QCD corrections to $F_L(x, Q^2)$

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Abstract: We perform a numerical study of the QCD corrections to the structure function $F_L(x, Q^2)$ in the HERA energy range. The K–factors are of $O(30\%)$ and larger in parts of the kinematic range. The relative corrections to $F_L^{c\overline{c}}$ turn out to be scale dependent and partially compensate contributions to the massless terms.

The longitudinal structure function in deep inelastic scattering, $F_L(x, Q^2)$, is one of the observables from which the gluon density can be unfolded. In leading order (LO) [1] it is given by

$$F_L^{ep}(x, Q^2) = \frac{\alpha_s(Q^2)}{\pi} \left\{ \frac{4}{3} c_{L,1}^q(x) \otimes F_2^{ep}(x, Q^2) + 2 \sum_q c_{L,1}^g(x, Q^2) \otimes [xG(x, Q^2)] \right\} \qquad (1)$$

with

$$c_{L,1}^q(x) = x^2 \qquad c_{L,1}^g(x) = x^2(1-x), \qquad (2)$$

and $\otimes$ denoting the Mellin convolution. Eq. (1) applies for light quark flavours. Due to the power behaviour of the coefficient functions $c_{L,1}^{q,g}(x)$, an approximate relation for the gluon density at small x

$$xG(x, Q^2) \simeq \frac{3}{5} \times 5.85 \left\{ \frac{3\pi}{4\alpha_s(Q^2)} F_L(0.4x, Q^2) - \frac{1}{2} F_2(0.8x, Q^2) \right\}, \qquad (3)$$

has been used to derive a simple estimate for $xG(x, Q^2)$ in the past [2]. Heavy quark contributions and the next-to-leading order (NLO) QCD corrections complicate the unfolding of the gluon density using $F_L(x, Q^2)$ and have to be accounted for in terms of K-factors. In the present note, these contributions are studied numerically for the HERA energy range.

The NLO corrections for the case of light quark flavours were calculated in ref. [3] and the LO and NLO contributions for the heavy flavour terms were derived in refs. [4] and [5], respectively. While in LO the heavy flavour part of $F_L(x, M^2)$ is only due to $\gamma^* g$ fusion, in NLO also light quark terms contribute. Moreover, the choice of the factorization scale M^2 happens to affect $F_L^{Q\overline{Q}}(x, M^2)$ substantially.

Light flavour contributions

The leading order contributions to $F_L(x, Q^2)$ are shown in Figure 1 for $x \geq 10^{-4}$ and $10 \leq Q^2 \leq 500$ GeV^2. Here and in the following we refer to the CTEQ parametrizations [6] and assume $N_f = 4$. We also show the quarkonic contributions which are suppressed by one order of magnitude against the gluonic ones in the small x range. The ratio of the NLO/LO contributions is depicted in figure 2. Under the above conditions, it exhibits a fixed point at $x \sim 0.03$. Below, the correction grows for rising Q^2 from $K = 0.9$ to 1 for $x = 10^{-4}$, $Q^2 \epsilon [10, 500]$ GeV^2. Above, its behaviour is reversed. The correction factor K rises for large values of x. For $x \sim 0.3$ it reaches e.g. 1.4 for $Q^2 = 10$ GeV^2. In NLO the quarkonic contributions are suppressed similarly as in the LO case at small x and contribute to F_L by 15% if only light flavours are assumed.

Heavy flavour contributions

The heavy flavour contributions to F_L are shown in figures 3 and 4, comparing the results for the choices of the factorization scale $M^2 = 4m_c^2$ and $M^2 = 4m_c^2 + Q^2$, with $m_c = 1.5$ GeV. Here we used again parametrization [6] for the description of the parton densities but referred to three light flavours only unlike the case in the previous section. The comparison of Figures 3a and 4a shows that the NLO corrections are by far less sensitive to the choice of the factorization scale than the LO results. Correspondingly the $K_{c\bar{c}}-$factors $F_L^{c\bar{c}}(NLO)/F_L^{c\bar{c}}(LO)$ are strongly scale dependent. Note that the ratios $K_{c\bar{c}}$ and K behave different and compensate each other partially. Thus the overall correction depends on the heavy-to-light flavour composition of $F_L(x, Q^2)$.

