The QCD dipole picture of small-x physics

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Abstract: The QCD dipole picture of BFKL dynamics provides an attractive theoretical approach to the study of the QCD (resummed) perturbative expansion of small-x physics and more generally to hard high-energy processes. We discuss applications to the phenomenology of proton structure functions in the HERA range and to the longstanding problem of unitarity corrections, and outline some specific predictions of the dipole picture.

1 Introduction

The dipole formulation [1, 2] is an approach to small-(Bjorken)x physics which for inclusive quantities can be shown [3] to be equivalent to the BFKL approach [4]. One starts with a $q\bar{q}$ state (onium), taken to be heavy enough to ensure the validity of perturbation theory. The main ingredients of the dipole picture of BFKL dynamics are the following

i) Choosing the quantisation in the infinite-momentum frame of the onium allows one to select the leading $\alpha \log 1/x$ terms of the QCD perturbative expansion of the onium wave-function.

ii) Changing the momentum representation into a mixed one $(\mathbf{b}, x)$, where $\mathbf{b}$ is the transverse coordinate, amounts to killing the contributions of the interference Feynman diagrams in the leading-log expansion. This results in a quasi-classical picture of the system of quarks and gluons in terms of probability distributions at the interaction time.

iii) Finally the $1/N_c$ limit leads to the emergence of a representation in terms of independent colourless dipoles, replacing the description in terms of soft, coloured gluons.

To illustrate these properties on a simple example, one constructs the component of the squared wave function that contains one soft gluon, as a function of the transverse positions $\mathbf{b}_0, \mathbf{b}_1$ (or impact parameter) of the onium quark and antiquark and $\mathbf{b}_2$ of the gluon, (see fig. 1). In the large-N_c limit, the original colour dipole of the onium state (of size b) effectively becomes two colour dipoles: one formed by qg (of size b_{02}) and the other by $g\bar{q}$ (of size b_{12}). So the addition of a gluon is equivalent to the branching of one dipole into two, and each of the produced dipoles can then branch independently — this leads to a cascade of dipoles developing when x becomes smaller and smaller, explaining the rise in the number of dipoles (or gluons) at small x.

To determine the gluon distribution, one must use some probe. One way is to measure the interaction cross section with a second onium. The evolution equation for the interaction cross-section (see fig. 1) of two $q\bar{q}$ states of sizes b and b' is

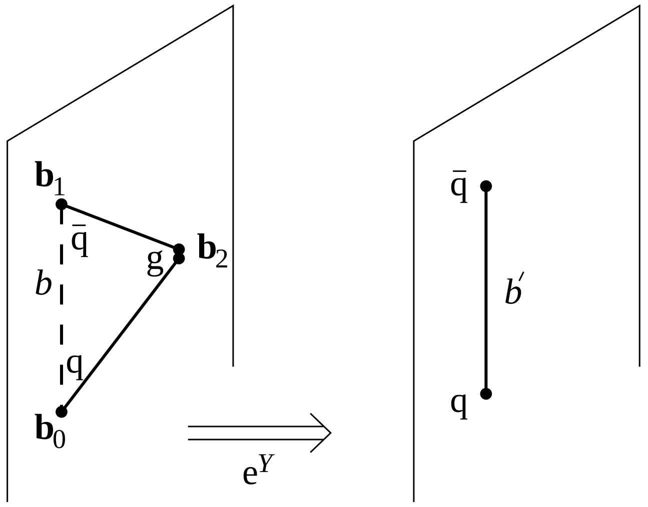

Figure 1: *Onium-onium interaction at first order via the one-soft-gluon component of the onium wavefunction; under the effect of the Lorentz boost* e^Y*, the original* $\bar{q}q$ *configuration of size* b *gives rise to a soft gluon component, or in the* $N_c \to \infty$ *limit, to two dipoles of sizes* b_{02} *and* b_{12} *interacting with the other onium of size* b'*.*

