Theoretical Uncertainties in the Determination of α_s from F_2^p at HERA

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Abstract: Uncertainties in the next-to-leading order evolution of deep–inelastic structure functions – arising from different prescriptions for truncating the perturbative series, and from the freedom the choose the renormalization and factorization scales – are briefly discussed, and their quantitative implications on α_s determinations from F_2^p at HERA are estimated.

1 Introduction

Deep–inelastic scattering (DIS), especially the proton structure function $F_2^p(x, Q^2)$, has been one of the classical tools for determining the strong coupling constant α_s. With the advent of HERA, the accessible kinematic range has been expanded to scales up to $Q^2 \approx 10^4$ GeV^2, and down to Bjorken-x values of $x \approx 10^{-4}$ in the deeply inelastic regime. Given a sufficient luminosity of HERA, this enhanced lever arm for measuring the scaling violations of F_2 can be used for obtaining improved α_s extractions from DIS.

For the anticipated experimental errors from such analyses, see refs. [1, 2]. In this contribution, we briefly summarize a recent study on present theoretical uncertainties pertaining the determination of α_S from F_2 scaling violations at HERA. For a detailed account the reader is referred to ref. [3].

2 QCD Evolution in Next-to-Leading Order

The complete understanding of the evolution of F_2 in perturbative QCD is currently limited to next-to-leading order (NLO). Generically, the evolution of the parton densities $f(x, M^2)$ at the fractional momentum x can be written as

$$\frac{\partial f(x, M^2)}{\partial \ln M^2} = \Big[a_s(M^2)\, P^{(0)}(x) + a_s^2(M^2)\, P^{(1)}(x) + O(a_s^3)\Big] \otimes f(x, M^2)\, , \qquad (1)$$

identifying M as the mass factorization scale and $a_s = \alpha_s/4\pi$. Here $\otimes$ stands for the Mellin convolution. The evolution equation (1) is understood to represent the non–singlet cases as well as the coupled singlet quark and gluon evolutions. $P^{(0)}$ and $P^{(1)}$ denote the corresponding leading order (LO) and NLO splitting functions, respectively.

The second important ingredient is the scale dependence of α_s, governed by

$$\frac{\partial a_s(R^2)}{\partial \ln R^2} = -\beta_0 a_s^2(R^2) - \beta_1 a_s^3(R^2) + O(a_s^4)\,, \tag{2}$$

where R stands for the renormalization scale. Only the first two terms in the series, β_0 and β_1, are kept at NLO. As already indicated in eq. (1), we set $R = M$ in this section.

Beyond LO, the parton densities are not observables. To obtain the structure functions from the (anti-) quark and gluon densities, the latter have to be convoluted with the appropriate coefficient functions, e.g.

$$F_2(x,Q^2) = \sum_{i=q,g} c_{2,i}(x,Q^2) \otimes f_i(x,Q^2)\,. \tag{3}$$

Here we have additionally identified the factorization scale by putting $M^2 = Q^2$. The coefficient functions c_i are also treated as an expansion in a_s, according to

$$c_{2,i}(x,Q^2) = c_{2,i}^{(0)}\,\delta_{i,q}\,\delta(1-x) + a_s(Q^2)\,c_{2,i}^{(1)}(x) + O(a_s^2)\,. \tag{4}$$

This completes the ingredients required for a complete NLO calculation. For an overview of the concrete calculations of $P^{(l)}(x)$ and $c^{(l)}(x)$, see ref. [4].

We now turn back to the evolution equation (1). Two approaches to its solution, which differ in their treatment of terms beyond NLO, have been widely used. The first approach is to solve eq. (1) numerically in x-space, see for example refs. [5, 6, 7]. An important alternative, employed in refs. [8, 9], is to transform to Mellin-N space via

$$A_N \equiv \int_0^1 dx\, x^{N-1} A(x)\,. \tag{5}$$

The main virtue of this transformation is that the convolution is reduced to a simple product. Hence eq. (1) turns into a system of ordinary differential equations at fixed N, which allows for further analytical developments. After rewriting the evolution in terms of $a_s \equiv a_s(M^2)$ using eq. (2), and expanding the resulting r.h.s. in a power series in α_s, one arrives in NLO at

$$\frac{\partial f_N(a_s)}{\partial a_s} = -\frac{1}{\beta_0 a_s}\left[P_N^{(0)} + a_s\left(P_N^{(1)} - \frac{\beta_1}{\beta_0}P_N^{(0)}\right) + O(a_s^2)\right] f_N(a_s)\,. \tag{6}$$

Its solution can be written in a closed form for the non–singlet cases, with $a_0 \equiv a_s(M_0^2)$, as

$$f_N(a_s) = \left[1 - \frac{a_s - a_0}{\beta_0}\left(P_N^{(1)} - \frac{\beta_1}{\beta_0}P_N^{(0)}\right) + O(a_s^2)\right]\left(\frac{a_s}{a_0}\right)^{P_N^{(0)}/\beta_0} f_N(a_0)\,. \tag{7}$$

For the notationally more cumbersome singlet evolution, see refs. [8, 9]. From these analytic solutions, one can acquire the x-space results by one contour integral in the complex N-plane.

