Review of Higher Order QCD Corrections to Structure Functions

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Abstract: A review is presented on all higher order QCD corrections to deep inelastic structure functions. The implications of these corrections for polarized and unpolarized deep inelastic lepton-hadron scattering will be discussed.

1 Introduction

The past twenty years have shown much progress in the field of perturbative calculations in strong interaction physics [1]. This in particular holds for the radiative corrections to the deep inelastic structure functions. Sometimes these corrections could be even extended up to third order in the strong coupling constant α_s. The structure functions we would like to discuss are measured in deep inelastic lepton-hadron scattering

$$l_1(k_1) + H(p) \rightarrow l_2(k_2) + \text{"}X\text{"} \tag{1}$$

where l_1, l_2 stand for the in- and outgoing leptons respectively. The hadron is denoted by H and "X" stands for any inclusive hadronic state. The relevant kinematical and scaling variables are defined by

$$q = k_1 - k_2 \qquad q^2 = -Q^2 > 0 \qquad x = \frac{Q^2}{2pq} \qquad y = \frac{pq}{pk_1} \tag{2}$$

with the boundaries

$$0 < y < 1 \qquad\qquad 0 < x \leq 1 \tag{3}$$

Reaction (1) proceeds via the exchange of one of the intermediate vector bosons V of the standard model which are represented by $V = \gamma, Z, W$. In the case of unpolarized scattering with $V = \gamma$ one can measure the structure functions $F_L(x, Q^2)$ (longitudinal) and $F_1(x, Q^2)$ (transverse) or the better known $F_2(x, Q^2)$ which is related to the former two via

$$F_2\,(x, Q^2) \;=\; 2x\,F_1\,(x, Q^2) \;+\; F_L\,(x, Q^2) \tag{4}$$

When $V = W$ or $V = Z$ one can in addition to F_1, F_2 and F_L also measure the structure function $F_3(x, Q^2)$ which is due to parity violation of the weak interactions. In the case the incoming lepton and hadron are polarized one measures besides the structure functions F_i $(i = 1, 2, 3, L)$ also the spin structure functions denoted by $g_i(x, Q^2)$ $(i = 1, \cdots 5)$. At this moment, because of the low Q^2 available, reaction (1) is only dominated by the photon $(V = \gamma)$ so that one has data for $g_1(x, Q^2)$ (longitudinal spin) and $g_2(x, Q^2)$ (transverse spin) only. The measurement of the structure functions at large Q^2 gives us insight in the structure of the hadrons. According to the theory of quantum chromodynamics (QCD) the hadrons consist out of quarks and gluons where the latter are carriers of the strong force. When Q^2 gets large one can probe the light cone behaviour of the strong interactions which can be described by perturbation theory because the running coupling constant denoted by $\alpha_s(Q^2)$ is small. In particular perturbative QCD predicts the Q^2-evolution of the deep inelastic structure functions mentioned above. Unfortunately the theory is not at that stage that it enables us to predict the x-dependence so that one has to rely on parametrizations which are fitted to the data. A more detailed description of the structure functions is provided by the parton model which can be applied if one can neglect power corrections of the type $(1/Q^2)^p$ (higher twist effects). Here one asumes that in the Bjorken limit ($Q^2 \to \infty$, x is fixed) the interaction between the hadron and lepton in process (1) proceeds via the partons (here the quarks and the gluons) of the hadron. If the scattering of the lepton with the partons becomes incoherent the structure function can be written as