$$\frac{\mathrm{d}\sigma(b',b,Y)}{\mathrm{d}Y} = \frac{\alpha N_c}{2\pi^2}\int \frac{b^2 \mathrm{d}^2\mathbf{b}_2}{b_{02}^2 b_{12}^2}[\sigma(b',b_{02},Y) + \sigma(b',b_{12},Y) - \sigma(b',b_{01},Y)], \quad (1)$$

where $Y \simeq \ln 1/x$ is known as the rapidity. The solution is

$$\sigma(b,b',Y) = \frac{8\pi\alpha^2 bb'}{\sqrt{\pi k Y}} \mathrm{e}^{(\alpha_{\mathbb{P}}-1)Y - \ln^2(b'/b)/kY} \quad (2)$$

with $(\alpha_{\mathbb{P}} - 1) = (4\ln 2)\alpha N_c/\pi$ and $k = \frac{\alpha N_c}{\pi} 14\zeta(3)$. Eq. (2) has some interesting features which deserve comment. First it reproduces exactly the high-energy ($\simeq$ small-x) behaviour associated with the BFKL "hard" Pomeron. Second, and more intriguing, a dependence appears on the scale-ratio b'/b between the two colliding onia. This is related to the property of BFKL dynamics that it "explores" a large region in the transverse-momentum plane, which is analogous to a classical diffusion mechanism.

2 Structure functions

The scale-ratio dependence obtained in formula (2) is of importance when considering another type of probe, a photon of virtuality Q^2, which corresponds on average[2] to a transverse size $1/Q$. In ref. [6], the (theoretical) process of deep-inelastic scattering on an onium state has been proposed to determine the origin of scaling violations of the structure function in the context of BFKL dynamics. Indeed from the viewpoint of the dipole picture, scaling violations are induced by a term analogous to the scale-ratio in eq. (2). One gets:

$$F_2^{onium} \propto \int \frac{d\gamma}{2i\pi}(bQ)^{2\gamma} \mathrm{e}^{\frac{\alpha N_c}{\pi}\chi(\gamma)\ln\frac{1}{x}} \propto bQ\ x^{-\left(\frac{4\alpha N_c \ln 2}{\pi}\right)} \frac{\exp\left(-\frac{1}{k\ln\frac{1}{x}}\ln^2(bQ)\right)}{\left(k\ln\frac{1}{x}\right)^{1/2}}, \quad (3)$$

where one uses the known BFKL analytic expression for the Mellin transform of the onium structure function, and $\chi(\gamma)$ is the corresponding kernel[4]. This expression leads to an interesting phenomenological extension to the proton structure functions, which has the property that it describes the scaling violations at small-x observed at HERA.

