Future Precision Measurements of $F_2(x, Q^2)$, $\alpha_S(Q^2)$ and $xg(x, Q^2)$ at HERA

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Abstract: The results are presented of a study of the accuracy one may achieve at HERA in measuring the strong coupling constant α_s and the gluon distribution $xg(x, Q^2)$ using future data of the structure function $F_2(x, Q^2)$ which are estimated to be accurate at the few % level over the full accessible kinematic region down to $x \simeq 10^{-5}$ and up to $Q^2 \simeq 50000$ GeV^2. The analysis includes simulated proton and deuteron data, and the effect of combining HERA data with fixed target data is discussed.

1 Introduction

Deep inelastic scattering is the ideal place to investigate the quark-gluon interaction. Previous fixed target experiments have lead to very precise tests of Quantum Chromodynamics in the kinematic range of larger $x \geq 0.005$ and lower $Q^2 \leq 300$ GeV^2. The first few years of experimentation at HERA extended this range to very low $x \simeq 0.0001$ and large $Q^2 \simeq 3000$ GeV^2 leading to remarkable results in the investigation of deep inelastic scattering [1, 2] including rather accurate measurements already of the proton structure function $F_2(x, Q^2)$. In this study an attempt has been made to estimate the accuracy of future measurements of F_2 at HERA and their possible impact on precision mesurements of the strong coupling constant $\alpha_s(Q^2)$ and the gluon distribution $xg(x, Q^2)$. The measurement of these quantities is a key task at HERA. Both can be determined in a number of different processes as deep inelastic jet production, charm and J/ψ production and with future measurements of the longitudinal structure function. The measurement of F_2, however, is expected to be the most precise way to determine α_s and xg from the scaling violations of F_2. Those are most prominent at very low x due to quark pair production from the gluon field and weaker at large $x \geq 0.1$ due to gluon bremsstrahlung. Both processes, and their NLO corrections, will be accessible with future high statistics data at HERA which is hoped to deliver a final luminosity figure near to $\mathcal{L} \simeq 1$ fb^{-1} during the next 8 years of operation.

The QCD analysis of the past and present F_2 structure function data lead to remarkable results already, more than listed here:

- A rather precise determination of $\alpha_s(Q^2)$ with an experimental error of 0.003 at $Q^2 = M_Z^2$ was performed using the SLAC and the BCDMS structure function data [3].
- Both H1 [4, 5] and ZEUS [6, 7] have determined the gluon distribution with an about 15% accuracy at $Q^2 = 20$ GeV^2 and $x \simeq 10^{-4}$ by using different sets of fixed target data [8, 9, 10] combined with the HERA results.
- The HERA deep inelastic structure function data have a big impact on global analyses and the determination of parton distributions [11].

The analysis presented in this paper will show that HERA will allow to reach the 1% level of determining α_s and xg. This represents a challenge to the theoretical understanding of deep inelastic scattering in perturbative QCD in the low x and low $Q^2 \sim M_p^2$ region. A precision measurement of the strong coupling constant will represent an important constraint to unified theories. As such it represents one fundamental reason to perform an extended long term programme of experimentation at HERA.

This paper is organized as follows. Section 2 presents the assumptions and the results of the simulation of F_2 structure function data. Section 3 contains the outline of the QCD analysis procedure and error treatment required for the analysis. The results of a detailed study of the α_s measurement accuracy are given in section 4. Similarly the determination of the gluon distribution is presented in section 5. A brief summary is given in section 6.

2 Accuracy of Future HERA Structure Function Data

Recent measurements of the proton structure function $F_2(x, Q^2)$ by the H1 and ZEUS collaborations [4, 6], based on data taken in 1994 with an integrated luminosity $\mathcal{L}$ of about 3 pb^{-1}, have reached a systematic error level of about 4-5% in the bulk region of the data, $10 \leq Q^2 \leq 100$ GeV^2. Exploratory measurements of the very low Q^2 region with about 15-20% accuracy were presented by H1 with 1995 shifted vertex data [12] and by ZEUS using a rear calorimeter installed near the beam pipe in backward direction [13]. Based on the experience of these analyses a study has been made in order to estimate what might be the ultimate accuracy of F_2 measurements at HERA. This is a difficult task: on one hand one can rather easily extrapolate the present knowledge of systematic errors and also calculate rather straightforward the effect of residual miscalibrations on the cross section measurement. On the other hand there will always be local, detector dependent effects in addition and, furthermore, one can not simulate the results to be expected from innovations of the structure function analyses. For example, it is likely that a low electron energy calibration, much below the kinematic peak, can be performed reconstructing the π_0 mass or, to give another one, the region of y below 0.01, which was considered to be not accessible due to calorimetric noise, may be accessed nevertheless by imposing a p_T balance constraint using the electron information. Therefore this simulation study may give valid estimates but the truth will be the result of data taking and analysis work over many years still to come.

For this analysis the following kinematic constraints have been imposed:

- $Q^2 \geq 1$ GeV^2 which may be the limit of applicability of the DGLAP evolution equations at low x [14];
- $\theta_e \leq 177^o$ which might be accessible with nominal energy running even after the luminosity upgrade;
- $y \leq 0.8$ a limit arising from large radiative corrections and a small scattered electron energy limit $E'_e \geq$ few GeV due to photoproduction background and electron identification limitations;
- $\theta_h \geq 8^o$, a hadron reconstruction limit imposed by the beam pipe which may differ somewhat finally.

A number of data sets was generated as summarized in table 1 and illustrated in fig.1. The maximum Q^2 of the data depends on the available luminosity and might reach values of up to 50000 GeV^2. The generation and systematic error calculation was performed with a numerical program written by one of us which was checked to be in good agreement with the Monte Carlo programs used for real data analyses.

number	nucleon	E_e	E_N	$\mathcal{L}/pb^{-1}$	Q^2_{min}	Q^2_{max}
I	proton	27.6	820	10	0.5	100
II	proton	27.6	820	1000	100	50000
III	proton	27.6	400	200	100	20000
IV	proton	15.0	820	10	0.5	100
V	deuteron	27.6	410	10	0.5	100
VI	deuteron	27.6	410	50	100	20000

Table 1: Summary of simulated data sets for this study, energy values are in GeV and Q^2 in GeV^2.