$$
\begin{aligned}
F^{V,V'}(x,Q^2) &= \int_x^1 \frac{dz}{z} \left[\sum_{k=1}^{n_f} \left(v_k^{(V)}\, v_k^{(V')} + a_k^{(V)}\, a_k^{(V')} \right) \left\{ \Sigma(\frac{x}{z},\mu^2)\, C_{i,q}^S\,(z,\frac{Q^2}{\mu^2}) \right. \right. \\
&+ \left. G\left(\frac{x}{z},\mu^2\right) C_{i,g}\left(z,\frac{Q^2}{\mu^2}\right) \right\} + \sum_{k=1}^{n_f} \left(v_k^{(V)}\, v_k^{(V')} + a_k^{(V)}\, a_k^{(V')} \right) \\
&\left. \Delta_k \left(\frac{x}{z},\mu^2\right) C_{i,q}^{NS}\left(z,\frac{Q^2}{\mu^2}\right) \right] \qquad i = 1,2,L
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
F_3^{V,V'}\left(x,Q^2\right) &= \int_x^1 \frac{dz}{z} \left[\sum_{k=1}^{n_f} \left(v_k^{(V)} a_k^{(V')} + a_k^{(V)}\, v_k^{(V')} \right) \right. \\
&\left. V_k \left(\frac{x}{z},\mu^2\right) C_{3,q}^{NS}\left(z,\frac{Q^2}{\mu^2}\right) \right]
\end{aligned} \tag{6}
$$

with similar expressions for the twist two contributions to the spin structure functions $g_1(x, Q^2)$ in which case we introduce the notations $\Delta\Sigma, \Delta G, \Delta C_{i,l}$ etc.. The vector- and axial-vector electroweak couplings of the standard model are given by $v_k^{(V)}$ and $a_k^{(V)}$ respectively with $V = \gamma, Z, W$ and $k = 1(u), 2(d), 3(s)$ .... . Further n_f denotes the number of light flavours and μ stands for the factorization/renormalization scale. The singlet (Σ) and non-singlet combinations of parton densities (Δ_k, V_k) are defined by

$$\Sigma\left(z,\mu^2\right) = \frac{1}{n_f} \sum_{k=1}^{n_f} \left(f_k\left(z,\mu^2\right) + f_{\bar{k}}\left(z,\mu^2\right) \right) \tag{7}$$

$$\Delta_k\left(z,\mu^2\right) = f_k\left(z,\mu^2\right) + f_{\bar{k}}\left(z,\mu^2\right) - \Sigma\left(z,\,\mu^2\right) \tag{8}$$

$$V_k\left(z,\,\mu^2\right) = f_k\left(z,\,\mu^2\right) - f_{\bar{k}}\left(z,\,\mu^2\right) \tag{9}$$

where $f_k, f_{\bar{k}}$ denote the quark and anti-quark densities of species k respectively. The gluon density is defined by $G(z, \mu^2)$. The same nomenclature holds for the coefficient functions

$C_{i,l}(l = q, g)$ which can also be distinguished in a singlet (S) and a non-singlet (NS) part. Like in the case of the structure functions the x-dependence of the parton densities cannot be determined by perturbative QCD and it has to be obtained by fitting the parton densities to the data. Fortunately these densities are process independent and they are therefore universal. This property is not changed after including QCD radiative corrections. It means that the same parton densities also show up in other so called hard processes like jet production in hadron-hadron collisions,direct photon production, heavy flavour production, Drell-Yan process etc. Another firm prediction of QCD is that the scale (μ) evolution of the parton densities is determined by the DGLAP [2] splitting functions P_{ij} $(i, j = q, \bar{q}, g)$ which can be calculated order by order in the strong coupling constant α_s. The perturbation series of P_{ij} gets the form

$$P_{kl} = a_s\, P_{kl}^{(0)} + a_s^2\, P_{kl}^{(1)} + a_s^3\, P_{kl}^{(2)} + \ldots \tag{10}$$

with $a_s = \alpha_s(\mu^2)/4\pi$. The splitting functions P_{ij} are related to the anomalous dimensions $\gamma_{ij}^{(n)}$ corresponding to twist two local operators $O_i^{\mu_1\cdots\mu_n}(x)$ of spin n via the Mellin transform

$$\gamma_{ij}^{(n)} = -\int_o^1 dz z^{n-1}\, P_{ij}(z) \tag{11}$$

These operators appear in the light cone expansion of the product of two electroweak currents which shows up in the calculation of the cross section of process (1)

$$J(x)\, J(0) \sim \sum_{n=0}^{\infty} \sum_{k} \tilde{C}_k^{(n)}\, (\mu^2 x^2)\, x_{\mu_1}..x_{\mu_n}\, O_k^{\mu_1\cdots\mu_n}(0, \mu^2) \tag{12}$$