In summary we note that the NLO corrections to F_L are large. Partial compensation between different contributions can emerge. For an unfolding of the gluon density from $F_L(x, Q^2)$ the NLO corrections are indispensable.

References

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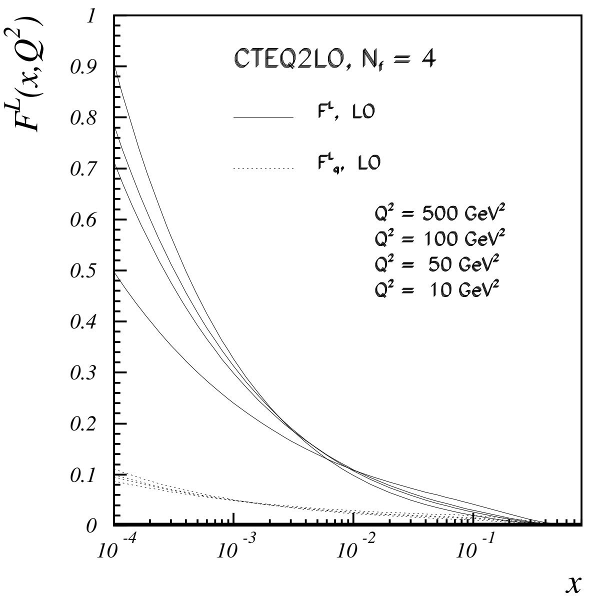

Figure 1 : Leading order contributions to $F_L(x, Q^2)$.

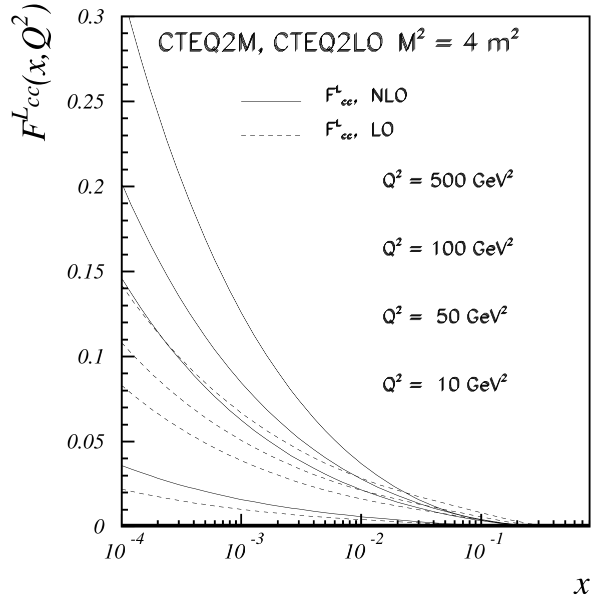

Figure 3a : LO and NLO $c\bar{c}$ contributions to $F_L(x, Q^2)$. The factorization scale is set to $M^2 = 4m_c^2$.

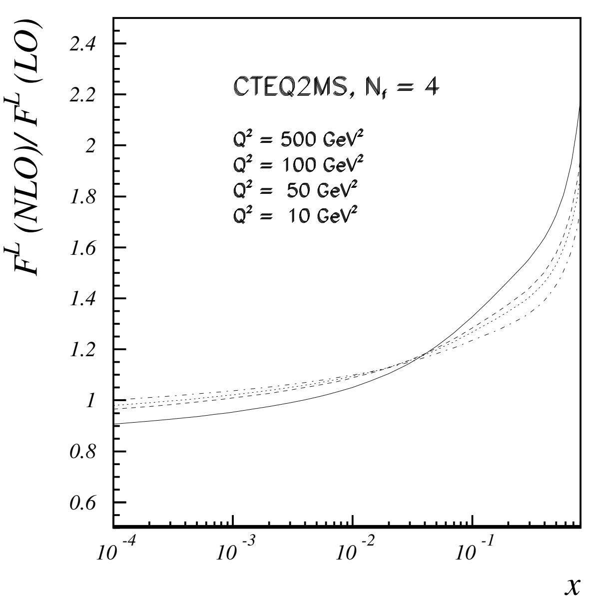

Figure 2 : The NLO correction factor for $F_L(x, Q^2)$ in the case of four light flavours.

