QED RADIATIVE PROCESSES IN ELECTRON-HEAVY ION COLLISIONS AT HERA

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Abstract: Elastic radiative tail corrections and beamstrahlung process are discussed in the context of heavy ions mode at HERA collider. The contribution of the radiative elastic tail are calculated without the use of the Born approximation and the multiphotons interactions with heavy nuclei/ions are taken into account. The numerical results for the cross sections are presented for the scattering on lead and tin for the lowest x-bins, where these contributions are important. A comment on beamstrahlung process is given. The effect is strongly machine design dependent and is found to be negligible for planned heavy ions mode at HERA.

1 Introduction.

The heavy ions mode at HERA collider which is actually considered as a one of the future possibilities will allow to study deep inelastic electron scattering on heavy targets with very high energy. The determination of the one photon cross section from the data demands excluding of a contribution from the other electroweek processes. These other contributions must be calculated theoretically and subtracted from the measured cross section. The set of theoretical calculations needed to experimental analysis is called *radiative corrections scheme*. There are many radiative corrections schemes on the market ready to use in the DIS on heavy ions at HERA (see e.g. [1, 2] and references therein).

In this paper I discuss these radiative processes which are not included (or included in Born approximation which is not correct in the case of heavy targets) in the standard radiative corrections schemes. There are two types of such procesess:

- The *radiative elastic tail*, the bremsstrahlung process from electron current in the elastic channel.
- The *beamstrahlung* process , originally considered for e^+e^- linear collider with TeV range of energy, where electron crossing a very high density positron bunch thus suffer strong acceleration and radiate a sizeable fraction of incident energy. In the case of heavy ions the electron is affected by heavy ions bunch. The heavy ions bunches (due to high electric charge) as well as dense positron bunches are a source of strong field in which electron can radiate.

The paper is organized as follows. In Section 2 the cross sections of the elastic tail for planned HERA heavy nuclei/ions mode are calculated for lead, tin and calcium in the Born one photon approximation as well as using the method in which the multiphotons interactions with heavy nuclei are taken into account. Section 3 contains a comment about beamstrahlung process. Finally, the summary is given in Section 4.

2 The contribution of the elastic radiative tail.

It is well known that the one of the more important contribution to the cross section measured in the DIS experiments in the small x bins proceeds from elastic tail. In the standard approach the only Born cross section is taken into account what means that the only one photons processes are considered (one photon emitted from lepton and one photon transmitted to target particle). To find the corresponding formulae the reader is reffered to the original papers [1, 2]. For the scattering on the heavy target the Born approximation is not correct because the effective coupling between virtual photon and heavy target particle is very strong due to the high electric charge Ze of the heavy nuclei/ions. Therefore the one photon exchange approximation is not valid and multiphotons exchange processes should be taken into account. The calculations presented in this paper are based on the method described in detail in [3] where the elastic radiative tail was calculated for NMC fixed target experiment for the muon scattering on heavy targets [4].

The basic concept of the method is to calculate the electron wave function in the presence of the "external" field generated by the heavy ion (Furry wave function). It can be done in the high energy approximation (Bethe et al., [5]). The cross section of the elastic radiative tail calculated using Furry electron wave function is given by the famous Bethe-Heitler formula [6] modified by extra factor R and multiplied by the formfactor of heavy ion [5, 3]. The calculations were done in the rest frame of the heavy ion's bunch using the potential approximation (the recoil effect of nucleus was neglected). Both used approximations (high energy and potential) are exellent approximations in the case of heavy nuclei at HERA . The one photon Born cross section in potential approximation is given exatly by Bethe-Heitler formula (R = 1) with heavy ion's formfactor.
The formulae for the elastic radiative tail cross section for the heavy targets are given in [3]. The

numerical calculations were done for scattering electrons with energy 27.6 GeV on the heavy nuclei/ions beam with energy 410 GeV per nucleon. The corresponding "fixed target" energy (energy in the heavy ion rest frame) is about 22 TeV. The cross sections for the elastic radiative tail for different x and y bins for lead and tin are presented in Fig. 1 and 2, respectively. For comparison the Born cross sections are also presented. The numbers in the brackets in Fig. 1 and 2 show how important the effect of multiphotons interactions is. The calculations were also done for the lighter nuclei (calcium) and the cross sections are very close to the cross sections calculated with Born approximation. The effect of multiphotons processes is really important only for the heaviest nuclei. The cross sections were calculated for the smallest x bins (0.0001 to 0.01) planned to reach at HERA, where the elastic tail is the important correction.