where $\tilde{C}_k^{(n)}$ (12) are the Fourier transforms of the coefficient functions $C_k^{(n)}$ (5),(6) $(k = q, g)$ in Minkowski space (x_μ). Like the splitting functions they are calculable order by order in α_s and the perturbation series takes the form

$$C_{i,k} = \delta_{kq} + a_s\, C_{i,k}^{(1)} + a_s^2\, C_{i,k}^{(2)} + a_s^3\, C_{i,k}^{(3)} + \ldots \tag{13}$$

with $i = 1, 2, 3, L$ and $k = q, g$. We will now review the higher order QCD corrections to the splitting functions and the coefficient functions which have been calculated till now.

2 Splitting Functions

The splitting functions are calculated by

1. $P_{ij}^{(0)}$ Gross and Wilczek (1974) [3]; Altarelli and Parisi (1977) [2].
2. $\Delta P_{ij}^{(0)}$ Sasaki (1975) [4]; Ahmed and Ross (1976) [5]; Altarelli and Parisi [2].
3. $P_{ij}^{(1)}$ Floratos, Ross, Sachrajda (1977) [6]; Gonzales-Arroyo, Lopez, Yndurain (1979) [7]; Floratos, kounnas, Lacaze (1981) [8]; Curci, Furmanski, Petronzio (1980) [9].

4. $\Delta P_{ij}^{(1)}$ Zijlstra and van Neerven (1993) [10]; Mertig and van Neerven (1995) [11]; Vogelsang (1995) [12].

Notice that till 1992 there was a discrepancy for $P_{gg}^{(1)}$ between the covariant gauge [6–8] and the lightlike axial gauge calculation [9] which was decided in favour of the latter by Hamberg and van Neerven who repeated the covariant gauge calculation in [12]. The DGLAP splitting functions satisfy some special relations. The most interesting one is the so called supersymmetric relation which holds in $\mathcal{N}=1$ supersymmetry [13]. Here the colour factors, which in $SU(N)$ are given by $C_F=(N^2-1)/2N$, $C_A=N$, $T_f=1/2$ become $C_F=C_A=2T_f=N$. The supersymmetric relation then reads

$$P_{qq}^{S,(k)}+P_{gq}^{(k)}-P_{qg}^{(k)}-P_{gg}^{(k)}=0 \tag{14}$$

$$\Delta P_{qq}^{S,(k)}+\Delta P_{gq}^{(k)}-\Delta P_{qg}^{(k)}-\Delta P_{gg}^{(k)}=0 \tag{15}$$

which is now confirmed up to first ($k=0$) and second ($k=1$) order in perturbation theory. The third order splitting functions $P_{ij}^{(2)},\Delta P_{ij}^{(2)}$ are not known yet. However the first few moments $\gamma_{ij}^{(2),(n)}$ for $n=2,4,6,8,10$ have been calculated by Larin, van Ritbergen, Vermaseren (1994) [14]. Besides exact calculations one has also determined the splitting functions and the anomalous dimensions in some special limits. Examples are the large n_f expansion carried out by Gracey (1994) [15]. Here one has computed the coefficients b_{21} and b_{31} in the perturbations series of the non-singlet anomalous dimension

$$\begin{aligned}\gamma_{qq}^{NS}\Big|_{n_f\to\infty} &= a_s^2\,\left[n_f\,C_F\,b_{21}\right]+a_s^3\,\Big[n_f^2\,C_F\,b_{31}\\ &+\; n_f\,C_A\,C_F\,b_{32}+n_f\,C_F^2\,b_{33}\Big]+...\end{aligned} \tag{16}$$

Further Catani and Hautmann (1993) [16] calculated the splitting functions $P_{ij}(x)$ in the limit $x\to 0$. The latter take the following form

$$P_{ij}^{(k)}(x)\Big|_{x\to 0}\sim\frac{ln^k x}{x}\to\gamma_{ij}^{(k),(n)}\Big|_{n\to 1}\sim\frac{1}{(n-1)^{k+1}} \tag{17}$$