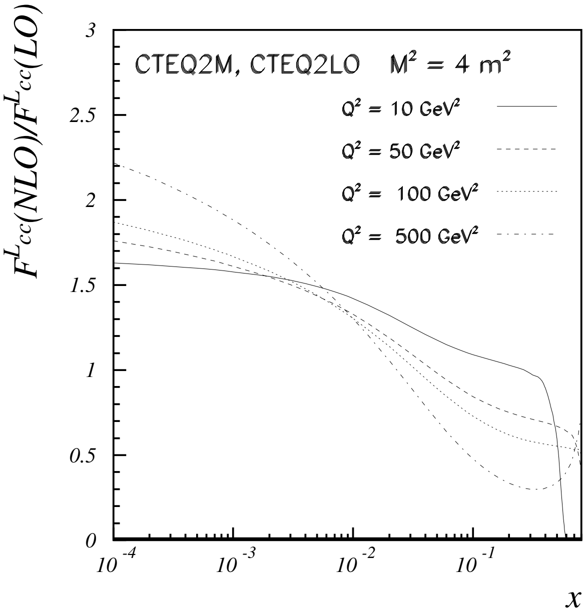

Figure 3b : Ratio of the NLO to LO $c\bar{c}$ contributions to $F_L(x, Q^2)$. The factorization scale is set to $M^2 = 4m_c^2$.

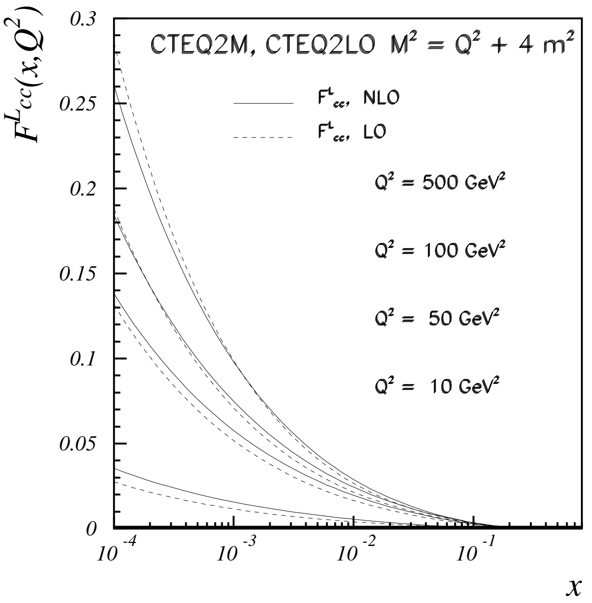

Figure 4a : Same as in Fig. 3a but for choosing the factorization scale $M^2 = 4m_c^2 + Q^2$.

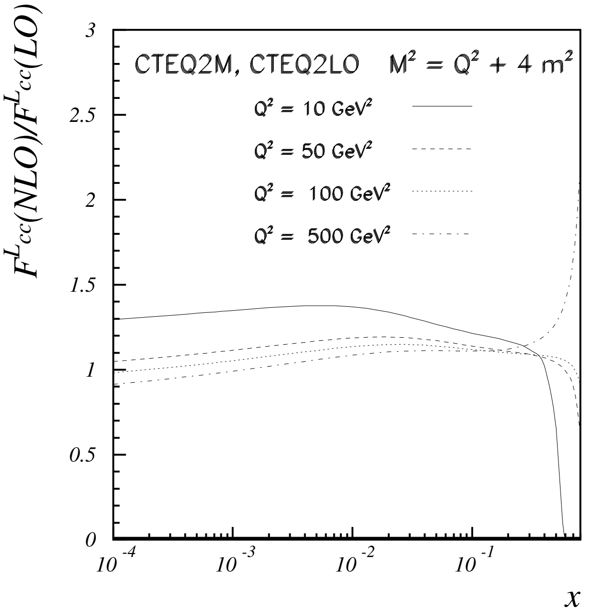

Figure 4b : Same as in Fig. 3b but for choosing the factorization scale $M^2 = 4m_c^2 + Q^2$.