To calculate R factor which modified Bethe-Heitler formula in the case of heavy targets the pure Coulomb potential was assumed [5]. For the case of nuclei the pure Coulomb potential is modified by the nuclear charge density (which is not point-like) and therefore the square of formfactor $F(q)^2$, the Fourrier transform of charge density, appeared in front of the formula

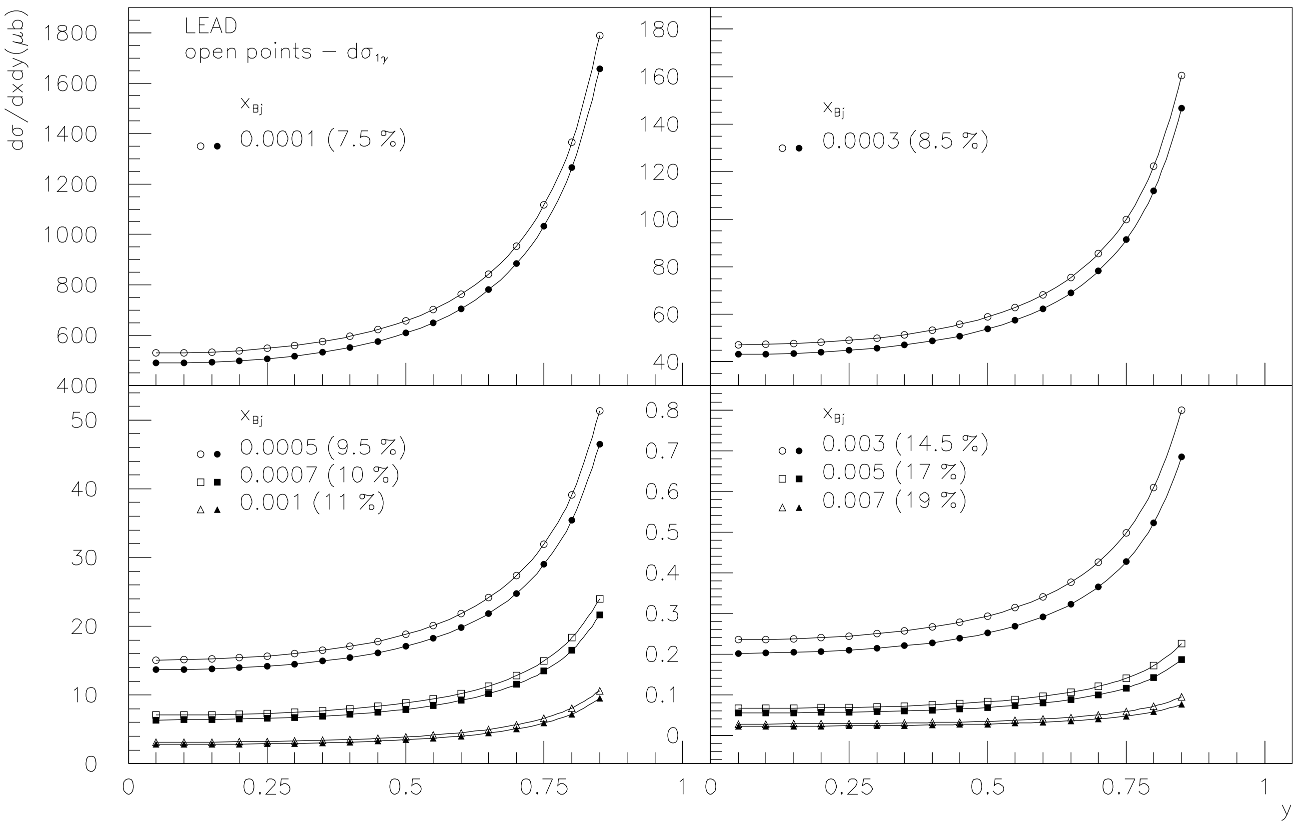

Figure 1: *The differential cross sections for the elastic radiative tail for lead nuclei/ions in Born approximation (open points) and with the multiphotons interactions taken into account according with the method proposed in [3]. Numbers in brackets show how large the differences between Born and multiphotons cross sections are.*