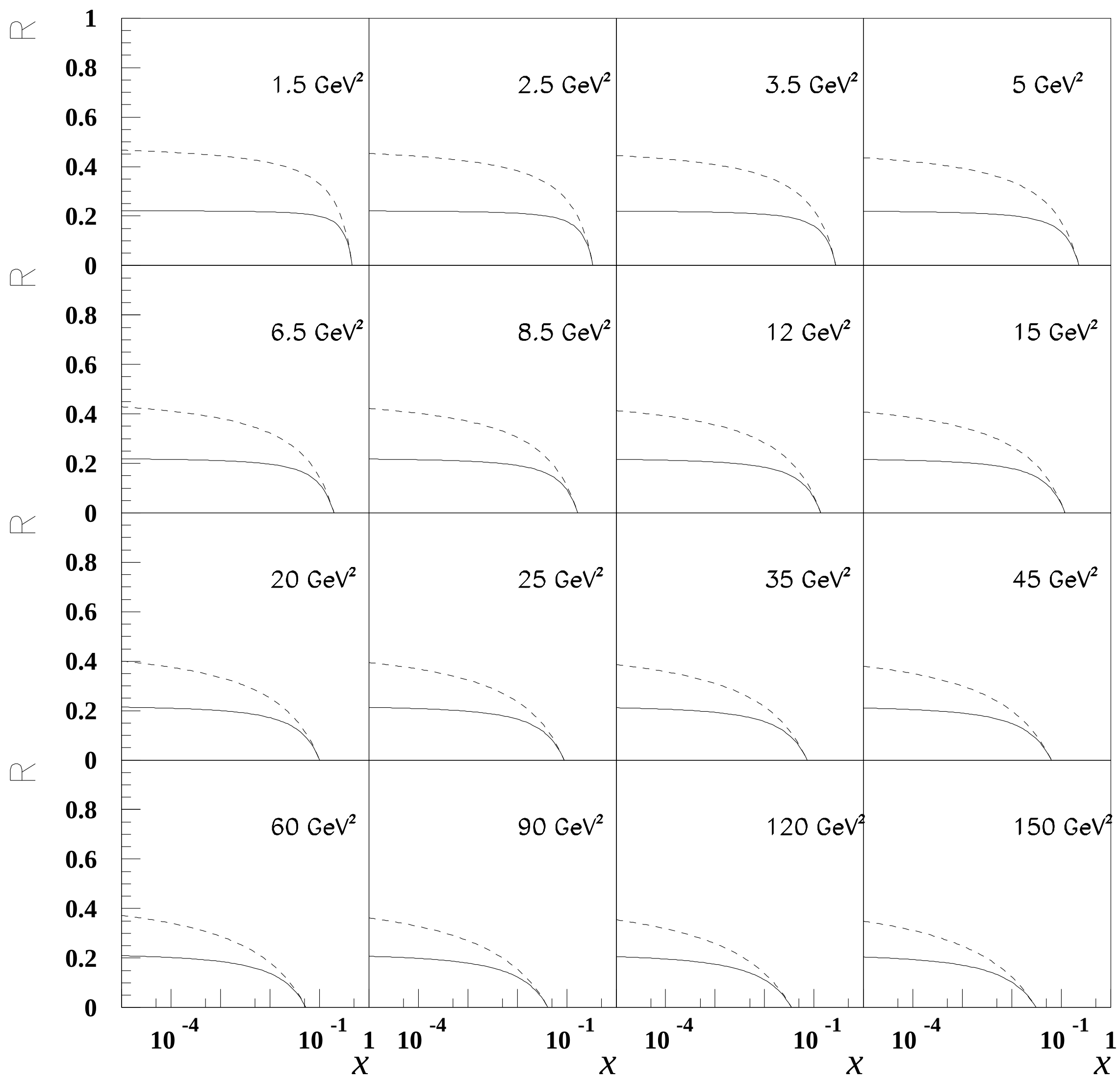

Figure 2: *Predictions for the ratio $R \equiv F_L/F_T$ in the dipole picture [6]. The full line describes the prediction based on a fit to F_2 data and, with the same parameters, a determination of the gluon structure function (not shown). The effect of the $\ln \frac{1}{x}$ resummation is seen by comparison with the one-loop approximation (dotted line). The prediction is significantly lower than the known DGLAP estimates, e.g. [8].*

Indeed, assuming k_T-factorisation properties[5] for high-energy scattering off a proton target, it is possible to extend the dipole model to deal with deep-inelastic scattering on a proton target[6]. Starting from formula (3), the Mellin integrand happens to be multiplied by $w(\gamma, b; Q_0)$ where w can be interpreted as the Mellin-transformed probability of finding a dipole of (small) transverse size b in the proton. $Q_0 >> b^{-1}$ is a typically non-perturbative proton scale. Noting that b is a small but arbitrary factorisation scale, the overall result has to be b-independent, provided it stays in the perturbative region. Hence, assuming renormalisation group properties to be valid[7], the b dependence of w has to match the $b^{-2\gamma}$ dependence in formula (3). One then writes

$$w(\gamma, b; Q_0) = w(\gamma)\ \ (bQ_0)^{2\gamma}\,. \tag{4}$$

This yields the final result[6]

