The Effect of Small-x Resummations on the Evolution of Polarized Structure Functions

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Abstract: The impact of the resummation of leading small-x terms in the anomalous dimensions is briefly summarized for the evolution of non–singlet and singlet polarized structure functions.

1 Introduction

The evolution kernels of both non–singlet and singlet polarized parton densities contain large logarithmic contributions for small fractional momenta x. The leading terms in this limit are of the form $\alpha_s^k \ln^{2k-2} x$ for both cases [1, 2]. The resummation of these contributions to all orders in the strong coupling constant α_s can be completely derived by means of perturbative QCD. The appropriate framework for investigating the resummation effects is provided by the renormalization group equations.

The impact of the resulting all–order anomalous dimensions on the behaviour of the deep–inelastic scattering (DIS) structure functions at small x thus depends as well on the non–perturbative input parton densities at an initial scale Q_0^2. Hence the resummation effects can only be studied via the evolution over some range in Q^2. This evolution moreover probes the anomalous dimensions also at medium and large values of x by the Mellin convolution with the parton densities. Therefore the small-x dominance of the leading terms over less singular contributions in the anomalous dimensions does not necessarily imply the same situation for observable quantities, such as the structure functions.

In the following we present a brief survey of quantitative results which shed light on the importance of these aspects. For full accounts, including the discussion of theoretical aspects, the reader is referred to refs. [3, 4] and [5] for the non–singlet and singlet evolutions, respectively. A recent review covering also the unpolarized cases can be found in ref. [6].

2 Quantitative Results

In leading (LO) and next–to–leading order (NLO) perturbative QCD, the complete anomalous dimensions are known. Hence the effect of the all–order resummation of the most singular parts of the splitting functions as $x \to 0$ concerns only orders higher than α_s^2. Due to the Mellin convolution terms less singular as $x \to 0$ may contribute substantially at these higher orders also. Since such contributions and further corrections are not yet known to all orders, it is reasonable to estimate their possible impact by corresponding modifications of the resummed anomalous dimensions $\Gamma(N, \alpha_s)$, where N denotes the Mellin variable. Plausible examples inspired by the behaviour of the full NLO results have been studied in refs. [3, 4, 5], including

$$\begin{array}{llll} \mathrm{A}: & \Gamma(N,\alpha_s) \to \Gamma(N,\alpha_s) - \Gamma(1,\alpha_s) & \mathrm{B}: & \Gamma(N,\alpha_s) \to \Gamma(N,\alpha_s)(1-N) \\ \mathrm{D}: & \Gamma(N,\alpha_s) \to \Gamma(N,\alpha_s)(1-2N+N^3)\,. & & \end{array} \tag{1}$$

Clearly the presently known resummed terms are only sufficient for understanding the small-x evolution, if the difference of the results obtained by these prescriptions are small.

2.1 Polarized non–singlet structure functions

This case has been investigated in refs. [3, 4] for the structure function combination $g_1^{ep} - g_1^{en}$ for two parametrizations of the non–perturbative initial distributions, see Figure 1. Results on the interference structure function $g_{5,\gamma Z}^{ep}(x, Q^2)$ can be found in ref. [6]. For the relatively flat CW input [7], the resummation effect on $g_1^{ep} - g_1^{en}$ reaches about 15% at $x = 10^{-5}$. However, in the restricted kinematical range accessible in possible future polarized electron–polarized proton collider experiments at HERA [9], it amounts to only 1% or less. For the steeper GRSV initial distributions [8], the effect is of order 1% or smaller in the whole x range. Hence the results do not at all come up to previous expectations of huge corrections up to factors of 10 or larger as anticipated in ref. [10].

