Experimental constraints on coefficients of α_S-expansion of Gottfried sum rule

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Abstract: The x- and Q^2-dependences of the Gottfried sum rule $S_G(x,Q^2)$ based on the experimental data on proton and deuteron structure functions are studied. For the first time, the coefficients c_1, c_2 of the expansion of $S_G(x,Q^2)$ in α_S/π up to second order are evaluated through the phenomenological analysis of NMC and H1 data. It is found that c_1 is negative while c_2 is positive. The obtained result is in disagreement with QCD predictions for these coefficients. We suggest measuring the low x-dependence of the F_2^D deuteron structure function at HERA in order to study the Q^2-dependence of the Gottfried sum rule.

1 Introduction

The experimental data on the proton and neutron structure functions (SF) are of great interest for verification of the theory of strong interaction, QCD. The relevant information can be used to extract the spin-dependent and spin-independent parton distributions, to estimate nonperturbative effects, to verify nucleon models and sum rules such as the Gottfried [1], Bjorken [2], Ellis-Jaffe [3], Gross-Llewellyn Smith [4] and Adler[5] ones.

New data on the deuteron SF F_2^D obtained at CERN, SLAC and Fermilab [6]-[9] stimulated great interest in theoretical studies of the deuteron structure.

The deuteron is an excellent neutron target and therefore the neutron structure functions F_2^n were usually extracted from the experimentally known proton and deuteron structure functions.

The extraction procedure of the neutron SF from deuteron and proton data is ambiguous and, therefore, the estimate of nuclear effects in the deuteron is extremely important not only to obtain new information on F_2^n but also to verify deuteron models and to perform a common QCD analysis of experimental data.

The Gottfried sum rule was verified by the NMC Collaboration [6] and the value of $S_G = 0.240 \pm 0.016$ was found to be below the parton model prediction. To study the sum rule in detail, the experimental data on the deuteron structure function F_2^D at low x and large Q^2 are necessary. Such measurements are possible to perform at HERA [10].

In the present paper, the phenomenological analysis of x and Q^2 dependences of the $S_G(x,Q^2)$ Gottfried sum rule is based on the NMC [7, 8], H1 [11] and ZEUS [12] parametrization of the proton structure function $F_2^p(x,Q^2)$. It is shown [13] that the available experimental data on

F_2^p and F_2^D allow one to estimate the α_S correction to $S_G(Q^2)$. The expansion coefficients of $S_G(Q^2)$ up to order $Q(\alpha^2)$ are estimated. It is found that the obtained results are in disagreement with the QCD predictions. To clarify the discrepancy, it was proposed to measure the $F_2^D(x,Q^2)$ deuteron structure function at low x and high Q^2 in order to extract precisely the coefficients of α_S-expansion of the Gottfried sum rule.

2 Deep-Inelastic Scattering on Deuteron

The cross section of deep-inelastic lepton-deuteron scattering in the one-photon approximation is expressed via the imaginary part of the forward scattering amplitude of the virtual photon or W-boson on the deuteron - $W^D_{\mu\nu}$. The latter is related to the deuteron spin-dependent - $g^D_{1,2}(\nu,Q^2)$ and spin-independent - $F^D_{1,2,3}(\nu,Q^2)$ structure functions as follows

$$W^D_{\mu\nu} = -(g_{\mu\nu} - q_\mu q_\nu/q^2)\cdot F_1^D + (p_\mu - q_\mu(pq)/q^2)(p_\nu - q_\nu(pq)/q^2)\cdot F_2^D/\nu$$

$$+i\epsilon_{\mu\nu\alpha\beta}q^\alpha\{s^\beta g_1^D/\nu + [s^\beta(qp) - p^\beta(sq)]M^{-1}g_2^D/\nu^2\} + i\epsilon_{\mu\nu\alpha\beta}\; q^\alpha p^\beta \cdot F_3^D/\nu. \qquad (1)$$