$$\begin{pmatrix} F_T \\ F_L \\ F_G \end{pmatrix} = \frac{2\alpha N_c}{\pi} \int \frac{d\gamma}{2i\pi} \left(\frac{Q^2}{{Q_0}^2}\right)^{\gamma} \mathrm{e}^{\frac{\alpha N_c}{\pi}\chi(\gamma)\ln(\frac{1}{x})} \begin{pmatrix} h_T \\ h_L \\ 1 \end{pmatrix} \frac{v(\gamma)}{\gamma} w(\gamma) \tag{5}$$

where $F_{T(L)}$ is the structure function corresponding to transverse(longitudinal) photons and F_G the gluon structure function. The known (resummed) coefficient functions $h_{T,L}(\gamma)$ are given in ref. [5], and the gluon-dipole coupling $v(\gamma)$ is derived in the second of refs. [6]. It is interesting that these formulae give a good fit of the HERA data on $F_2 = F_T + F_L$ in the small-x range and in a large domain of $Q^2 \leq 150\ GeV^2$. Moreover it leads to a gluon structure function in agreement with the H1 determination based on the next-leading order DGLAP evolution[8]. Note also that the ratios F_G/F_2 and $R \equiv F_L/F_T$ are independent of the non-perturbative function $w(\gamma)$. In relation to this a remark is in order for the future prospects of experimentation at HERA: As shown in fig.2, the predictions for R are rather low ($R < 2/9$) which appears to be in contradiction with the phenomenological estimate[8] based on the renormalisation group evolution for F_L. Indeed, as shown in fig. 2, the resummation of the leading $\alpha \log 1/x$ terms of the QCD perturbative expansion is crucial for obtaining the final prediction. This may give a hint for an experimental discrimination of DGLAP versus BFKL evolution equations which is difficult to achieve from the study of F_2 and F_G alone.

Another series of interesting phenomenological results apply to hard diffraction at HERA. The QCD dipole picture leads to two distinct dynamical components of diffraction by a virtual photon. One component, dominant at large diffractively produced masses, is analogous to the triple-(hard)Pomeron coupling and can be explicitly derived from the inelastic interaction of dipoles from both the photon and the proton sides [9]; A second component results from the *quasi-elastic* interaction of the primary dipole coupled to the photon to the proton target and is dominant at smaller diffractive masses [10]; The quantitative predictions from these two components are strongly correlated with the fits for F_2, giving a nice interrelation between the different aspects of deep-inelastic processes at HERA and the possibility to rely on perturbative QCD to get a coherent description for them.

3 Unitarity corrections

When the centre-of-mass energy becomes very high, the BFKL equation yields a scattering amplitude which violates the unitarity bound, or equivalently conservation of probability. The dipole formulation offers a well-defined way of alleviating the problem. One considers the scattering of two onia in the centre of mass frame. Schematically the scattering amplitude is just related to the probability that there will be an interaction between a parton in one onium and a parton in the other. The usual small-x growth of the cross section relies on the idea that the interaction cross section is proportional essentially to the product of the number of partons in each onium. This is only valid when the overall likelihood of an interaction is low. When there are many partons in each onium, multiple interactions become common[11], and the interaction probability then depends on the details of how the partons are distributed in transverse position (for example if they are clumped together, then multiple interactions are much more likely than if they are uniformally spread out). These multiple-scattering corrections are equivalent to multiple t-channel pomeron exchange diagrams[1, 12].

To obtain the probabilities of different gluon distributions inside the onium, one can use OEDIPUS (Onium Evolution, Dipole Interaction and Perturbative Unitarisation Software) [13]. This simulates the small-x dipole branching producing random dipole configurations with the correct weights. It determines the interaction (both with and without multiple-scattering corrections) between pairs of these random configurations and then averages over the configurations. It is important that one averages over configurations only *after* taking into account multiple interactions — doing the averaging before taking into account the multiple interactions (the eikonal approximation) tends to wash out the correlations between gluons, and causes one to underestimate the point where corrections set in by up to two orders of magnitude in x.