The following systematic error sources were considered in the analysis the effect of which is illustrated in fig.2:

- An electron energy calibration error of 0.5% in the backward region ($\theta_e \geq 160^o$) and 1% in the central barrel and forward region of the detectors.
- An electron polar angle uncertainty of 0.5 mrad backwards and 1 mrad in the central part of the detector ($\theta_e \leq 165^o$).
- A 2% uncertainty of the hadronic energy scale which is important at lower $y \leq 0.1$ where the kinematics cannot be determined solely with the electron variables E'_e and θ_e because of divergencies of the resolution $\propto 1/y$. The energy scales E'_e and E_h may be cross calibrated by comparing cross section mesurements in different parts of the detector [15] once there is high statistics available in the barrel part, and using the electron and track information in the detector.

- The photoproduction background may cause a 1-2% error at large $y \geq 0.5$ and for $Q^2 \leq 100$ GeV^2. This requires an about 10% control of its shape and normalization which can be envisaged with the electron taggers, the hadronic calorimeter sections and using tracking information in front of the calorimeters which suppresses the π_0 part of the contamination.

- Radiative corrections can be controled to 1%, perhaps 2% at highest $y \geq 0.7$, using the hadronic and electron information which overconstrains the kinematics. The Monte Carlo [16] and numerical calculations [17] are known to be in very good agreement. This F_2 simulation assumes the radiative corrections to be performed, including the electroweak part which at high y and Q^2 modifies the cross section at the $\sim 20\%$ level.

- Beam background and various efficiencies are assumed to introduce an overall error of 2%.

- A luminosity error of 1% is assumed.

These systematic errors are about one half of those presently reached in the high statistics domain of the F_2 measurements. If the kinematic dependence of the correlated systematic errors is sufficiently well known, it can be taken into account in QCD fits, see below. Note in this respect that the required luminosity is not simply given by the statistical errors per bin but rather by the statistics needed for detailed systematic studies. However there will always be residual local and higher order effects which we represent here by a random systematic error of 1%.

3 Analysis Procedure

The generated data were analysed using the H1 [18] and ZEUS [19] QCD fitting programs. Elsewhere in these proceedings both programs are shown to be in good agreement [20]. In order to simplify the analysis the data were replaced by the QCD model (see below) so that the fits immediately converged to the minimum $\chi^2 = 0$ and CPU time was effectively spent only on the calculation of the covariance matrix of the fitted parameters. The errors on the gluon distribution and on α_s are then obtained from standard error propagation.

3.1 QCD Model

The QCD prediction for the F_2 structure function can be written as

$$F_2^{QCD}(x, Q^2) = F_2^{uds}(x, Q^2) + F_2^c(x, Q^2), \tag{1}$$

where F_2^{uds} obeys the NLO QCD evolution equations for $f = 3$ light flavours and the charm contribution F_2^c is calculated according to [21]. The light flavour contribution in turn is decomposed into a singlet and a non-singlet part:

$$F_2^{uds}(x, Q^2) = F_2^S(x, Q^2) + F_2^{NS}(x, Q^2). \tag{2}$$

The singlet structure function is related to the singlet quark momentum distribution, $x\Sigma = \sum_f x(q_f + \bar{q}_f)$, which obeys an evolution equation coupled to the gluon distribution xg. The main contribution to F_2^{NS} comes from the difference of up and down quarks and antiquarks: $x\Delta_{ud} = x(u+\bar{u}) - x(d+\bar{d})$. We remark here that Δ_{ud} is constrained by the difference $F_2^p - F_2^d$ of proton and deuteron structure functions.

At the input scale $Q_0^2 = 4$ GeV^2 the parton distributions were parametrised as

$$\begin{aligned} xG(x, Q_0^2) &= A_G x^{B_G}(1-x)^{C_G} \\ x\Sigma(x, Q_0^2) &= A_S x^{B_S}(1-x)^{C_S}(1 + D_S x + E_S\sqrt{x}) \\ x\Delta_{ud}(x, Q_0^2) &= A_{NS} x^{B_{NS}}(1-x)^{C_{NS}}. \end{aligned} \tag{3}$$

The input parameters for the gluon and the singlet distributions were obtained from a fit to the simulated data whereas the non-singlet parameters and their uncertainties were taken from [7]. The input value of α_s was set to $\alpha_s(M_Z^2) = 0.113$ corresponding to $\Lambda^{(4)}_{\overline{\rm MS}} = 263$ MeV [3].

3.2 Definition of the χ^2 and Fit Procedure

The two fitting programs of H1 and ZEUS have been used in parallel and all important numbers were cross checked. The ZEUS program uses a step by step (à la Runge Kutta) procedure to solve the DGLAP evolution equations. The H1 program projects the DGLAP equations on a functional basis where they are solved exactly [18, 22]. Both programs use MINUIT to make the fitting. In addition the H1 program has the possibility to use an independent set of routines (called LSQFIT) which performs a least chisquare fit. In LSQFIT the χ^2 function to be minimised is recognized to be the sum of the square of deviations and the derivatives of the deviations are computed by finite differences. Both MINUIT and LSQFIT can compute the second order derivatives of the χ^2 with respect to the parameters: these may be used for the error computation as will be shown in the following.

The χ^2 is defined as

$$\chi^2 = \sum_i \left(\frac{F_i(p,s) - f_i}{\Delta f_i}\right)^2 + \sum_l (s_l)^2 \tag{4}$$

where F_i is the model prediction, f_i the measured F_2 value, Δf_i its statistical error and the sum runs over all data points (i). In addition to the set of parton distribution parameters $\{p\}$, including α_s, we have introduced the parameter set $\{s\}$ which takes into account the systematic errors of the measurements. The relation between the model prediction and the QCD prediction for F_2 is written as:

$$F_i(p,s) = F_i^{QCD}(p) \;\; (1 - \sum_l s_l \Delta_{li}^{syst}) \tag{5}$$

where Δ_{li}^{syst} is the relative systematic error on data point (i) belonging to the source (l). We assume that the parameters s_l are gaussian distributed with zero mean and unit variance so that the Δ_l correspond to a one standard deviation systematic error[1].

3.3 Systematic Error Evaluation

Given the χ^2 definition of the previous section there are essentially three methods to evaluate the systematic errors on the fitted parameters:

[1] Asymmetric errors can be taken into account by adding terms quadratic in s_l in eq. 5.

- Repeat the fit with several values of the systematics variables s_l, either chosen at random or giving to each variable in turn the value 1. The systematic errors are then obtained by adding all the deviations from the central value in quadrature.