Here q,p are momenta of the photon and deuteron; M is the deuteron mass; $\nu=(pq)$; the 4-vector s_α describes the deuteron spin. The symmetric part of the deuteron tensor $W^D_{\mu\nu}$ can be written as $W^D_{\mu\nu} = W^{\alpha\beta}_{\mu\nu}\cdot\rho^{(S)}_{\alpha\beta}$

$$W^{\alpha\beta}_{\mu\nu} = \int\frac{d^4k}{(2\pi)^4 i}\delta(m^2-k^2)\theta(k_0)\theta(p_+ - k_+)\; Sp\{w^N_{\mu\nu}\cdot\bar\psi^\alpha(k_1)\cdot(m+\hat k)\cdot\psi^\beta(k_1)\}. \qquad (2)$$

Here the θ-function and light-cone variables - $(k_\pm, k_\perp)$ are used. The tensor $\rho^{(S)}_{\alpha\beta}$ is the symmetric part of the deuteron polarization density matrix. The antisymmetric part of the deuteron tensor $W^D_{\mu\nu}$ is expressed in a form similar to (2). The procedure to construct the relativistic deuteron wave function (RDWF) ψ_α was proposed and RDWF was obtained in [14].

3 Deuteron and Neutron Structure Functions

The deuteron SF F_2^D in the light-cone variables is expressed as follows

$$F_2^D(\alpha,Q^2) = \int_\alpha^1 dx\; d^2k_\perp\; p(x,k_\perp)\cdot F_2^N(\alpha/x,Q^2). \qquad (3)$$

The nucleon SF $F_2^N = (F_2^p + F_2^n)/2$ is defined by the proton and neutron ones. The function $p(x,k_\perp)$ describes the probability that the active nucleon carries away the fraction of the deuteron momentum $x = k_{1+}/p_+$ and the transverse momentum $k_\perp$ in the infinite momentum frame. It is expressed via the RDWF

$$p(x,k_\perp) \propto Sp\{\bar\psi^\alpha(k_1)\cdot(m+\hat k_1)\cdot\psi^\beta(k_1)\cdot\hat q/\nu\cdot\rho^{(S)}_{\alpha\beta}\}. \qquad (4)$$

The nuclear effect in the deuteron is described by the ratio $R_F^{D/N} = F_2^D/F_2^N$. It was shown in [15] that the effect of relativistic Fermi motion grows with x and the ratio $R_F^{D/N}$ reaches 6% at $x\simeq 0.7$. The dependence of the ratio $R_F^{D/N}$ on x resembles the nuclear EMC effect and it is practically independent of Q^2. Using the universal behaviour of the ratio it is possible to extract the neutron SF F_2^n

$$F_2^n(x,Q^2) = 2\cdot[R_F^{D/N}(x)]^{-1}\cdot F_2^D(x,Q^2) - F_2^p(x,Q^2). \qquad (5)$$

We would like to note that for a reliable estimate of other contributions to F_2^D from nuclear effects such as nuclear shadowing, meson exchanges etc., the data at low x such as the E665 data [9] are required.

4 Gottfried Sum Rule

The extracted neutron SF can be used to verify the Gottfried sum rule [1]:

$$\int_0^1 [F_2^p(x) - F_2^n(x)]dx/x = 1/3. \qquad (6)$$

The Gottfried integral as a function of x and Q^2 is defined as follows

$$S_G(x,Q^2) = \int_x^1 [F_2^p(y,Q^2) - F_2^n(y,Q^2)]dy/y. \qquad (7)$$

To verify the sum rule, not only the x-dependence of SF's also Q^2-dependence in a wide kinematical range are necessary. The realistic comparison of the experimental results with theoretical predictions based on QCD is more argued at high Q^2.

As has been reported in [6], the value of S_G at $Q^2 = 4.0\ (GeV/c)^2$ obtained from the measurements of F_2^D and F_2^p is considerably below the value of the naive quark-parton model equal to 1/3: $S_G = 0.240 \pm 0.016$. This result in the parton model is usually interpreted as the violation of the isospin symmetric sea. As will be shown later, the symmetry violation demonstrates the strong Q^2-dependence.

