
NMC $F_2^{\mu d}$	129	139	129	142
NMC $F_2^{\mu n}/F_2^{\mu p}$	85	136	132	144
E665 $F_2^{\mu p}$	53	8	8	8
CCFR $F_2^{\nu N}$	66	41	56	49
CCFR $xF_3^{\nu N}$	66	51	47	64

the leading order, including leading $\alpha_s(Q^2)\ln(1/x)$ terms, expressions. (For more details see [3]) The full LO(x) (where we use the abbreviation LO(x) to mean leading order including leading $\alpha_s(Q^2)\ln(1/x)$ terms) expressions involve leading $\ln(1/x)$ terms which are in the form of Catani's factorization scheme independent anomalous dimensions.[4] These appear in the expressions for both the inputs and evolution, and give a theoretical prediction for the small x form of $F_2(x,Q_0^2)$ in terms of $F_L(x,Q_0^2)$ (or *vice versa*, of course).

2 Results

We obtain a good fit for $F_2(x,Q^2)$ data using the LO(x) scheme independent expression and the one–loop coupling constant with $\Lambda^{n_f=4}=90MeV$. Satisfying the theoretical relationship between the small x forms of $F_2(x,Q_0^2)$ and $F_L(x,Q_0^2)$, forces Q_0^2 to be roughly in the range $20-100GeV^2$ (with little sensitivity). Within this range the theoretical prediction for $F_L(x,Q_0^2)$ at very small x is in fact a little ($\sim$ 10-15 %) larger than that obtained from the best fit. Thus, the agreement is qualitatively fairly good. The discrepency may be, at least partially, due to the naive treatment of heavy quark thresholds in this fit, or is perhaps simply due the the fact that this is, after all, really only a leading order treatment. The LO(x) fit can be compared to pair of fits using the standard two–loop expressions[5] where R_1 allows $\Lambda_{\overline{MS}}^{n_f=4}$ to be free (giving $\Lambda_{\overline{MS}}^{n_f=4}=241MeV$) and R_2 fixes $\Lambda_{\overline{MS}}^{n_f=4}$ at the higher value of $344MeV$, forcing a better fit to the HERA data. The results are shown in table 1 (full references for the experimental data can be found in [5]). The full LO(x) scheme independent fit to the HERA data is markedly better than the two–loop fits, and this is combined with a much better fit to the BCDMS data. The fit to the rest of the data is similar in all three cases.

A stronger test of which of the two approaches (if either) is correct comes when the fit to $F_2(x,Q^2)$ is used to predict $F_L(x,Q^2)$. Predictions using the full LO(x) scheme independent expression (the prediction is different to that in [6] because in this previous treatment Q_0^2 was too low to be consistent with the theoretical relationship between inputs) and one of the two–loop expressions are shown in fig. 1. (the two–loop curve corresponds to R_1: that for R_2 is very similar.) A direct measurement of $F_L(x,Q^2)$ at HERA will clearly help a great deal in determining whether the approach advocated in this paper is correct.

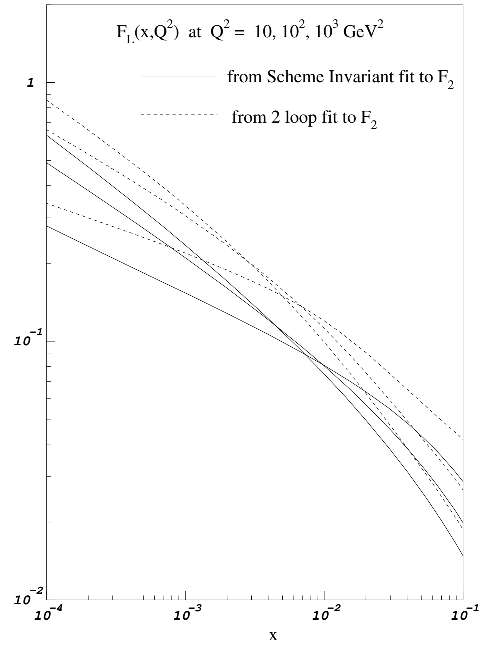

Figure 1: Comparison of predictions for $F_L(x,Q^2)$ using LO(x) fit and two–loop (MRSR1) fit.

Acknowledgments

I would like to thank R.G. Roberts for continual help during the period of this work and for the use of the MRS fit program.

References

[1] H1 collaboration: *Nucl. Phys.* B **470**, 3 (1996).

[2] ZEUS collaboration: M. Derrick *et al*, *Z. Phys.* C **69**, 607 (1996); preprint DESY 96-076 (1996), to be published in *Z. Phys.* C.

[3] R.S. Thorne, preprint RAL-TR-96-065 (to appear).

[4] S. Catani, talk presented at the UK Workshop on HERA Physics (Durham, September 1995), preprint DDF 248/4/96 (to appear); proceedings of International Workshop on Deep Inelastic Scattering and Related Phenomena, DIS 96, (Rome, April 1996).

[5] A.D. Martin, R.G. Roberts and W.J. Stirling, preprint DTP/96/44 or RAL-TR-96-037, to be published in *Phys. Lett.* B.

[6] R.S. Thorne, proceedings of International Workshop on Deep Inelastic Scattering and Related Phenomena, DIS 96, (Rome, April 1996).