- Leave the systematic parameters fixed to zero but propagate the errors on s_l (assumed to be 1) to the covariance matrix of the fitted parameters [23]. If the deviations are linear functions of the systematic variables it is easy to compute directly the errors from the second derivatives of the χ^2. Let us introduce the following matrices:

$$M = \sum_i \frac{\partial F_i}{\partial p} \frac{\partial F_i}{\partial p} \frac{1}{\Delta m_i^2} \approx 1/2 \frac{\partial^2 \chi^2}{\partial p \partial p} \tag{6}$$

$$C = \sum_i \frac{\partial F_i}{\partial p} \frac{\partial F_i}{\partial s} \frac{1}{\Delta m_i^2} \approx 1/2 \frac{\partial^2 \chi^2}{\partial p \partial s} \tag{7}$$

$V^{stat} = M^{-1}$ is the $\{p\}$ statistical error matrix and C is the matrix which expresses statistical and systematic correlations. One can show that [23]

$$V^{syst} = M^{-1} C C^T M^{-1} \tag{8}$$

is the $\{p\}$ systematic error matrix. The LSQFIT program determines these matrices and also the function error bands. For MINUIT a fit has to be performed where the systematic parameters are left free and the inverse of the resulting covariance matrix contains the matrices M and C. Reinverting M and using eq.(8) yields the statistical and systematic errors in case all systematic parameters are kept fixed.

- The $\chi^2 = \chi^2_{min} + 1$ method
If the correlations between parameters are big and/or the dependence of the deviations with respect to the parameters is highly non–linear, it is more appropriate to compute the error on a specific parameter by considering an increase of the χ^2 by one, all the other parameters being optimised. Both MINUIT and LSQFIT can provide this calculation. This method has been used also to draw error bands with LSQFIT which is faster than the MINOS option of MINUIT. In this case the value of the function itself at some fixed x and Q^2 point is taken as a parameter. Then the equation say $G(x, Q^2) = G$ is used to eliminate the more sensitive parameter.

The three different methods and the two different programs have been compared in detail leading to consistent results. Fits were performed on data randomly offset both statistically *and* systematically. Taking as the error the r.m.s. of the fitted α_s values this appeared to be in agreement with the standard error calculation. This method gives the most reliable error estimate, but it is clearly too elaborate to be of practical use in a study like the one undertaken here.

3.4 Fitting the Systematics

If the kinematical dependence of a systematic error, like the $1/y$ behaviour of the electron energy scale uncertainty, is well known, a contribution, $\sum_l s_l^2$, can be added to the χ^2 and a fit can be performed determining an extended set of parameters $\{p, s\}$. The interest of such a procedure is obviously that here full knowledge of the experiment enters to improve the measurement accuracy. Such a procedure was adopted in [3] to reduce the influence of the main experimental errors, the magnetic field calibration of the BCDMS spectrometer for example, on the value and error of α_s. The method to find the resulting errors is practically the same as for the statistical error treatment. LSQFIT and MINUIT deliver the complete error matrix which in the case of MINUIT is exactly the one used in method 2.

4 Results on α_s

4.1 Introduction

The data and the fitting procedures as described above were used to determine the expected error of $\alpha_s(M_Z^2)$, and of xg in the subsequent section. Three types of fits were performed:

- A - Fits to HERA proton data alone.
- B - Fits to HERA proton and deuteron data.
- C - Fits to HERA proton data with inclusion of fixed target data, in most of the cases those from SLAC [8] and BCDMS [9].

In the fits the systematic error parameters were left free. The input values $s_l = 0$ were always reproduced while the input errors $\Delta_{s_l} = 1$ were typically reduced by a factor of two. The correlation coefficients between the systematic error parameters were well below unity. In the following we denote the error on α_s from these fits by $\Delta\alpha_{fit}$. The statistical error ($\Delta\alpha_{stat}$) and the systematic error ($\Delta\alpha_{syst}$) for fixed systematic errors were calculated from the covariance matrix as described above.

4.2 α_s with HERA Data only

As a starting point of the investigation fits were made to HERA high energy proton data alone (sets I and II in table 1). The low Q^2 sample (set I) covers an x range of $1.4 \times 10^{-5} < x < 4.3 \times 10^{-2}$ whereas the high Q^2 sample (set II) covers $2.4 \times 10^{-3} < x < 0.65$. For the nominal fits integrated luminosities of 10 and 500 pb^{-1} were assumed for the low and the high Q^2 data set respectively.

The strong coupling constant and the parameters describing the input singlet (5 parameters) and gluon distributions (3 parameters) at $Q_0^2 = 4$ GeV^2 were left free in the fit. The gluon normalization was calculated by imposing the momentum sumrule. The non-singlet contribution to F_2 was kept fixed since it is not well constrained by proton data alone.

Besides α_s and the parton distribution parameters five systematic error parameters were introduced as described in section 3. In addition the assumed random systematic error was added in quadrature to the statistical error.

Recent analyses of ZEUS [7] and H1 [4] F_2 data have shown that perturbative QCD might be applicable down to $Q^2 \simeq 1$ GeV^2 at very low x, see also [14]. In figure 3a and table 2 the α_s error is given as a function of Q_c^2 which is the lowest Q^2 considered in the fit. The statistical error (which includes the 1% random systematic error) increases from $\Delta\alpha_{stat} = 0.0024$ to

0.0053 with increasing Q_c^2. When all systematic errors are fitted, $\Delta\alpha_{fit}$ is almost identical to the statistical error for $Q_c^2 = 1$ GeV^2 but increases more rapidly to 0.0075 at $Q_c^2 = 8$ GeV^2. When the systematics are not fitted, their contribution to $\Delta\alpha_s$ rises very strongly to about 0.012 above $Q_c^2 = 2$ GeV^2. The same tendency is observed when all the systematic errors are scaled down by a factor of two (figure 3b) though, as expected, the uncertainty of α_s is reduced almost by a factor of two.

To investigate the impact of the high Q^2 data on the result, the luminosity of dataset II was varied between $\mathcal{L} = 10$ and 1000 pb^{-1}. For a Q_c^2 cut of 1 GeV^2 the variation of the α_s errors with L is fairly modest as shown in figure 4a and table 2. However, the dependence on the high Q^2 luminosity becomes stronger if the Q_c^2 cut is raised. This is illustrated in figure 4b for $Q_c^2 = 3$ GeV^2. Here a factor of 10 increase in luminosity decreases the α_s errors by about 40%. On the other hand, increasing the luminosity of the low Q^2 sample (dataset I) from 10 to 50 pb^{-1} gave an insignificant improvement ($\approx 10\%$) on the uncertainty in α_s.

An improvement of the result is obtained if the lower energy data are included, sets III and IV in table 1. For example, for $Q_c^2 = 3$ GeV^2 the nominal data set yields an error of 0.0061, see table 2 while the inclusion of the lower energy data reduces that error to 0.0046, if in both cases the systematics is fitted.