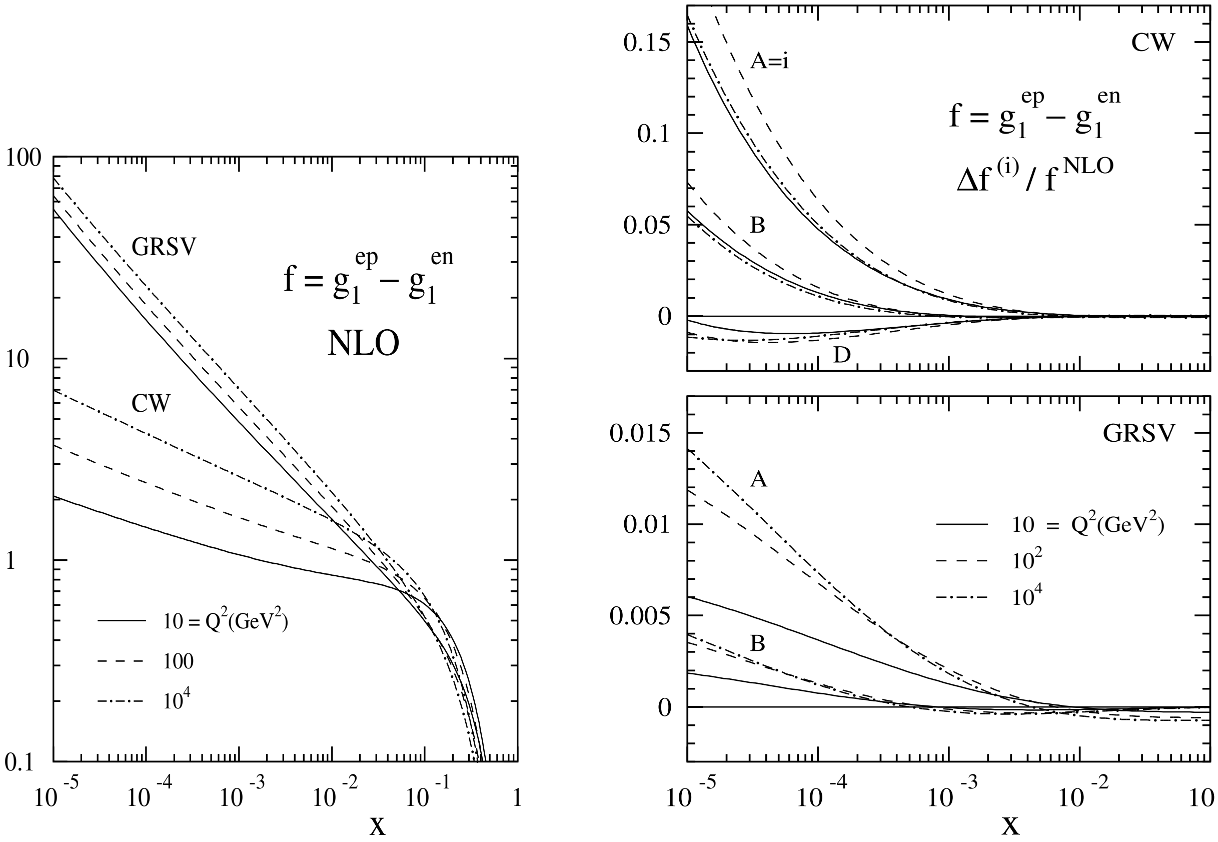

Figure 1: The NLO small-x evolution of the polarized non–singlet structure function combination $g_1^{ep} - g_1^{en}$, and the relative corrections due to the resummed kernels, for the initial distributions of refs. [7] and [8]. The dependence on possible less singular terms is illustrated by the prescriptions 'A', 'B', and 'D' of eq. (1). The figure has been adapted from ref. [6].

2.2 Polarized singlet structure functions

The numerical consequences of the small-x resummation for the evolution of $g_1^{ep,en}(x,Q^2)$ and the parton densities have been given for different input distributions in ref. [5]. Figure 2 shows an example. The effects are much larger here than for the non–singlet structure functions. Also illustrated in these figures [by the results for the prescription '(B)' in eq. (1)] is the possible impact of the yet uncalculated terms in the higher–order anomalous dimensions which are down by one power of N with respect to the resummed leading pieces as $N \to 0$. As in the non–singlet case considered before, the effect of these additional terms can be very large, even the sign of the deviation from the NLO evolution cannot be taken for granted.

