Gluon distributions from the CCFM equation.

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Abstract: We solve a unified integral equation (the CCFM equation) for the gluon distribution of a proton in the small x regime. The equation generates a gluon with a steep $x^{-\lambda}$ behaviour, with $\lambda \sim 0.5$. We compare the solution with that of the double-leading-logarithm approximation to Altarelli-Parisi evolution and that of the BFKL equation.

1 Introduction

A theoretical framework which gives a unified treatment of both the DGLAP ($\ln(Q^2)$) and BFKL ($\ln(1/x)$) evolution has been provided by Catani, Ciafaloni, Fiorani and Marchesini [1]. It is based on the coherent radiation of gluons, which leads to an angular ordering of the gluons along a chain of multiple emissions. The CCFM equation is defined in terms of a scale (Q) dependent *unintegrated* gluon density $F(x, k_T^2, Q^2)$, which specifies the chance of finding a gluon with longitudinal momentum fraction x and transverse momentum of magnitude k_T.

$$\begin{aligned} F(x,k_T^2,Q^2) &= F^0(x,k_T^2,Q^2) + \\ &\int_x^1 dz \int \frac{d^2q}{\pi q^2}\Theta(Q-zq)\Delta_S(Q,zq)\tilde{P}(z,q,k_T)F\left(\frac{x}{z},k_T'^2,q^2\right). \end{aligned} \tag{1}$$

The inhomogeneous or "no-rung" contribution, F^0, may be regarded as the non-perturbative driving term. The function $\tilde{P}$ is the gluon-gluon splitting function

$$\tilde{P} = \overline{\alpha}_S\left[\frac{1}{1-z} + \Delta_R\frac{1}{z} - 2 + z(1-z)\right] \tag{2}$$

where $\overline{\alpha}_S \equiv 3\alpha_S/\pi$. The multiplicative factors Δ_S and Δ_R cancel the singularities manifest as $z \to 1$ and $z \to 0$ respectively. Their explicit form can be found in, for example, refs. [1, 2].

2 Numerical solution of the CCFM equation

In this contribution we are interested in the CCFM equation at small x. In this region we may simplify the equation (1) as follows [2]

$$F(x,k_T^2,Q^2) = F^0(x,k_T^2,Q^2) + \overline{\alpha}_S\int_x^1 \frac{dz}{z}\int\frac{d^2q}{\pi q^2}\Theta(Q-zq)\Delta_R(z,q,k_T)F\left(\frac{x}{z},(\boldsymbol{k}_T+\boldsymbol{q})^2,q^2\right) \tag{3}$$

We take the scale of the running coupling, α_s, to be k_T^2 and we choose F^0 such that it would generate a "flat" gluon, $xg \sim 3(1-x)^5$, in the absence of angular ordering and the Δ_R correction term [2]. With these choices we solve (3) by iteration from the starting distribution. Fig. 1 shows our solution in terms of the integrated gluon distribution, $xg(x, Q^2)$. Also shown are the corresponding solutions for the BFKL equation and the double-leading-logarithm (DLL) approximation to the DGLAP equations.