As described above, the nonsinglet distribution is input to the fit of the proton data. Taking its uncertainty from the QCD analysis of [7] a contribution of about 0.004 is estimated to the uncertainty on α_s. However, when both proton and deuteron data are available the nonsinglet contribution is constrained by the difference $F_2^p - F_2^d$ and the 0.004 error gets eliminated. Therefore, a low and a high Q^2 deuteron data sample (set V and VI in table 1) with modest lumionosities were included in the fit. Apart from the highest Q^2 region these data have roughly the same kinematic coverage as the proton data.

In the combined proton and deuteron QCD fit the three parameters which describe the nonsinglet input distribution were left free. Furthermore one normalization parameter and five independent systematic parameters for the deuteron data were added. The resulting α_s errors are given in table 2 for a Q_c^2 cut of 3 GeV^2. It is seen that the error on α_s is reduced by about 25% compared to the corresponding fit on proton data only although the number of fit parameters had to be increased and the non-singlet distribution is as well fitted.

4.3 Inclusion of High x Fixed Target Experiment Data

The published QCD analysis [3] of SLAC and BCDMS proton and deuteron structure function data yielded an experimental error of 0.003 on $\alpha_s(M_Z^2)$. The natural question to be answered is whether the combination of the low x and high Q^2 HERA data with the fixed target experiment data can improve this result significantly.

As a first necessary step it was studied if our fits can reproduce the error quoted above. The following conditions were applied to mimic the analysis of [3] as closely as possible:

- A cut of $W^2 > 10$ GeV^2 was imposed to effectively remove the region at high x and low Q^2 dominated by higher twist effects.
- The parameters of the input singlet, gluon and nonsinglet distributions were left free except B_S and B_G which describe the low x behaviour of $x\Sigma$ and xG (the SLAC/BCDMS data extend only down to $x = 0.07$).

- One normalisation parameter was kept fixed (BCDMS deuteron) whereas those of the remaining three datasets were left free. In addition two systematic parameters for the BCDMS data were left free.
- No momentum sumrule was imposed.
- Being interested in the derived error only, the SLAC and BCDMS data were replaced by the model input.
- The quoted errors of ref [3] correspond to an increase of the χ^2 by 9 units which was taken into account in the estimate of the statistical errors.

The fit defined above on the SLAC/BCDMS data alone yielded as a result $\Delta\alpha_s = 0.0030$, exactly as published.

Using all high energy HERA proton data in addition the error on α_s was reduced to $\Delta\alpha_{fit} = 0.0016$ with $Q_c^2 = 3$ GeV^2. Adding the latest data of the NMC experiment with a preliminary treatment of the systematic errors of this measurement reduces this number to 0.0013. In table 2 results are given on $\Delta\alpha_s$ for various choices of the Q_c^2 cut, the size of the systematic errors and the luminosity of the high Q^2 data sample. It turns out that $\Delta\alpha_{fit}$ ranges from about 0.001 to 0.002 and is thus fairly insensitive to these choices. Compared to fits on HERA proton data alone, the error on α_s is much less sensitive as to whether the systematic parameters are left free or kept fixed and the dependence on the minimum Q^2 is less severe. With fixed systematic errors the combined statistical and systematic error ranges from $\Delta\alpha_s = 0.002$ to about 0.003.

4.4 Double Logarithmic Scaling and the Error of α_s

With high precision data the low x behaviour of F_2 will be much better understood. If the data further support the double logarithmic approximation [24] of the low x, large Q^2 behaviour of F_2, then a precision of α_s to 0.001 or even better can be reached with HERA data alone. Already with the present H1 data only, such an analysis [25] lead to a value of $\alpha_s(M_Z^2) = 0.113 \pm 0.002(stat) \pm 0.006(syst)$. The advantage of this approach is obvious as it likely requires only the low x data of HERA and depends on two scale parameters, Q_o^2, x_o, a normalization constant and α_s only. This is in contrast to the QCD analysis of HERA and fixed target data, considered here, which has to include the full parametrization of two nonsinglet, the singlet and the gluon distribution leading to typically 15 parameters to be simultaneously controled. Further theoretical understanding of the double scaling approach is necessary, however.

5 Determination of the Gluon Distribution

5.1 HERA Proton Data Only

The previous determinations of the gluon distribution from the scaling violation of F_2 at low x by H1 [4, 5] and ZEUS [6, 7] were performed combining the HERA results with fixed target data and treating α_s as an extra parameter. Since the scaling violations essentially are proportional to the product of $\alpha_s \cdot xg$, a large data range in x and Q^2 is required to disentangle these two basic quantities.

Fit	L(LQ)	L(HQ)	sfac	qmin	sigf	stat	syst	total
A	10	500	1.0	1.0	.0024	.0021	.0052	.0056
A	10	500	1.0	2.0	.0050	.0036	.0107	.0113
A	10	500	1.0	3.0	.0061	.0041	.0114	.0120
A	10	500	1.0	5.0	.0069	.0047	.0114	.0123
A	10	500	1.0	8.0	.0075	.0052	.0105	.0117
A	10	500	0.5	1.0	.0015	.0013	.0024	.0028
A	10	500	0.5	2.0	.0033	.0026	.0055	.0061
A	10	500	0.5	3.0	.0040	.0031	.0060	.0067
A	10	500	0.5	5.0	.0046	.0035	.0060	.0070
A	10	500	0.5	8.0	.0050	.0040	.0054	.0067
A	10	10	1.0	1.0	.0028	.0023	.0045	.0050
A	10	100	1.0	1.0	.0026	.0022	.0048	.0053
A	10	1000	1.0	1.0	.0023	.0020	.0053	.0057
A	10	10	1.0	3.0	.0096	.0080	.0126	.0149
A	10	100	1.0	3.0	.0073	.0055	.0123	.0134
A	10	1000	1.0	3.0	.0057	.0037	.0110	.0116
B	10	500	1.0	3.0	.0043	.0030	.0087	.0092
C	10	500	1.0	1.0	.0014	.0011	.0031	.0033
C	10	500	1.0	8.0	.0016	.0012	.0032	.0034
C	10	500	0.5	1.0	.0011	.0009	.0018	.0020
C	10	500	0.5	8.0	.0014	.0011	.0018	.0022
C	10	10	1.0	3.0	.0020	.0018	.0022	.0028
C	10	100	1.0	3.0	.0017	.0015	.0023	.0027
C	10	1000	1.0	3.0	.0015	.0011	.0030	.0032

Table 2: Errors on α_s for fits with HERA proton data only (A), proton and deuteron data (B) and combinations of simulated HERA proton data with fixed target experiment data (C) from SLAC and BCDMS, see text.

