IR-Renormalon Contribution to the Longitudinal Structure Function F_L

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Measurements of the ratio

$$R(x,Q^2) = \frac{\sigma_{\rm L}(x,Q^2)}{\sigma_{\rm T}(x,Q^2)} = \frac{F_2(x,Q^2)}{F_2(x,Q^2)-F_L(x,Q^2)}\left(1+\frac{4M^2x^2}{Q^2}\right)-1 \tag{1}$$

which so far had only rather limited precision should improve substantially over the next years $F_L = F_2 - 2xF_1$. Phenomenological fits to the existing data suggest surprisingly large higher twist contributions. It would be interesting to get an estimate of the twist-4 part of F_L, which has been analyzed in the framework of operator product expansion. The corresponding matrix elements could not yet be evaluated, by relatively reliably methods like lattice gauge calculations or QCD sum rules. In this contribution we will derive an estimate for the nonsinglet part of the longitudinal structure function, denoted by F_L^{NS}, in the framework of the renormalon calculus [1].

The forward Compton-scattering-amplitude is calculated in the Borel-plane with an effective gluon propagator, taking only one gluon exchange into account. In the Landau gauge the effective gluon propagator resums arbitrary many quark loop insertions in the gluon line, which is an exact procedure in QED and reads in the Borel representation:

$$\mathcal{B}_{1/a_s}[a_s D^{ab}_{\mu\nu}(k)](u) \;=\; \delta^{ab}\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}}{k^2}\left(\frac{\mu^2 e^{-C}}{-k^2}\right)^{\beta_0 u} \quad , \tag{2}$$

$a_s = \alpha_s/(4\pi)$, C corrects for the renormalization scheme dependence and u is the Borel transformation parameter. In QCD the restriction to one gluon exchange is an exact procedure only in the large N_f-limit, where N_f is the number of quark-flavors. The next-to-leading N_f-order terms are approximated by naive-nonabelianization (NNA), which means to replace the one loop beta-function of QED by the QCD beta-function $\beta_0 = 11 - \frac{2}{3}N_f$ and corresponds to the replacement $N_f \to N_f - \frac{33}{2}$. The quality of this approximation has to be checked by comparing the NNA-perturbative-coefficients with the known exact ones.

The IR-renormalon-poles reflect the factorial growth of the perturbative series, which has to be interpreted as an asymptotic expansion. It has to be truncated after the minimal term, which determines the best accuracy one may achieve using perturbative expansion. As the Q^2 dependence of this ambiguity is power-like, it has been suggested to use it as an estimate of higher-twist contributions. Despite the fact that the conceptual basis for this approach is controversial, the procedure has given reasonable estimates [2, 3].

		Exact results	NNA approximants
$N=2$	$N_f=3$	$43.1254\ a_s^2 + 1386.59\ a_s^3$	$61.3333\ a_s^2 + 2168.\ a_s^3$
	$N_f=4$	$38.5822\ a_s^2 + 1032.7\ a_s^3$	$56.7901\ a_s^2 + 1858.71\ a_s^3$
$N=4$	$N_f=3$	$37.75\ a_s^2 + 1472.58\ a_s^3$	$46.08\ a_s^2 + 1984.51\ a_s^3$
	$N_f=4$	$34.3367\ a_s^2 + 1155.64\ a_s^3$	$42.6667\ a_s^2 + 1701.4\ a_s^3$
$N=6$	$N_f=3$	$32.9091\ a_s^2 + 1433.24\ a_s^3$	$36.3918\ a_s^2 + 1726.31\ a_s^3$
	$N_f=4$	$30.2134\ a_s^2 + 1152.07\ a_s^3$	$33.6961\ a_s^2 + 1480.03\ a_s^3$
$N=8$	$N_f=3$	$29.1822\ a_s^2 + 1373.83\ a_s^3$	$30.1249\ a_s^2 + 1519.45\ a_s^3$
	$N_f=4$	$26.9507\ a_s^2 + 1117.97\ a_s^3$	$27.8934\ a_s^2 + 1302.68\ a_s^3$

Table 1: *NNA approximants compared to the exact results [2].*

To obtain the NNA-approximated coefficient function $\tilde{C}_{L,N}(a_s)$ we have to calculate the first order a_s correction to the Compton scattering amplitude using the effective propagator (2). We get

$$\begin{aligned}
\mathcal{B}[\tilde{C}_{L,N}(a_s)](s) &= C_F \left(\frac{\mu^2 e^{-C}}{Q^2}\right)^s \frac{8}{(2-s)(1-s)(1+s+N)} \frac{\Gamma(s+N)}{\Gamma(1+s)\Gamma(N)} \\
&= C_F \left(\frac{\mu^2 e^{-C}}{Q^2}\right)^s 8 \frac{\Gamma(s+N)}{\Gamma(1+s)\Gamma(N)} \\
&\quad \times \left(\frac{1}{(2+N)(1-s)} - \frac{1}{(3+N)(2-s)} + \frac{1}{(2+N)(3+N)(1+s+N)}\right) \qquad (3)
\end{aligned}$$

Two IR-renormalons appear at $s=1$ and $s=2$ in the Borel-representation. The position of the UV-renormalon $s=-(1+N)$ depends on the moment N one is dealing with.