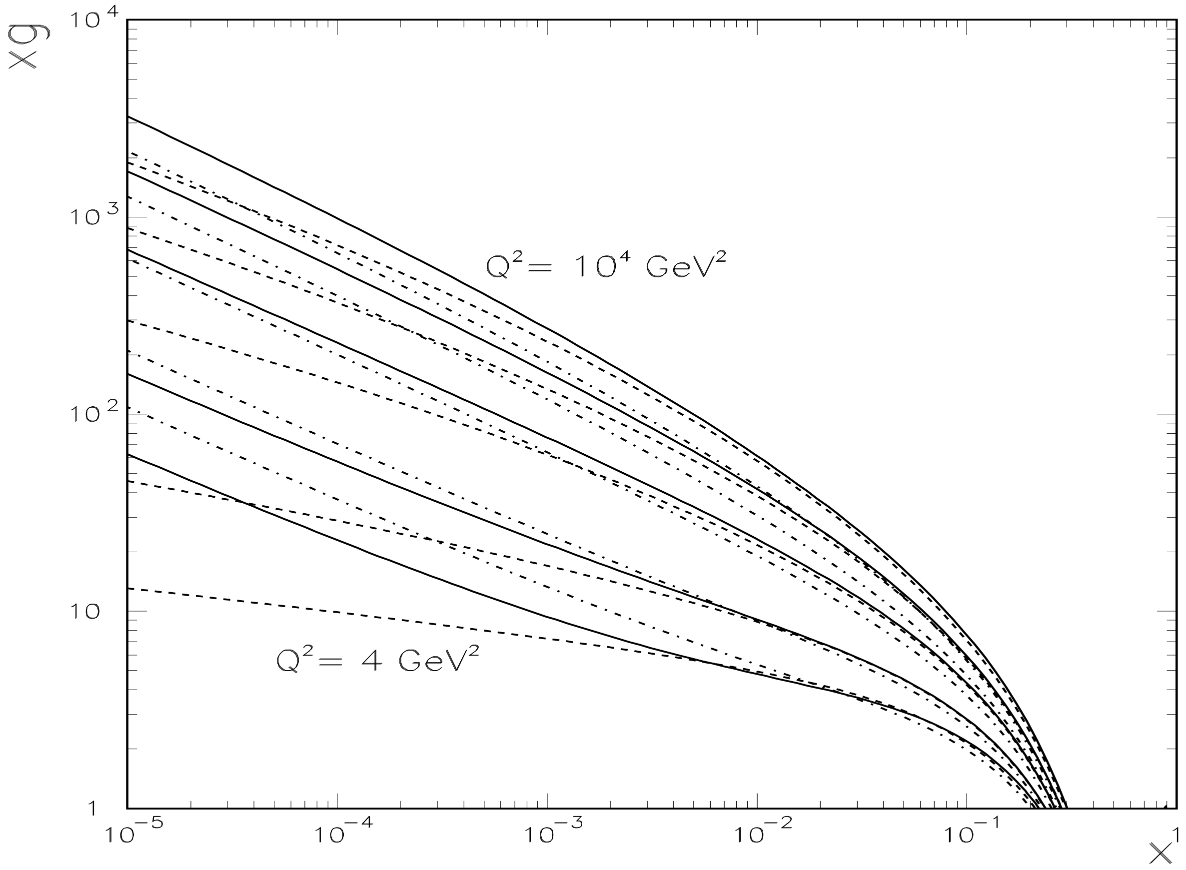

Figure 1: *The integrated gluon distribution xg versus x, obtained from the CCFM (continuous curves), BFKL (dot-dash curves) and the DLL (dashed curves) integral equations, for $Q^2 = 4, 10, 10^2, 10^3$ and 10^4 GeV^2. Our solutions are obtained from a "flat" gluon input [2]*

To quantify the increase in xg, we show in Fig. 2 the effective value of λ, defined by

$$xg(x, Q^2) = Ax^{-\lambda}. \tag{4}$$

For small x we see that the solutions converge to a typical $x^{-0.5}$ behaviour, approximately independent of Q^2, which is consistent with that obtained from the solution of the BFKL equation, although the onset of the $x^{-\lambda}$ form is more delayed for the CCFM solution.

The gluon distribution itself is, of course, not an observable. However, the behaviour of the gluon feeds through into physical quantities such as the structure functions. In deep inelastic scattering the virtual photon couples to the gluon via the $g \to q\overline{q}$ transition. We have therefore calculated the structure function F_2 from the unintegrated gluon distribution F using the k_T-factorization theorem. Details of this calculation can be found in ref. [3].

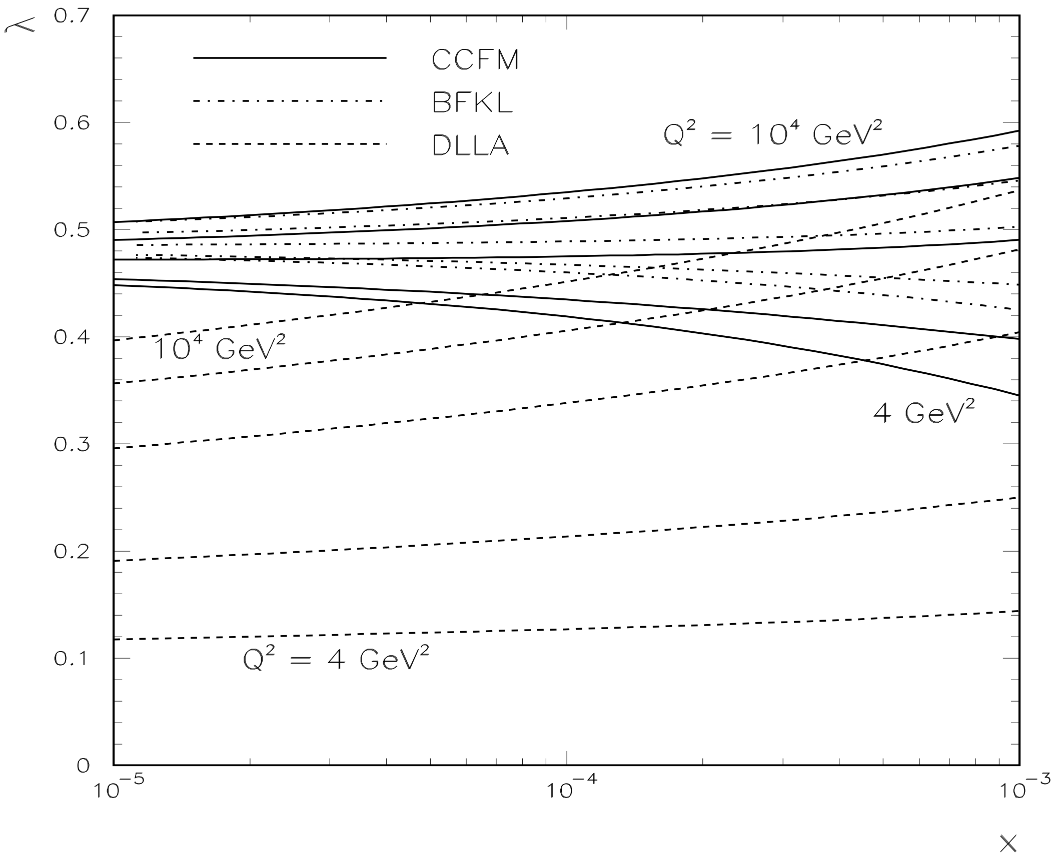

Figure 2: *The effective values of* λ*, defined by* $xg = Ax^{-\lambda}$*. The CCFM values (continuous curves) are compared with those obtained from the BFKL (dot-dashed curves) and DLL approximations (dashed curves). In each case we show curves corresponding to five different values of* Q^2*.*

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