Fig.5 shows the result of a QCD fit to the simulated F_2 data, sets I and II in table 1, without fitting the systematics but determining xg and the singlet distribution $x\Sigma$. In this fit α_s was fixed considering an α_s uncertainty of 0.005 for the calculated gluon error. The inner dark error band is the statistical error while the total error is shown as the outer grey band for all Q^2 values. The error bands were drawn using the LSQFIT routines and the $\chi^2 = \chi^2_{min} + 1$ method. At $Q^2 = 20$ GeV^2 and $x = 0.0001$ the total error amounts to 11% somewhat better than the present result which included the fixed target experiments. If α_s is allowed to vary and the systematics is fitted as described in section 3 the gluon determination gets very accurate with an estimated error of 3% at the same x and Q^2 values. This is illustrated in figure 6.

5.2 HERA Proton and Deuteron Data

The consideration of deuteron data of $\mathcal{L} = 50$ pb^{-1} allowed finally to perform a complete fit of all distributions. In particular one has to notice that the non–singlet distributions are not well constrained with proton data only. In section 3 the up–down quark distribution difference was introduced as the non–singlet quantity to be determined assuming the strange distribution was fixed, see [7]. Generally there are two non–singlet distributions which may be written as $u^+ = u + \bar{u} - \Sigma/3$ and similarly d^+. The deuteron data allow to determine these distributions. Fig.7 shows the calculated accuracy of the u^+ distribution at $Q_o^2 = 2$ GeV^2 from a complete fit to all distributions, α_s and with free systematic error parameters. This represents an interesting result as apparently the non–singlet distributions will be measurable with high accuracy down to very low x, and the predicted weak Q^2 dependence can be verified. The gluon belonging to this fit is determined with an error of only 1-2% at the x, Q^2 point chosen for comparison.

Further studies of the importance of HERA deuteron data are necessary. For example, this data will have an additional few % uncertainty due to shadowing corrections at low x which was neglected here. Constraints on the non–singlet distributions are available already from the fixed target deuteron data, but at higher x. A more thorough discussion of these aspects has been beyond the scope of this study. Independently of theoretical preassumptions, however, one may regard this data as important to measure the low x behaviour of the up-down quark distribution difference which requires modest luminosity only. With larger luminosity $\mathcal{L} \simeq 50$ pb^{-1} it will help to decompose the flavour contents of the nucleon as was discussed already ten years ago [26].

6 Summary

According to this analysis HERA will have an important impact on the measurement of α_s and the gluon distribution. A precision near 0.001 for $\alpha_s(M_Z^2)$ and of almost 1% for xg is in reach if

- F_2 measurements in the full HERA range will become available with systematic and statistical errors of a few % only,
- the systematic errors are thoroughly studied at the per cent level as functions of x and Q^2 such that their gross effects can be absorbed in the QCD analysis,

- and the HERA data can be combined reliably with the fixed target experiment results on F_2.

Such an accuracy represents a great challenge for the experimental programme at HERA. The HERA data should be complete, i.e. comprise a high luminosity data set with $\mathcal{L} \geq 300$ pb^{-1} and modest luminosity data sets: i) at lowered proton energy $\mathcal{L} \geq 50$ pb^{-1} to reach highest x, as close as possible to the fixed target data region, ii) at lowered electron beam energy $\mathcal{L} \geq 3$ pb^{-1} to cover the x dependence at smallest Q^2 and iii) possibly also deuteron data with $\mathcal{L} \simeq 50$ pb^{-1}. Note that no attempt was made to optimize these luminosity values, in particular, since the necessary level of systematic error control is competing with the requirements coming from simple statistical error considerations.

Completion of this programme also requires a major theoretical effort to calculate the 3-loop coefficient functions since the present theoretical uncertainty of α_s of ~ 0.005 [27] exceeds most of the estimated experimental α_s errors discussed in this study. It is obvious that a precision at the level of 0.002 for $\alpha_s(M_Z^2)$ will lead to a very precise study of its Q^2 dependence and resolve the question of the compatibility of the deep inelastic α_s values with those from e^+e^- scattering.

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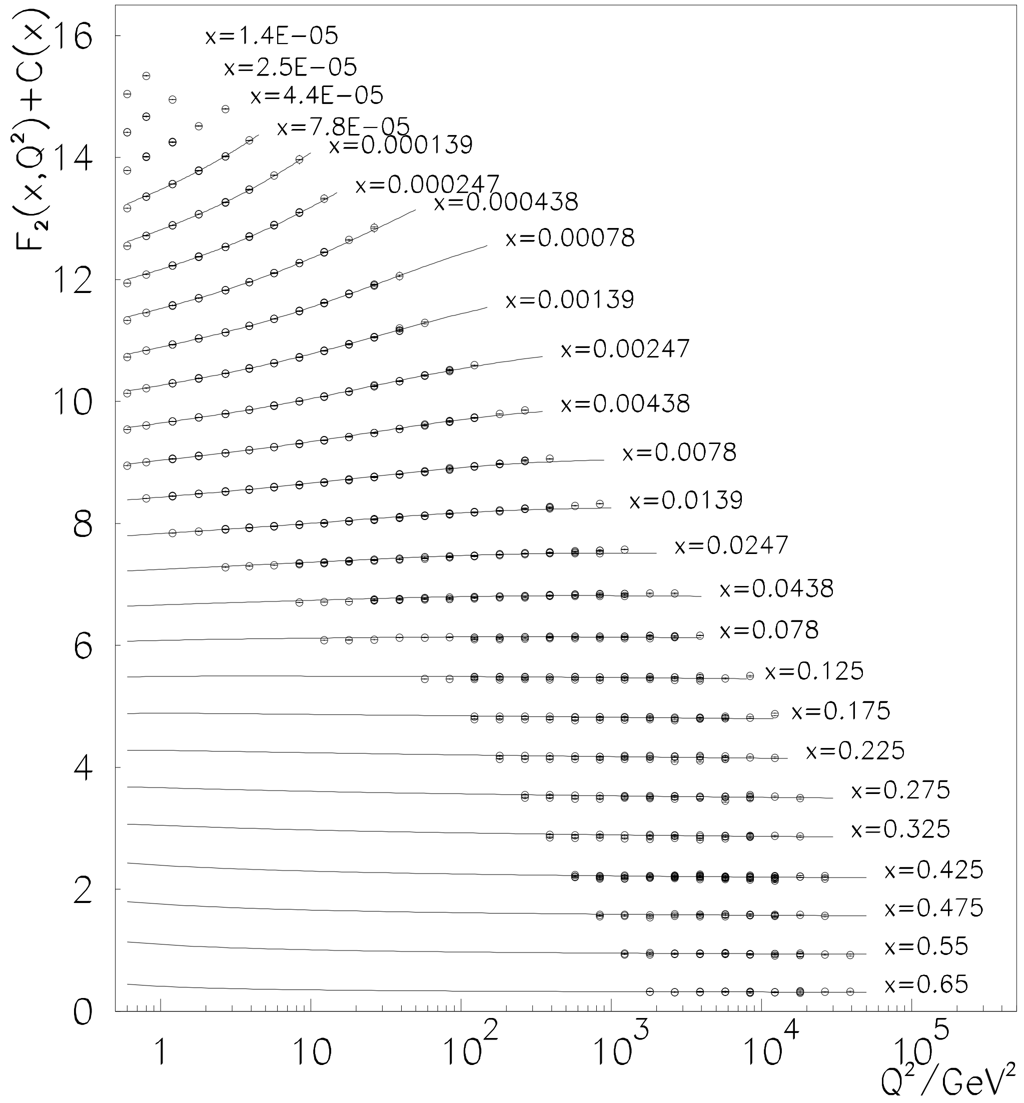