The above expressions follow from the BFKL equation [17] and k_T-factorization [18]. Some results are listed below. The leading terms in $\gamma_{gg}^{(n)}$ are given by

$$\gamma_{gg}^{(n)}\Big|_{n\to 1}=\left[C_A\frac{a_s}{n-1}\right]+2\zeta(3)\,\left[C_A\frac{a_s}{n-1}\right]^4+2\zeta(5)\,\left[C_A\frac{a_s}{n-1}\right]^6 \tag{18}$$

where $\zeta(n)$ denotes the Riemann zeta-function. Further we have in leading order $1/(n-1)$

$$\gamma_{gq}^{(n)}\Big|_{n\to 1}=\frac{C_F}{C_A}\;\gamma_{gg}^{(n)}\Big|_{n\to 1} \tag{19}$$

$$\begin{aligned}\gamma_{qg}^{(n)}\Big|_{n\to 1} &= a_s T_f\frac{1}{3}\left[1+1.67\left\{\frac{a_s}{n-1}\right\}+1.56\left\{\frac{a_s}{n-1}\right\}^2\right.\\ &\left.+\;3.42\left\{\frac{a_s}{n-1}\right\}^3+5.51\left\{\frac{a_s}{n-1}\right\}^4+...\right]\end{aligned} \tag{20}$$

$$\gamma_{qq}^{S,(n)}\Big|_{n\to 1} = \frac{C_F}{C_A}\left[\gamma_{qg}^{(n)}\Big|_{n\to 1} - \frac{1}{3}\, a_s T_f\right] \tag{21}$$

Kirschner and Lipatov (1983) and Blümlein and Vogt (1996) have also determined the subleading terms in the splitting functions (anomalous dimensions). They behave like

$$P_{ij}^{(k)}(z)\Big|_{z\to 0} \sim ln^{2k}\, z \qquad\qquad \gamma_{ij}^{(k),(n)}\Big|_{n\to 0} \sim \frac{1}{n^{2k+1}} \tag{22}$$

The same logarithmic behaviour also shows up in ΔP_{ij} and $\Delta\ \gamma_{ij}^{(n)}$. In the latter case the expressions in (22) become the leading ones since the most singular terms in (17) decouple in the spin quantities. The expressions in (22) have been calculated for the spin case by Bartels, Ermolaev, Ryskin (1995) [20] and by Blümlein and Vogt (1996) [21] who also investigated the effect of these type of corrections on the spin structure function $g_1(x,Q^2)$. Finally the three-loop anomalous dimension $\Delta\gamma_{qq}^{S,(1)}$ is also known (see Chetyrkin,Kühn (1993) [22] and Larin (1993) [23]). It reads

$$\Delta\gamma_{qq}^{S,(1)} = a_s^2\,[-6n_f\, C_F] + a_s^3\left[\left(18C_F^2 - \frac{142}{3}C_A C_F\right) n_f + \frac{4}{3}\, n_f^2 C_F\right] \tag{23}$$

Notice that the second order coefficient was already determined by Kodaira (1980) [24].

3 Coefficient Functions

The higher order corrections to the coefficient functions are calculated by

1. $C_{i,q}^{(1)}$, $C_{i,g}^{(1)}$ $\quad i = 1,2,3,L$ — Bardeen, Buras, Muta, Duke (1978) [25], see also Altarelli (1980) [26].
2. $\Delta C_q^{(1)}$, $\Delta C_g^{(1)}$ — Kodaira et al. (1979) [27], see also Anselmino, Efremov, Leader (1995) [28]

Together with the splitting functions $P_{ij}^{(k)}$, $\Delta P_{ij}^{(k)}$ $(k = 0,1)$ one is now able to make a complete next-to-leading (NLO) analysis of the structure functions $F_i(x,Q^2)\,(i = 1,2,3,L)$ and $g_1(x,Q^2)$. The second order contributions to the coefficient functions are also known