The NNA-approximated perturbative coefficients can be derived from (3) to all orders in a_s (setting the renormalization point to $\mu^2 = Q^2$ and $C = -5/3$ for $\overline{\text{MS}}$-scheme) and then be compared with the exact results derived by Larin et al. for the non singlet moments $N = 2,\ 4,\ 6,\ 8$ (see table 1). In each order the leading N_f-term $N_f^{n-1} a_s^n$ agrees exactly, so that the result is exact for the first order a_s correction, which is not shown. The N_f-subleading terms are NNA approximated. It is interesting to observe that NNA is for higher moments a consistently better approximation than for lower ones. This is due to the fact that the most problematic property of NNA is to neglect multiple gluon emission. As such processes are important for small x we cannot expect our NNA structure functions to be correct in this region. Ever higher moments of the structure functions are less and less sensitive to their small-x behaviour and therefore the NNA should systematically improve.

As already mentioned the IR-renormalon poles in (3) reflect a factorial growth of the pertrubative series. This means that the perturbative expansion at best can be regarded as an asymptotic expansion. The general uncertainty in the perturbative prediction is then of the order of the minimal term in the expansion, which can be estimated either directly or by taking the imaginary part $\Im/\pi$ of the Borel transform. The leading IR-renormalon at $s=1$ gives rise to a Λ^2/Q^2 ambiguity for the asymptotic series. This uncertainty is easily transformed from the momentum-space to Bjorken-x:

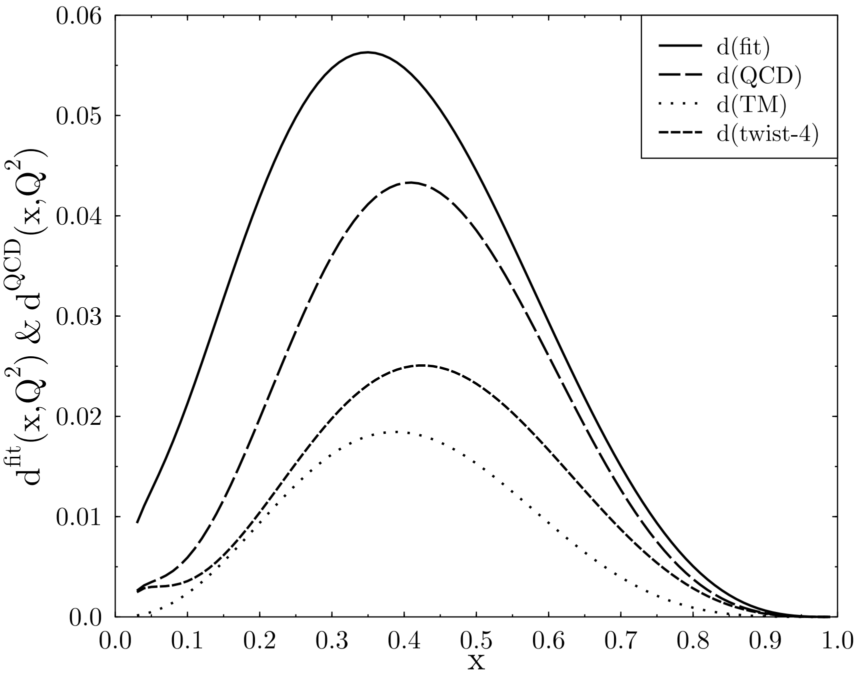

Figure 1: $\Lambda_{\overline{MS}} = 250$ MeV, $Q^2 = 5$ GeV *and* $N_f = 4$ *[2].*

$$F_L^{p-n}(x,Q^2) + \frac{4x^2M^2}{Q^2}F_2^{p-n}(x,Q^2) = 4C_F a_s(Q^2)\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2 F_2^{p-n}(y,Q^2) + \mathcal{O}(a_s^2)$$

$$+4x^3\frac{M^2}{Q^2}\int_x^1 \frac{dy}{y}F_2^{p-n}(y,Q^2) \pm \frac{8C_F}{\beta_0}\frac{\Lambda_{\overline{MS}}^2 e^{5/3}}{Q^2}\left[F_2^{p-n}(x,Q^2) - 2\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^3 F_2^{p-n}(y,Q^2) + \mathcal{O}(a_s)\right] \tag{4}$$

The first two terms are the standard convolution and the target mass correction respectively. The last term with undetermined sign is the calculated ambiguity, which we will use as an estimate of the twist-4 correction to F_L^{NS}.

In Figure 1 the experimental fit of the power corrections d^{fit} is compared with the QCD-calculation d^{QCD} (Eq. (4)) using an experimental fit for F_2, where we have shown target mass d^{TM} and twist-4 contribution $d^{twist-4}$ separately. We observe a rather large contribution coming from the IR renormalon estimate for the twist-4 part, which accounts for more than half of the discrepancy between the experimental fit and a prediction which takes into account the target mass correction only. In view of the fundamental objections concerning the used IR-renormalon method, the prediction of a large twist-4 contibution to F_L^{NS} in agreement with experimental results is a surprising success. It further supports the idea that, while the rigorous QCD calculations of higher twist contribution to F_L are not yet available, calculations like the one presented here may be used to predict the order of magnitude of power suppressed corrections.

References

[1] References may be found in [2, 3].

[2] E. Stein, M. Meyer-Hermann, L. Mankiewicz, A. Schäfer, Phys.Lett.B376 (1996) 177.

[3] M. Meyer-Hermann, M. Maul, L. Mankiewicz, E. Stein, A. Schäfer, *IR-Renormalon Contribution to the Polarized Structure Function* g_1, hep-ph/9605229, to appear in Phys.Lett.B.