Figure 1: Simulated structure function data sets. The huge luminosity of 1 fb^{-1} will lead to precise data even at very high Q^2. For $Q^2 \geq 10000$ GeV^2 about 2000 events may be available. The largest bins shown are made with 20-50 events. With $\mathcal{L} = 10$ pb^{-1} for $Q^2 \geq 0.5$ GeV^2 about 10^7 events are occuring which will be prescaled at lowest Q^2. The curve represents a NLO QCD fit. The high x, low Q^2 region can not be accessed with HERA but is almost completely covered by the fixed target experiment data, not shown here.

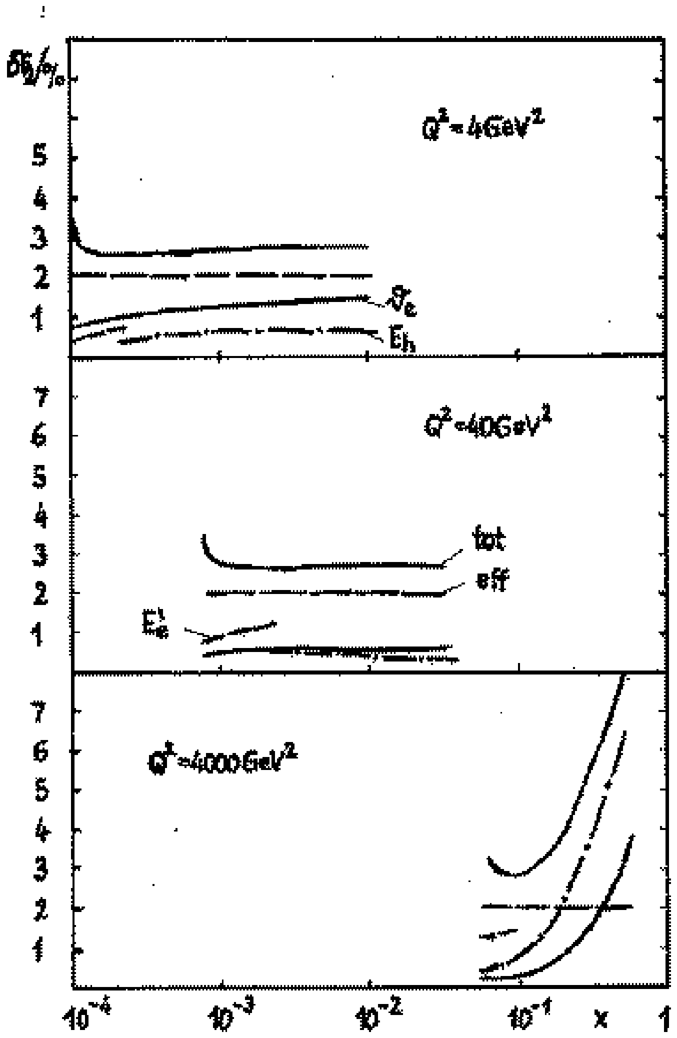

Figure 2: Estimated systematic errors of the F_2 measurement for different Q^2 as a function of x. Dashed line: effect of error on the scattered electron energy E_e', dashed-dotted line: effect of error on the hadronic energy scale E_h, solid line: effect of error on the polar angle θ_e; long dashed line: 2% efficiency error. Not drawn are the effect of photoproduction background at high y and the radiative correction error. Both have been added to the other error sources which gives a total error drawn as the upper solid line. The authors apologize for the workshop style of this figure.

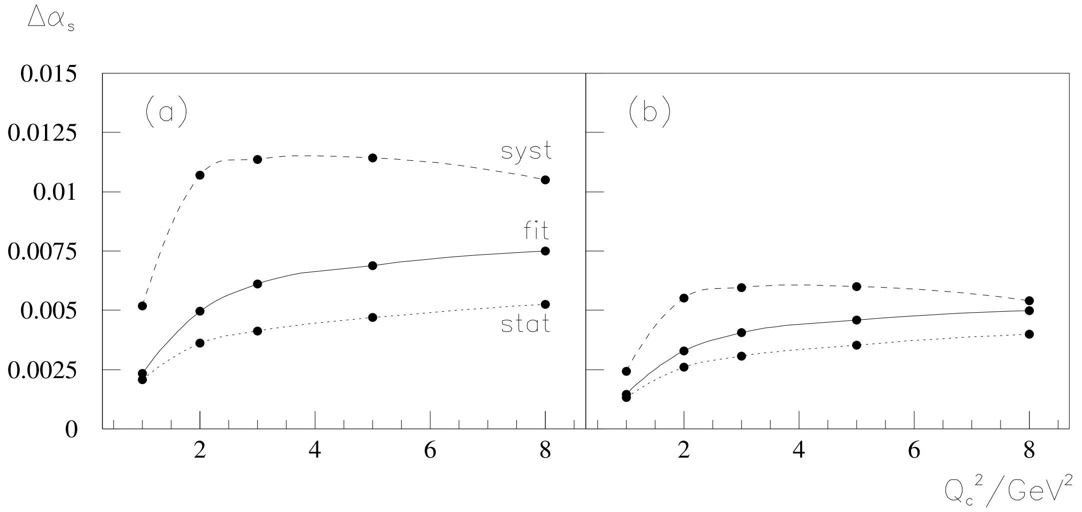

Figure 3: The error on $\alpha_s(M_Z^2)$ from fits to the HERA high energy proton data as a function of Q_c^2: (a) with full systematics included; (b) with systematics further reduced by a factor of two. The (dotted, solid, dashed) curves correspond to the errors ($\Delta\alpha_{stat}$, $\Delta\alpha_{fit}$, $\Delta\alpha_{syst}$) described in the text.