1. $C_{i,q}^{(2)}, C_{i,g}^{(2)}$ $\quad i = 1,2,3,L$ — Zijlstra and van Neerven (1991) [29]
2. $\Delta C_q^{(2)}, \Delta C_g^{(2)}$ — Zijlstra and van Neerven (1993) [10]

The first few moments of $C_{i,k}^{(2)}$ $(i = 2,L; k = q,g)$ were calculated by Larin and Vermaseren (1991) [30] and they agree with Zijlstra and van Neerven [29]. The first moment of $\Delta C_q^{(2)}$ was checked by Larin (1993) [31] and it agrees with the result of Zijlstra and van Neerven [10]. The third order contributions to the coefficient functions are not known except for some few moments. They are given by

1. $C_{1,q}^{(3),(1)}$ (Bjorken sum rule) Larin, Tkachov, Vermaseren (1991) [32];
2. $C_{3,q}^{(3),(1)}$ (Gross-Llewellyn Smith sum rule) Larin and Vermaseren (1991) [33];
3. $\Delta C_{q}^{(3),(1)}$ (Bjorken sum rule) Larin and Vermaseren (1991) [33];
4. $C_{i,q}^{(3),(n)}$ $(i = 2, L)$ $n = 2, 4, 6, 8$ Larin, van Ritbergen, Vermaseren (1994) [14], (see also [34]).

Since the three-loop splitting functions $P_{ij}^{(2)}$, $\Delta P_{ij}^{(2)}$ are not known, except for a few moments, it is not possible to obtain a full next-to-next-to-leading order (NNLO) expression for the structure functions. However recently Kataev et al. (1996) [35] made a NNLO analysis of the structure functions $F_2(x, Q^2)$, $F_3(x, Q^2)$ (neutrino scattering) in the kinematical region $x > 0.1$ which is based on $\gamma_{qq}^{NS,(2),(n)}$ for $n = 2, 4, 6, 8, 10$ [14]. Like in the case of the DGLAP splitting functions Catani and Hautmann (1994) [16] also derived the small x-behaviour of the coefficient functions. At small x the latter behave like

$$C_{i,k}^{(l)}\Big|_{x\to 0} \sim \frac{ln^{l-2}x}{x} \qquad C_{i,k}^{(l),(n)}\Big|_{n\to 1} \sim \frac{1}{(n-1)^{l-1}} \qquad (l \geq 2) \tag{24}$$

[3mm] The ingredients of the derivation are again the BFKL equation [17] and k_T-factorizaton [18]. from [16] we infer the following Mellin-transformed coefficient functions.

$$\begin{aligned} C_{L,g}^{(n)}\Big|_{n\to 1} &= a_s\, T_f\, n_f\, \frac{2}{3}\left[1 - 0.33\left\{\frac{a_s}{n-1}\right\} + 2.13\left\{\frac{a_s}{n-1}\right\}^2\right. \\ &+ \left. 2.27\left\{\frac{a_s}{n-1}\right\}^3 + 0.43\left\{\frac{a_s}{n-1}\right\}^4 + \ldots\right] \end{aligned} \tag{25}$$

$$\begin{aligned} C_{2,g}^{(n)}\Big|_{n\to 1} &= a_s\, T_f\, n_f\, \frac{1}{3}\left[1 + 1.49\left\{\frac{a_s}{n-1}\right\} + 9.71\left\{\frac{a_s}{n-1}\right\}^2\right. \\ &+ \left. 16.43\left\{\frac{a_s}{n-1}\right\}^3 + 39.11\left\{\frac{a_s}{n-1}\right\}^4 + \ldots\right] \end{aligned} \tag{26}$$

$$C_{L,q}^{S,(n)}\Big|_{n\to 1} = \frac{C_F}{C_A}\left[C_{L,g}^{(n)}\Big|_{n\to 1} - \frac{2}{3}\, a_s\, n_f\, T_f\right] \tag{27}$$

$$C_{2,q}^{S,(n)}\Big|_{n\to 1} = \frac{C_F}{C_A}\left[C_{2,g}^{(n)}\Big|_{n\to 1} - \frac{1}{3}\, a_s\, n_f\, T_f\right] \tag{28}$$