(12) in [3]. To take into account the nuclear charge density the Furry wave function should be calculated in the presence of *modified* Coulomb field. Unfortunately for real nucleus density it is not possible to solve the Dirac equation for the Furry wave function. The presence of F^2 in formula (12) in [3] is justified only for Born one photon cross section. When the multiphotons processes are taken into account (R factor) it is not clear how the formfactors should be put into the cross section formula. [1] However the formfactor is not taken into account correctly there are some arguments that the obtained results for the cross sections are correct [3]. To check how important for the cross section's calculations the shape of formfactor is, the different shapes of formfactor was probed in the cross section's multiphotons part (the difference between Born and the multiphotons cross section; such difference of cross sections contains also interference

[1]e.g. if it is formally considered double photons exchange Feynmann diagrams instead of F^2 the combination of three F's (interference terms) and four F's should be integrated. In the presented method all diagrams (processes) are summed up and such procedure is not possible.

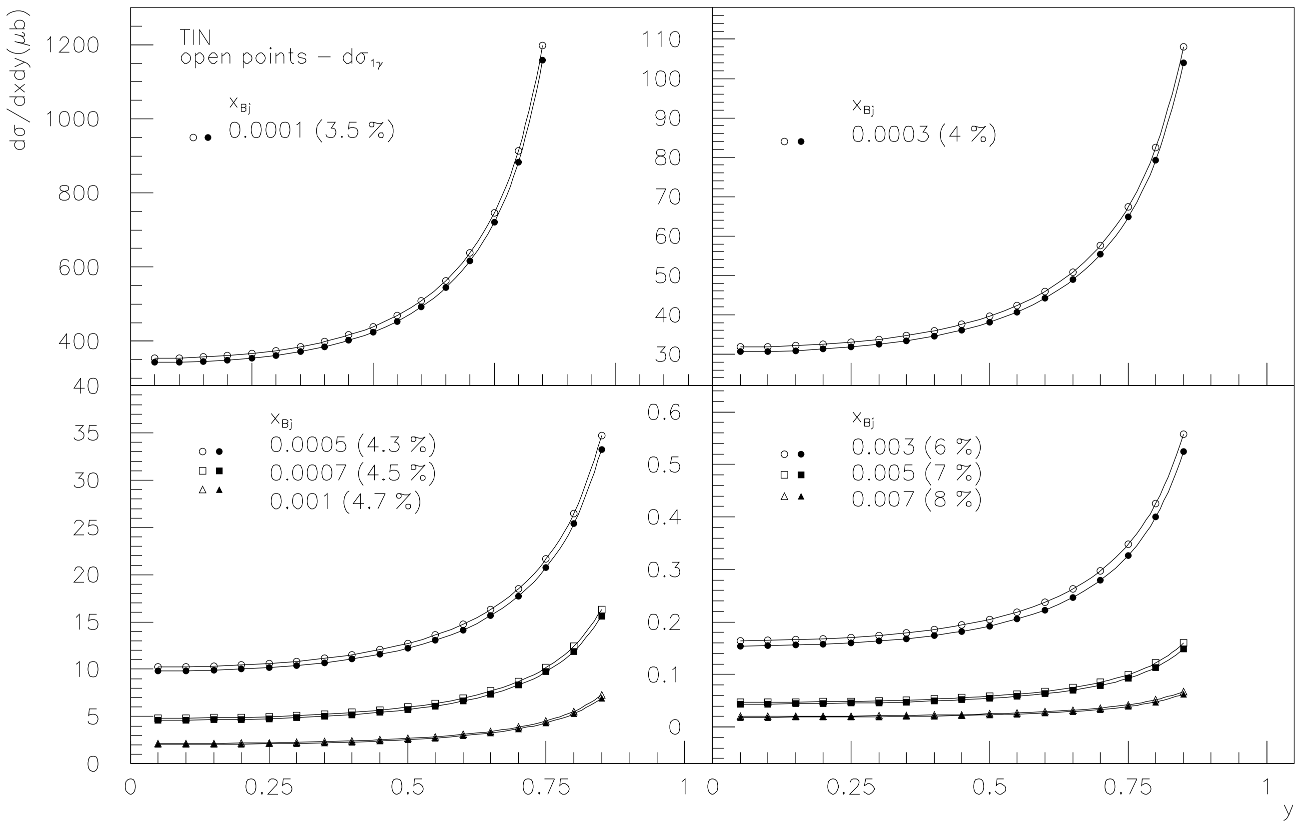

Figure 2: *The differential cross sections for the elastic radiative tail for tin nuclei/ions .*

terms) while the Born part become unchanged with true, physical formfactor, F.