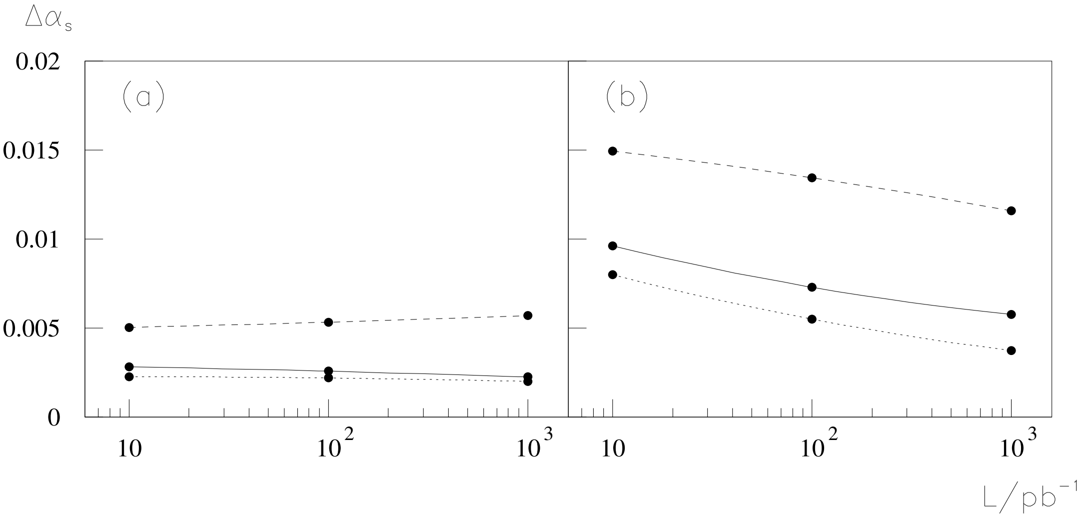

Figure 4: The error on $\alpha_s(M_Z^2)$ as a function of the luminosity of the high Q^2 sample: (a) for $Q_c^2 = 1$ GeV^2; (b) for $Q_c^2 = 3$ GeV^2. The (dotted, solid, dashed) curves correspond to the errors ($\Delta\alpha_{stat}$, $\Delta\alpha_{fit}$, $\Delta\alpha_{syst}$) described in the text.

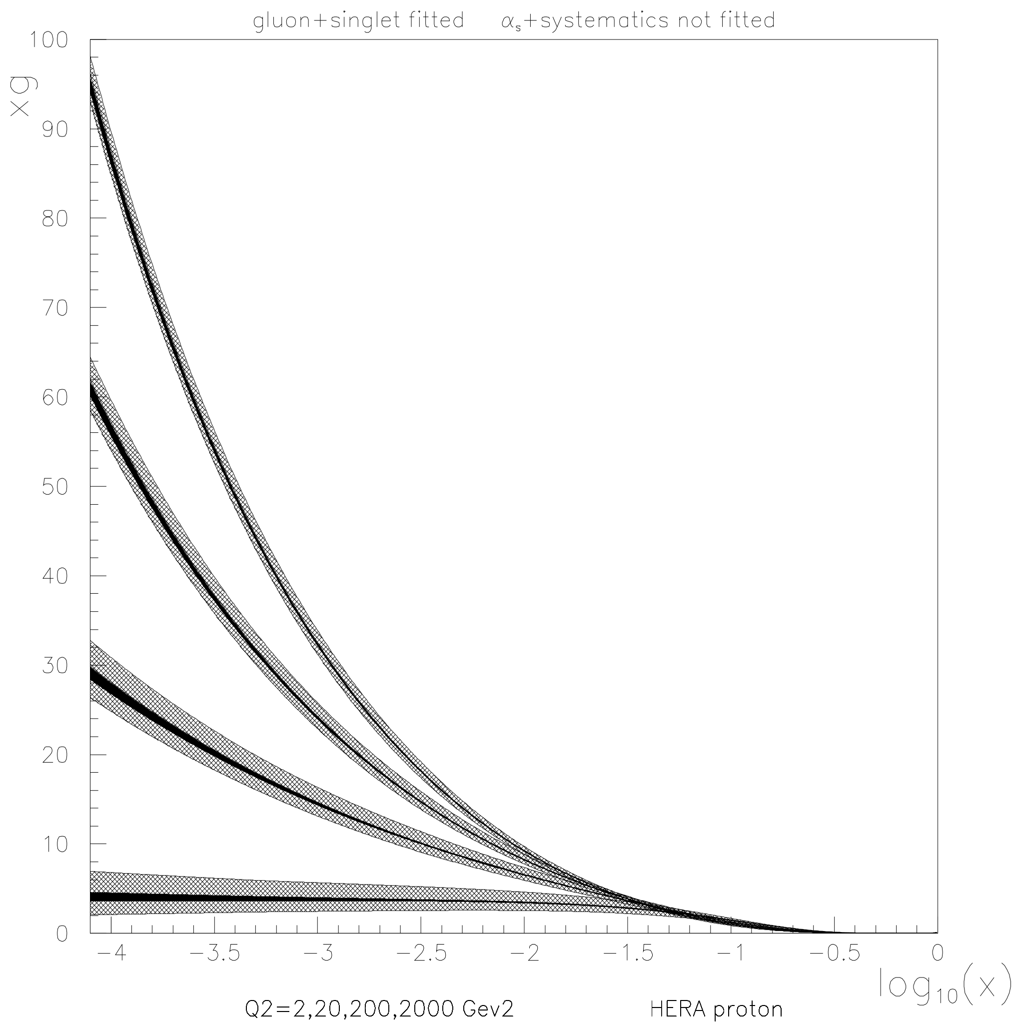

Figure 5: Determination of the gluon distribution using future F_2 data from electron-proton scattering with fixed systematic error parameters. The inner band is the statistical error. Note that for simplicity the gluon is shown outside the allowed region of $x \leq Q^2/10^5$.