The truncation of the perturbative series at NLO in either eq. (1) [5, 6, 7] or eq. (6) [8, 9] leads to noticeably different evolutions, see refs. [3, 10]. Fitting the same data, in the HERA kinematical range for $Q^2 \geq 4$ GeV^2, with the two approaches leads to central values for $\alpha_s(M_Z^2)$ which differ by $\Delta\alpha_s(M_Z^2) \simeq 0.003$ [3]. This shift is very similar to the differences observed in earlier non–singlet fits with various programs to the large-x BCDMS data [11].

3 Renormalization and Factorization Scale Dependence

In the previous section, the condition $M^2 = R^2 = Q^2$ was imposed on the renormalization and factorization scales. In general, however, those scales can be chosen differently. The stability of the results against variations of R and M provides a measure of the quality of the perturbative approximation under consideration. For general choices of the scales, the NLO evolution equations for the parton densities read

$$\frac{\partial f(x;M^2,R^2)}{\partial \ln M^2} = a_s(R^2)\Big[P^{(0)}(x) + a_s(R^2)P^{(1)}(x) + a_s(R^2)\beta_0 P^{(0)}(x)\ln\left(\frac{R^2}{M^2}\right)\Big] \otimes f(x;M^2,R^2)\,. \tag{8}$$

This relation differs from eq. (1), where the parton densities are expressed as functions of one scale, only by terms beyond NLO.

In ref. [3] the scale–choice dependence of F_2 has been studied for two cases. The impact of varying the renormalization scale R has been considered for $M^2 = Q^2$. In this case, the structure function is given by

$$F_2(x,Q^2) = \sum_{i=q,g} f_i(x;Q^2,R^2) \otimes \left[c_{2,i}^{(0)}\,\delta_{i,q}\,\delta(1-x) + a_s(R^2)c_{2,i}^{(1)}(x)\right]. \tag{9}$$

The dependence on the factorization scale, on the other hand, has been investigated for $R^2 = M^2$. Then the parton densities obey eq. (1), and F_2 is determined via

$$F_2(x,Q^2) = \sum_{i,j=q,g} \left[c_{2,i}^{(0)}\,\delta_{i,q}\delta_{ij}\,\delta(1-x) + a_s(M^2)\left\{c_{2,i}^{(1)}(x)\,\delta_{ij} + P_{ij}^{(0)}(x)\ln\left(\frac{Q^2}{M^2}\right)\right\}\right] \otimes f_j(x,M^2)\,. \tag{10}$$

The uncertainty of $\alpha_s(M_Z^2)$, as determined in fits to future high–precision F_2 data from HERA, due to the arbitrariness of R and M has been estimated in the following way: a sample data set has been constructed for $R^2 = M^2 = Q^2$, using typical kinematic cuts, see fig. 3 of ref. [3]. Then one-parameter fits of $\alpha_s(M_Z^2)$ to these data have been performed, with R^2 and M^2 varied up and down up to factors of four in the manner described above. The resulting shifts from the assumed central value of $\alpha_s(M_Z^2) = 0.112$ are displayed in Table 1 for characteristic additional Q^2 cuts on the data.

Q^2 Cut (GeV^2)	$M^2 = Q^2/4$	$M^2 = Q^2/2$	$M^2 = 2Q^2$	$M^2 = 4Q^2$
4	–	–	-0.0068	-0.012
20	$+0.0067$	$+0.0029$	-0.0024	-0.0044
50	$+0.0032$	$+0.0015$	-0.0015	-0.0029
Q^2 Cut (GeV^2)	$R^2 = Q^2/4$	$R^2 = Q^2/2$	$R^2 = 2Q^2$	$R^2 = 4Q^2$
4	-0.0076	-0.0037	$+0.0032$	$+0.0059$
20	-0.0067	-0.0032	$+0.0027$	$+0.0049$
50	-0.0061	-0.0028	$+0.0023$	$+0.0042$

Table 1. Theoretical shifts on $\alpha_s(M_Z^2)$ from variations of the factorization scale M (upper part) and the renormalization scale R (lower part), for F_2 data in the HERA kinematic range. The reference data was generated for $\alpha_s(M_Z^2) = 0.112$ and imposing $M^2 = R^2 = Q^2$.

4 Summary

The theoretical uncertainties have been briefly outlined, which arise from the truncation of the (twist–2) perturbative series for the scaling violations of the structure function F_2 at NLO. Their effect on the value of $\alpha_s(M_Z^2)$ as extracted from data in the HERA regime has been investigated. An uncertainty of

$$\Delta\alpha_s(M_Z^2) = \left.\begin{array}{c} +0.004 \\ -0.006 \end{array}\right|_R \left.\begin{array}{c} -0.003 \\ +0.003 \end{array}\right|_M .$$

has been estimated originating in the freedom of choice of the renormalization scale R and the factorization scale M, if only data above $Q^2 = 50$ GeV^2 are taken into account. Different prescriptions for truncating the series at $M^2 = R^2 = Q^2$ lead to shifts of about $\Delta\alpha_s(M_Z^2) = 0.002$ under these conditions. Lower Q^2 cuts, as possibly experimentally desirable, lead to larger theoretical uncertainties. All these uncertainties can only be significantly reduced, if the analyses reach the next-to-next-leading order level. To achieve this goal, however, the extremely demanding calculation of the 3–loop splitting functions has to be carried out.

Acknowledgements : This work was supported on part by the EC Network 'Human Capital and Mobility' under contract No. CHRX–CT923–0004 and by the German Federal Ministry for Research and Technology (BMBF) under No. 05 7WZ91P (0).

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