The order α_s^2 coefficients were already obtained via the exact calculation performed by Zijlstra and van Neerven (1991) [29]. The subleading terms given by

$$C_{2,k}^{(l)}\Big|_{x\to 0} \sim \ln^{2l-1} x \qquad (l \geq 1) \tag{29}$$

were investigated by Blümlein and Vogt (1996) [21]. The most singular terms shown in (24) do not appear in the spin coefficient functions $\Delta C_k^{(l)}$ because the Lipatov pomeron decouples in polarized lepton-hadron scattering. Therefore the most singular behaviour near $x=0$ is given by (29) (see [10],[21]). Besides the logarithmical enhanced terms which are characteristic of the low x-regime we also find similar type of logarithms near $x=1$. Their origin however is completely different from the one determining the small x-behaviour. The logarithmical enhanced terms near $x=1$, which are actual distributions, originate from soft gluon radiation. They dominate the structure functions F_i and g_i near $x=1$ because other production mechanisms are completely suppressed due to limited phase space. Following the work in [36] and [37] the DGLAP splitting functions and the coefficient functions behave near $x=1$ like

$$P_{qq}^{NS,(k)} = \Delta\, P_{qq}^{NS,(k)} \sim \left(\frac{1}{1-x}\right)_+ \qquad P_{gg}^{(n)} = \Delta P_{gg}^{(k)} \sim \left(\frac{1}{1-x}\right)_+ \tag{30}$$

$$\Delta C_q^{NS,(k)} = C_{i,q}^{NS,(k)} \sim \left(\frac{\ln^{2k-1}(1-x)}{1-x}\right)_+ \qquad (l=1,2,3) \tag{31}$$

Notice that the above corrections cannot be observed in the kinematical region ($x<0.4$) accessible at HERA. Furthermore the behaviour in (30) is a conjecture (see [7]) which is confirmed by the existing calculations carried out up to order α_s^2.

4 Heavy Quark Coefficient Functions

The heavy quark coefficient functions have been calculated by

1. $C_{i,g}^{(1)}(x,Q^2,m^2)$ $(i=2,L)$ Witten (1976) [38];
2. $\Delta C_g^{(1)}(x,Q^2,m^2)$ Vogelsang (1991) [39];
3. $C_{i,g}^{(2)}(x,Q^2,m^2)$, $C_{i,q}^{(2)}(x,Q^2,m^2)$ $(i=2,L)$ Laenen, Riemersma, Smith, van Neerven (1992) [40].

where m denotes the mass of the heavy quark. The second order heavy quark spin coefficient functions $\Delta C_g^{(2)}(x,Q^2,m^2)$ and $\Delta C_q^{(2)}(x,Q^2,m^2)$ are not known yet. Due to the presence of the heavy quark mass one was not able to give explicit analytical expressions for $C_{i,k}(i=2,L;k=q,g)$. However for experimental and phenomenological use they were presented in the form of tables in a computer program [41]. Analytical expressions do exist when either $x\to 0$ or $Q^2 \gg m^2$. In the former case Catani, Ciafaloni and Hautmann [42] derived the general form

$$C_{i,k}^{(l)}\Big|_{x\to 0} \sim \frac{1}{x}\ln^{l-2}(x)\, f(Q^2,m^2) \qquad (l\geq 2,\ i=2,\ L;\ k=q,g) \tag{32}$$

Like for the light parton coefficient functions (see (24)) the above expression is based on the BFKL equation [17] and k_T-factorization [18]. In second order Buza et al. (1996) [43] were able to present analytical formulae for the heavy quark coefficient functions in the asymptotic limit $Q^2 \gg m^2$. This derivation is based on the operator product expansion and mass factorization.