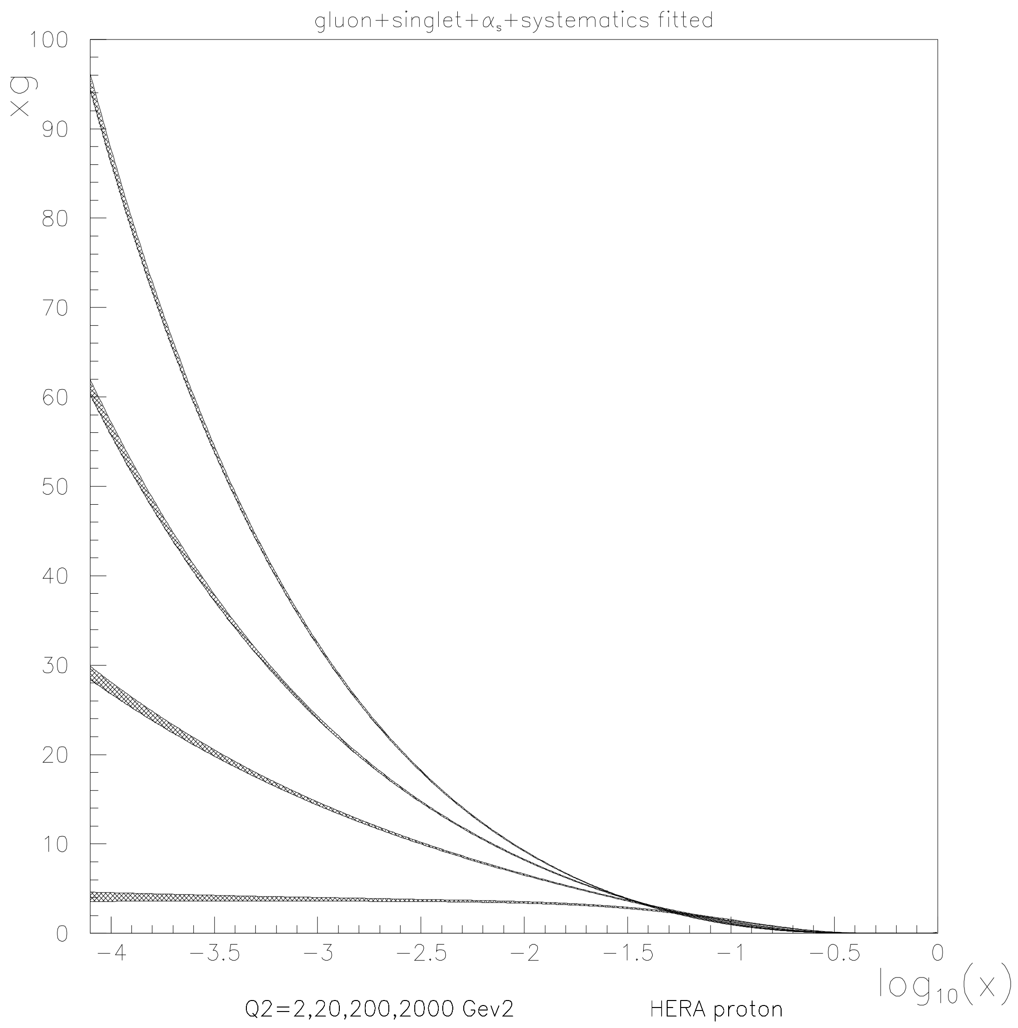

Figure 6: Determination of gluon distribution using future F_2 data from electron-proton scattering with fitted systematic error parameters. Note that for simplicity the gluon is shown also outside the allowed region of $x \leq Q^2/10^5$.

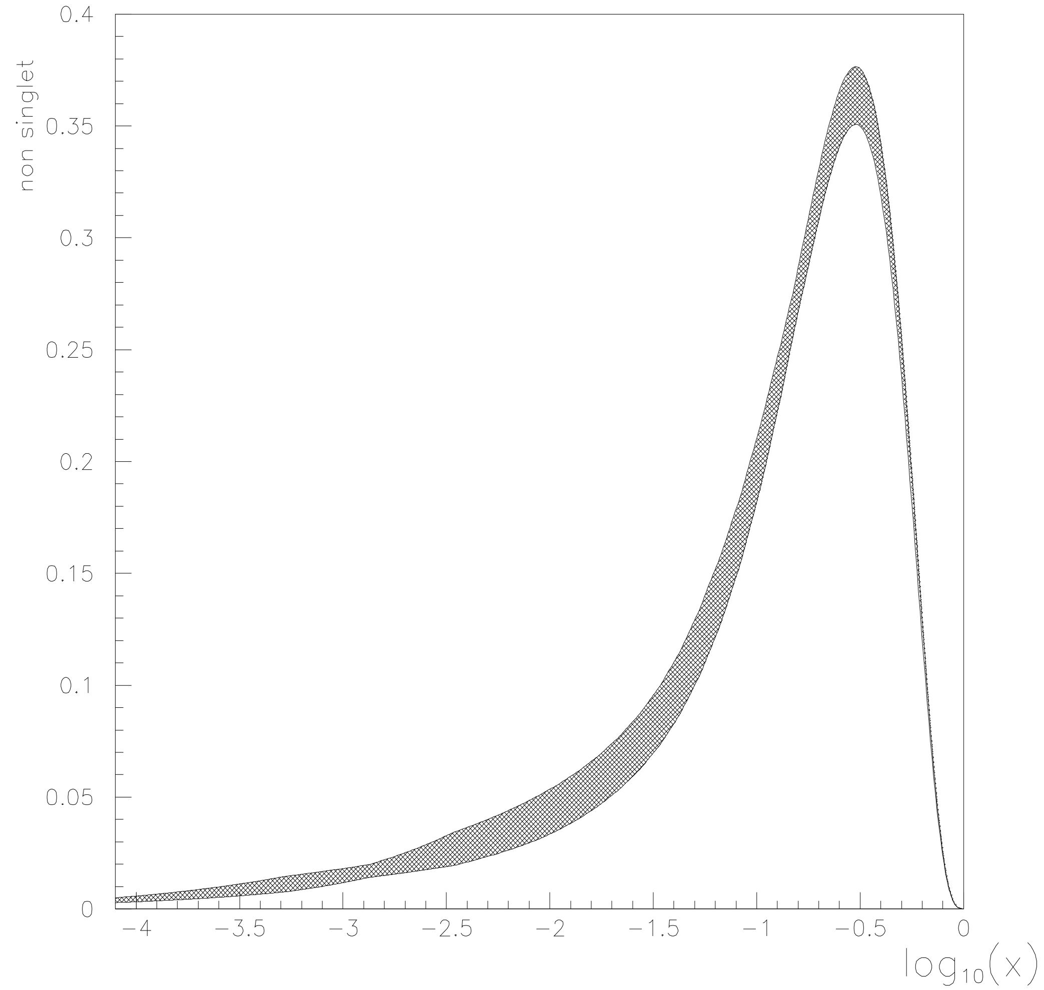

Figure 7: Determination of the non-singlet distribution $u^+ = u + \bar{u} - \Sigma/3$ from a QCD fit to the simulated proton and deuteron data (sets I,II and V,VI in table 1).