Comparison of the results allowed to estimate how crucial the shape of formfactor is in the cross section calculations. The numerical results were obtained for lead and are presented in Fig. 3. The formfactor's formula used in calculations was taken from [7].

To varying shape of the formfactor the γ parameter was changed (see [7]). For two cases the shape was assumed to be only exponential factor; the sum of the oscillations were neglected (modified in Fig. 3).
From Fig. 3 it is clear that the differences (numbers in brackets) are smaller than 1% except two unphysical situations where the formfactor is nearly constant (point like charge density) and very "narrow" (flat charge density in the nucleus). In these cases the differences are a little larger for higher x bins where the contribution from elastic tail is smaller as it is seen from Fig. 1 and 2. The obtained results justify the multiphotons formula for the tail cross section with the formfactor treatment as in the Born approximation.

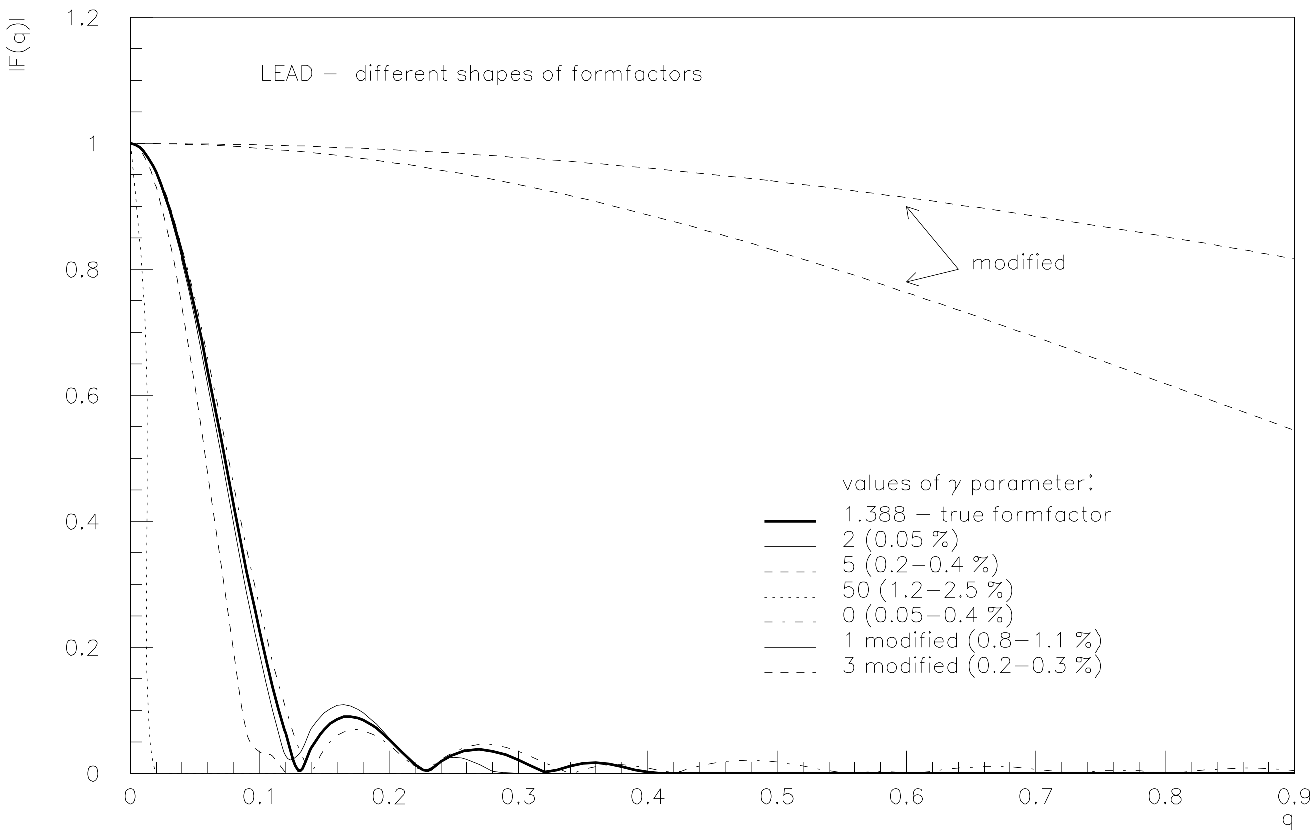

Figure 3: *The different shapes (determined by γ) of the lead formfactor used in the estimation of the uncertainties of the proposed method of calculations. The two curves with label modified are only the exponential term (see [7]). The numbers in brackets show the differences between the cross sections with different formfactor's shape taken into account in the multiphotons interactions.*

3 Comment on beamstrahlung process in the heavy ions mode at HERA.

The beamstrahlung process is a radiation of photons due to interactions between electron crossing target particles bunch. The whole bunch is a source of strong electromagnetic field which affects the electron. The effect was estimated to be quite large in the case of very dense electrons and positrons bunches which were considered in the context of the future electron-positron linear collider in TeV range of energy (see [8, 9] and references therein).

The question is if the beamstrahlung effect can be important in the case of the electron scattering on heavy ions bunch. The bunches of ions at HERA will not so dense as in planned e^+e^- linear colliders but - due to the high electric charge the heavy ions bunch maybe significant source of classical electromagnetic field.

The effect depends strongly on the machine designe: the shapes of the bunches, density and energy. The machine's parameters determin also if the classical or quantum beamstrahlung should be considered. In both cases the set of neaded formulae is known. In the quantum case the calculations were done in one photon approximation (one real photon emission) as well as the real multiphotons emissions were taken into account [9].