The QCD corrections of order $O(\alpha_s)$ [16] and $O(\alpha_s^2)$ [17] are estimated for the Gottfried sum rule in the case of flavor-symmetric sea $\bar{u} = \bar{d}$:

$$S_G(Q^2) = \frac{1}{3}\cdot(1 + \bar{c}_1\cdot(\alpha_S/\pi) + \bar{c}_2\cdot(\alpha_S/\pi)^2). \qquad (8)$$

The coefficients $\bar{c}_1, \bar{c}_2$ are equal to 0.036, 0.72 for $n_f = 3$ and 0.038, 0.55 for $n_f = 4$, respectively. Thus, the coefficients $\bar{c}_1$, $\bar{c}_2$ are found to be positive and relatively small, and as mentioned in [17], the QCD corrections cannot explain the deviation of the theoretical prediction from the experimental result of the NMC collaboration without the assumption that the light quark sea is flavor asymmetric.

5 Procedure to Extract Neutron Structure Function

The method to extract $F_2^n(x,Q^2)$ from proton and deuteron experimental data was proposed and realized in [15].

The procedure includes the items: **1.** experimental data on the ratio $R_F^{D/p} = F_2^D/F_2^p$ and structure functions F_2^p, F_2^D; **2.** the relativistic deuteron model [14]; **3.** the choice of the parametrization of the F_2^n neutron structure function and the determination of free parameters to describe the ratio $R_F^{D/p} = F_2^D/F_2^p$; **4.** the comparison of the absolute values of the experimental and theoretical structure function $F_2^D(x,Q^2)$; **5.** the determination of the ratio

$R_F^{D/N} = F_2^D/F_2^N$ describing the nuclear effect in the deuteron; **6.** the extraction of the neutron structure function from experimental data using the formula

$$F_2^n(x,Q^2) = 2\cdot[R_F^{D/N}]^{-1}\cdot F_2^D(x,Q^2) - F_2^p(x,Q^2). \quad (9)$$

In [15] the NMC data [6, 7] on the ratio $R_F^{D/p} = F_2^D/F_2^p$, F_2^p and the relativistic deuteron model were used to extract the neutron SF F_2^n. It was shown that the calculated results for ratio $R_F^{D/p}$ and $F_2^D(x,Q^2)$ are in good agreement with the available experimental data [6, 7, 18, 19].

Thus, the conclusion was made that the extraction procedure proposed for $F_2^n(x,Q^2)$ is self-consistent because it provides a good description of higher statistics experimental data on the ratio $R_F^{D/p}$ and F_2^D over a wide kinematic range of x and Q^2.

We would like to emphasize that the nuclear effect of Fermi motion was only used in [15]. The shadowing effect [20] should be also included into the procedure if the deuteron data in the low x- and Q^2- range are used (for example, E665 data). In that case, the factor $R^{D/N}$ should be corrected at low x.

6 Results and Discussion

Figure 1(a) shows the dependence of the Gottfried integral $S_G(x,Q^2)$ on x and Q^2. The parametrization of the proton structure function $F_2^p(x,Q^2)$ for H1 data is taken from [11]. It was shown in [13] that the x- and Q^2-dependences of $S_G(x,Q^2)$ for the parametrizations [7, 8] are similar to H1 one [11]. The NMC parametrization [7] was used in[15] for calculating the deuteron structure function $F_2^D(x,Q^2)$ and a good agreement with experimental data SLAC, BCDMS, NMC was obtained both for a low and high x range. It is assumed [15] that the ratio $f(x) = F_2^n(x,Q^2)/F_2^p(x,Q^2) \to 1$ as $x \to 0$. The parametrization of the $f(x)$ function was obtained and will be used later for the determination of the neutron SF from the H1 and ZEUS proton structure function parametrization.