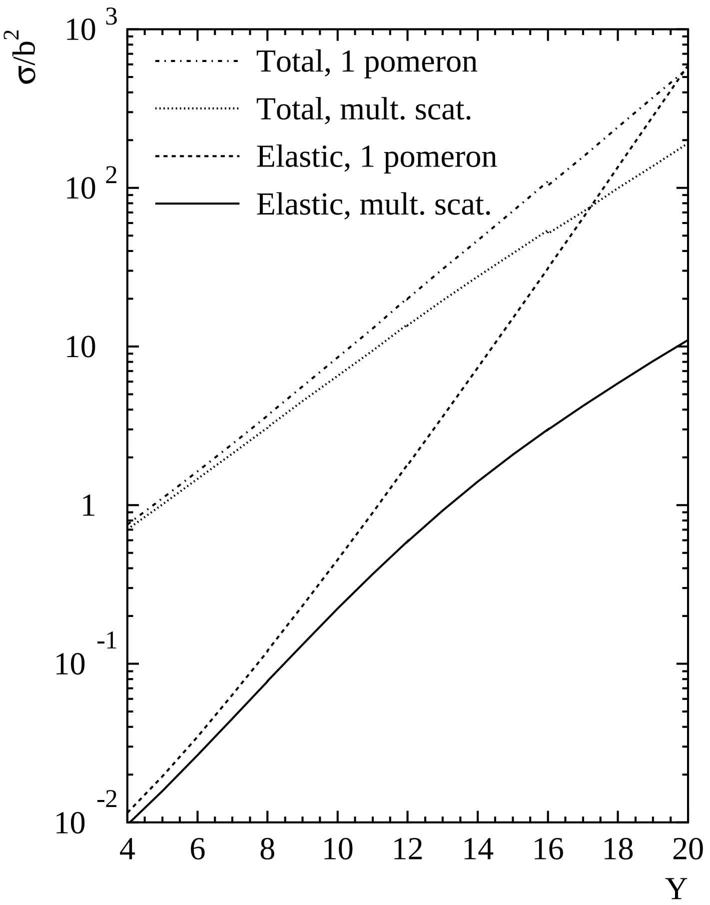

Figure 3: *The elastic and total cross sections for onium-onium scattering, as a function of rapidity, showing both the one-pomeron approximation and the results including multiple-scattering corrections.*

The results [12] are shown in figure 3. The rapidity Y corresponds roughly to $\ln 1/x$, and b is the onium size. The most striking point is that corrections to the total cross section set in very slowly, whereas the elastic cross section is subject to very strong modifications. The reason is that the total cross section is proportional to the integral over impact parameter, $\mathbf{r}$, of the amplitude $F(\mathbf{r})$, whereas the elastic cross section is proportional to the integral of the square of the amplitude:

$$\sigma_{\mathrm{tot}}(Y) = 2\int \mathrm{d}^2\mathbf{r}\, F(\mathbf{r}), \qquad \sigma_{\mathrm{el}}(Y) = \int \mathrm{d}^2\mathbf{r}\, |F(\mathbf{r})|^2. \tag{6}$$

Because of BFKL diffusion, for moderate $r = |\mathbf{r}|$ the leading dependence of the amplitude is $F(r) \sim 1/r^2$. Therefore the elastic cross section is dominated by small impact-parameters, where the amplitude is large and there are strong multiple-scattering corrections. The total cross section comes from a wide range of r, where the amplitude will on average be smaller, and so the corrections are less important. Effectively the total cross section carries on growing through an increase in area of interaction. More details can be found in [12]

Onium-onium scattering is a good theoretical laboratory because it ensures that it is safe to use perturbative QCD. For DIS one can expect two major qualitative differences: (a) infra-red effects will constrain the maximum size of the dipoles, limiting the growth of the total cross section, and altering the balance between total and elastic cross sections; (b) the presence of two different scales means that different kinds of dipole configurations will dominate the scattering, tending to reduce the multiple-interaction effects.

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