To distinguish which case (classical or quantum) should be considered in the case of heavy ions at HERA the parameter Υ was defined:

$$\Upsilon = \frac{\omega_c}{3p} = \frac{2Er_e\lambda_e}{\sqrt{3}m\sigma_z(\sigma_x + \sigma_y)}N. \tag{1}$$

where p, E, r_e, λ_e and m are momentum, energy, classical radius, Compton wave length and mass of electron; σ_z, σ_x and σ_y are the parameters of the heavy ion's bunch (elliptical) and N is a number of ions inside the bunch. The ω_c is a classical so-called *critical* frequency up to which the spectrum of radiated photons is important. The meaning od the Υ is the classical average ratio of the radiated energy and electron incident energy. The Υ parameter should be much smaller than one in the classical case. If the Υ is greater than one the quantum calculations are needed. It is exactly the case in planned future e^+e^- machines.
The assumed parameters of HERA heavy ions beam for calculating Υ were the following:

energy of ions	- 410 GeV/nucleon ,
bunch spacing	- 160 bunches/ 96 ns ,
typical current	- 50 mA ,
$\sigma_z, \sigma_x, \sigma_y$	- 0.11 m, 70 μm , 300 μm

These parameters give N $\simeq 10^{10}$ ions and $\Upsilon \simeq 10^{-7}$. It is now clear that the HERA is operating in classical regime. The energy loss due to beamstrahlung process can be calculated using the following classical formula

$$\delta = \frac{4N^2Er_e^3}{3\sqrt{3}m\sigma_z(\sigma_x + \sigma_y)^2} \simeq 10^{-9}. \tag{2}$$

This result shows that the beamstrahlung process at HERA with heavy ions beams is not important in contrast to the collisions of very dense electrons-positrons beams considered in the new generation of the linear electrons-positrons colliders.

4 Summary

In this paper the elastic radiative tail correction and the beamstrahlung process were calculated for the future DIS process with electrons and heavy ions at HERA collider. The multiphotons interactions, which are important in the heaviest nuclei, were taken into account and compared with Born standard approach. The method originally proposed by Bethe et al. [5] was modified and applied to improve the standard radiative corrections schemes in the case of heavy targets [3]. The effect is quite large in the HERA kinematics (up to 19%). The uncertainties connected with the formfactor treatment were discussed and estimated to be less than 1% .
The beamstrahlung process was also discussed. The HERA machine parameters and kinematics allow to consider this process classically. The numerical results show that the electron interactions with the whole heavy ions bunch (beamstrahlung) is negligible.

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