We would like to note that there is the crossover point x_0 (in particular, $x_0 \sim 0.01$ for the H1 parametrization) separating two ranges: one - with decreasing $S_G(x,Q^2)$ and the other - with increasing $S_G(x,Q^2)$ with Q^2, respectively. A similar dependence of the $S_{GLS}(x,Q^2)$ Gross-Llewellyn Smith integral on Q^2 with the crossover point $x_0 =\simeq 10^{-2}$ is predicted in [21]. Note that the main part of the $S_G(x,Q^2)$ integral is given by integration over the interval of relatively large x ($10^{-2} < x < 1.$) and determines the first term in expansion (8).

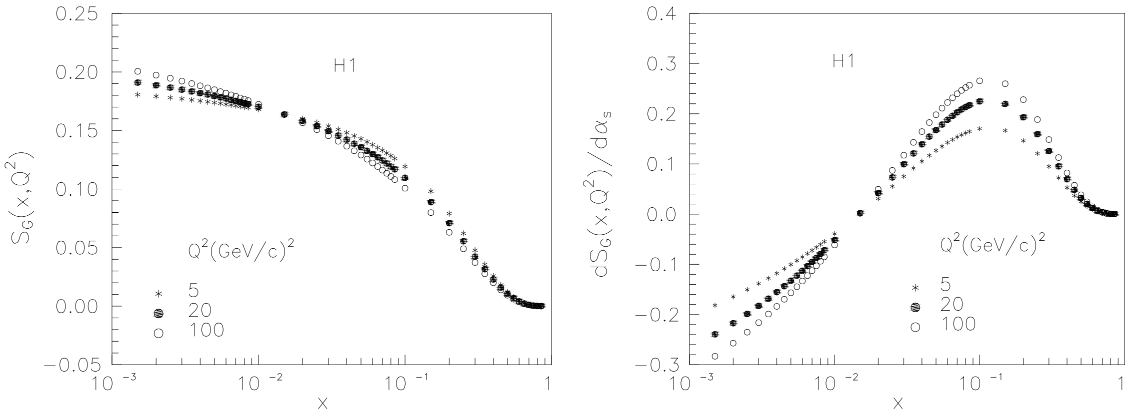
Figure 1(b) shows the dependence of the derivative $dS_G(x,Q^2)/d\alpha_S$ on x and Q^2. The 3-loop perturbative QCD expression of α_S with $n_f = 4$ is used. The derivative grows up to $x \simeq 0.1 - 0.2$ then decreases and changes in sign for $Q^2 > 100\ GeV^2$ for NMC and even for smaller Q^2 for H1 parametrization. It should be stressed that *the negative value of the derivative $dS_G(x,Q^2)/d\alpha_S$ is due to the small x contribution to the Gottfried integral.*

Figure 2 shows the dependence of $S_G(Q^2)$ on α_S/π at $x = 10^{-3}$ in (7). The α_S-dependence could be parametrized by the parabola:

$$S_G(Q^2) = S_0\cdot(1 + c_1\cdot(\alpha_S/\pi) + c_2\cdot(\alpha_S/\pi)^2). \quad (10)$$

The values of S_0, c_1, c_2 are presented in Table 1. They could be considered as α_S corrections to the Gottfried sum rule. It was shown in [13] that the derivative $dS_G(Q^2)/d\alpha_S$ is negative at

$\alpha_S/\pi < 0.05$ for all parametrizations F_2^p, and the H1 curve crosses the NMC one in the range $Q^2 = 5 - 10\ (GeV/c)^2$.



a) b)

Figure 1: *The Gottfried integral $S_G(x, Q^2)$ and the derivative $dS_G(x, Q^2)/d\alpha_S$ as a function of x and Q^2.*