5 Phenomenology at low x

Since the calculation of the higher order corrections to the DGLAP splitting functions P_{ij} and the coefficient functions C_{ik} is very cumbersome various groups have tried to make an estimate of the NNLO corrections to structure functions in particular to $F_2(x,Q^2)$. The most of these estimates concerns the small x-behaviour. In [44] Ellis, Kunszt and Levin Hautman made a detailed study of the Q^2-evolution of F_2 using the small x-approximation for P_{ij} (17) and $C_{2,k}^{(2)}$ (24). Their results heavily depend on the set of parton densities used and the non leading small x-contributions to $P_{ij}^{(2)}$. The latter are e.g. needed to satisfy the momentum conservation sum rule condition. Large corrections appear when for $x \to 0$ the gluon density behaves like $xG(x,\mu^2) \to$ const. whereas they are small when the latter has the behaviour $xG(x,\mu^2) \to x^{-\lambda}(\lambda \sim 0.3 - 0.5$; Lipatov pomeron).
However other investigations reveal that the singular terms at $x = 0$, present in P_{ij} and $C_{i,k}$, do not dominate the radiatve corrections to $F_2(x,Q^2)$ near low x. This became apparent after the exact coefficient functions or DGLAP splitting functions were calculated.
In [45] Glück, Reya and Stratmann (1994) investigated the singular behaviour of the second order heavy quark coefficient functions (32) in electroproduction and they found that its effect on F_2 was small.
Similar work was done by Blümlein and Vogt (1996) [21] on the effect of the logarithmical terms (22),(29) on $g_1(x,Q^2)$ which contribution to the latter turned out to be negligable.
Finally we would like to illustrate the effect of the small x-terms, appearing in the coefficient functions $C_{2,k}^{(2)}$ and $C_{L,k}^{(2)}$, on the structure functions $F_2(x,Q^2)$ and $F_L(x,Q^2)$. For that purpose we compute the order α_s^2 contributions to F_2 and F_L. Let us introduce the following notations. When the exact expressions for the coefficient functions $C_{i,k}^{(2)}$ are adopted the order α_s^2 contributions to F_i will be called $\delta F_i^{(2),exact}$. If we replace the exact coefficient functions by their most singular part which is proportional to $1/x$ (see (24)) the order α_s^2 contributions to F_i are denoted by $\delta F_i^{(2),app}$. The results are listed in the table below. Further we have used the parton density sets MRS(D0) ($xG(x,\mu^2) \to const.$ for $x \to 0$) and MRS(D-) ($xG(x,\mu^2) \to x^{-\lambda}$ for $x \to 0$) [46]

	MRS(D0)			MRS(D-)		
x	F_2^{NLO}	$\delta F_2^{(2),exact}$	$\delta F_2^{(2),app}$	F_2^{NLO}	$\delta F_2^{(2),exact}$	$\delta F_2^{(2),app}$
10^{-3}	0.67	-0.069	0.088	0.99	-0.084	0.116
10^{-4}	0.82	-0.088	0.158	2.29	-0.226	0.349
10^{-5}	1.00	-0.092	0.251	5.99	-0.665	1.059
x	F_L^{NLO}	$\delta F_L^{(2),exact}$	$\delta F_L^{(2),app}$	F_L^{NLO}	$\delta F_L^{(2),exact}$	$\delta F_L^{(2),app}$
10^{-3}	0.149	-0.029	-0.040	0.263	0.008	-0.052
10^{-4}	0.210	-0.062	-0.071	0.780	0.031	-0.156
10^{-5}	0.281	-0.102	-0.113	2.370	0.105	-0.475

From the table above we infer that a steeply rising gluon density near $x = 0$ (MRS(D-)) leads to small corrections to F_2 and F_L. On the other hand if one has a flat gluon density (MRS(D0)) the corrections are much larger in particular for F_L. A similar observation was made for F_2 in [44]. However the most important observation is that the most singular part of the coefficient functions gives the wrong prediction for the order α_s^2 contributions to the structure functions except for F_L provided the set MRS(D0) is chosen. This means that the subleading terms are important and they cannot be neglected. Therefore our main conclusion is that only exact calculations provide us with the correct NNLO analysis of the structure functions. The asymptotic expressions obtained in the limits $x \to 0, x \to 1$ and $Q^2 \gg m^2$ can only serve as a check on the exact calculations of the DGLAP splitting functions and the coefficient functions.

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