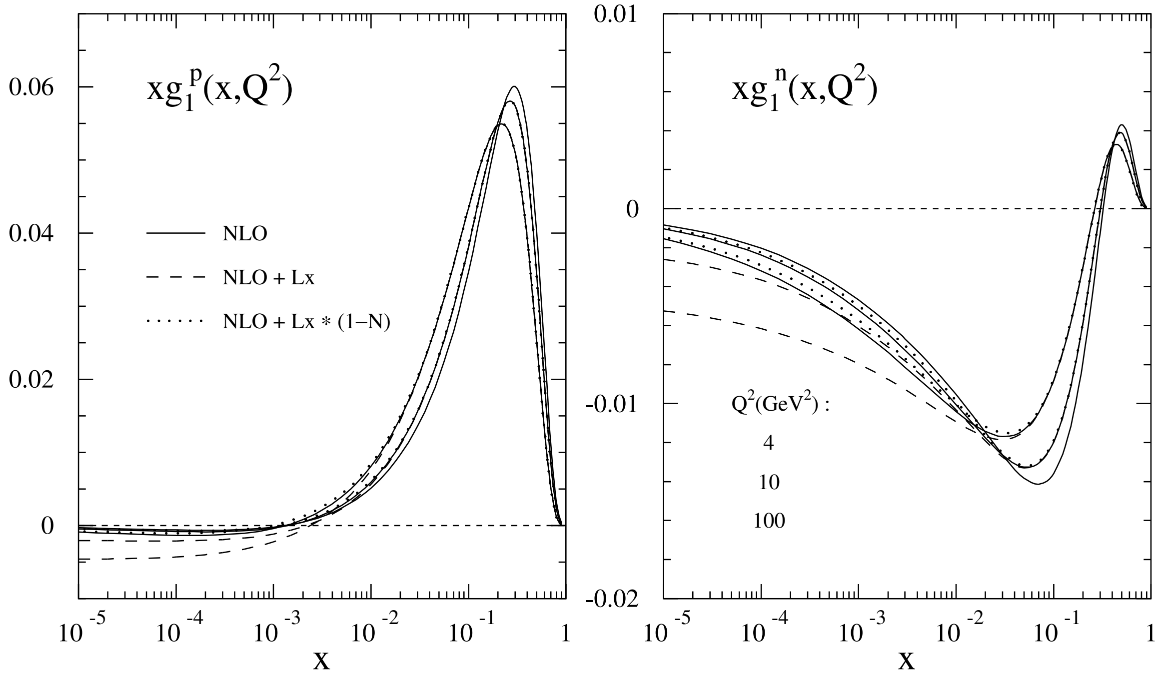

Figure 2: The x and Q^2 behaviour of the polarized proton and neutron structure functions $g_1^{p,n}(x,Q^2)$ as obtained from the GRSV standard distribution [8] at $Q_0^2 = 4\,\mathrm{GeV}^2$. The results are shown for the NLO kernels (full), the leading small-x resummed kernels (dashed), and the modification 'B' of eq. (1) of the latter by possible less singular terms (dotted). The figure has been taken from ref. [5].

3 Conclusions

The effects of the resummation of the leading small-x terms in the polarized non–singlet and singlet anomalous dimensions have been summarized. For non–singlet structure functions the corrections due to those $\alpha_s(\alpha_s \ln^2 x)^l$ contributions are about 1% or smaller, in the kinematical ranges probed so far as well as the regime accessible at a polarized version of HERA [3, 4]. In the singlet case very large corrections are obtained. As in the non–singlet cases, however, possible less singular terms in higher order anomalous dimensions, are hardly suppressed against the presently resummed leading terms in the evolution: even a full compensation of the resummation effects cannot be excluded [5]. To draw firm conclusions on the small-x evolution of also the singlet structure functions, the next less singular terms as well as the complete three–loop anomalous dimensions are needed.

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