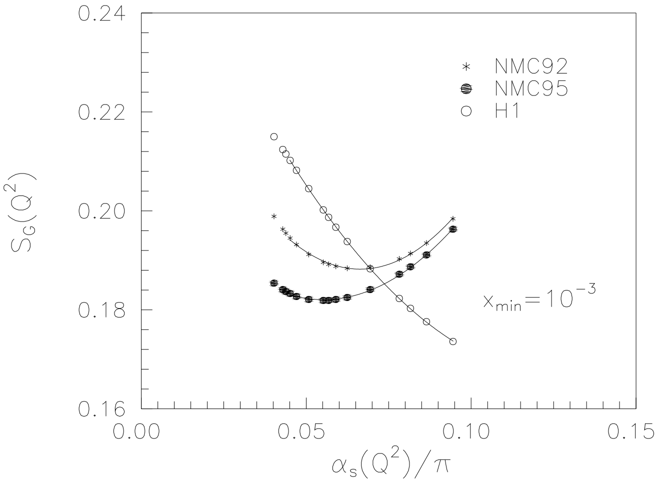


Figure 2: *The Gottfried integral $S_G(, Q^2)$ as a function of α_S at $x = 10^{-3}$. The lines present the paraboloic fit of $S_G(, Q^2)$ for different parameterisations F_2^p.*

Despite different kinematical regions of NMC and H1 experiments used for fits of experimental data one can see from Table 1 the reasonable quantitative and good qualitative agreement between the corresponding coefficients $S_0,\ c_1,\ c_2$ for parametrizations of the proton SF under consideration. Note that the F_2^p parametrization of H1 is combined with the data from NMC and BCDMS experiments and a smooth transition between different F_2^p data is obtained. The kinematic range covers almost four orders of magnitude in x and Q^2.

Table 1. The coefficients of the $S_G(Q^2)$ Gottfried integral expansion in α_S/π.

	S_0	c_1	c_1
NMC92 [7]	0.246	-7.06	53.2
NMC95 [8]	0.210	-4.87	44.2
H1 [11]	0.271	-6.04	23.7
average value	0.242± 0.21	-6.00± 0.74	40.4± 11.1

The average values for the coefficients r_i $(i = S_0, c_1, c_2)$ are calculated by $< r >= (\sum_i r_i)/n$ and $< \Delta r >= (\sum_i | < r > -r_i|)/n$ and presented in Table 1. The errors obtained could be considered as a crude estimation of systematic uncertainties.

It should be noted that the parametrization of the ZEUS data [12] for F_2^p provides qualitatively the same results: $S_0 = 0.383$, $c_1 = -12.9$ and $c_2 = 76.2$.

We would like to emphasize that the values of coefficients obtained from the phenomenological analysis of experimental data are essentially different from the theoretical QCD predictions for $\bar{c}_1$ and $\bar{c}_2$. The coefficient c_1 is found to be negative, in contrast to $\bar{c}_1$. Both c_1, c_2 are many times larger in the absolute value than $\bar{c}_1$, $\bar{c}_2$.

7 Conclusions

The analysis of Q^2-dependence of the Gottfried sum rule based on experimental data on the proton F_2^p and deuteron F_2^D structure functions in the framework of the covariant approach in the light-cone variables and relativistic deuteron model was performed:

- the procedure to extract the neutron SF $F_2^n(x, Q^2)$ is described and used to analyze the parametrizations of NMC, BCDMS, SLAC, H1, ZEUS data on F_2^D and F_2^p SF's
- the increase of the $S_G(Q^2)$ Gottfried sum rule for small $\alpha_S(Q^2)$ is a general feature for the NMC92, NMC95, H1 parametrizations. This behaviour is connected with the *negative value of the first order α_S correction* to the GSR
- it is shown that the results obtained for c_1 and c_2 are in disagreement with the calculation made in the framework of QCD assuming the flavour symmetry of sea quarks ($\bar{u} = \bar{d}$)
- the measurements of the deuteron structure function F_2^D at HERA, extraction the neutron SF F_2^n and verification of the Q^2-dependence of $S_G(x, Q^2)$ at low x are necessary to determine the α_S-corrections to GSR more reliably.

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