Interplay of Soft and Hard Interactions in Nuclear Shadowing at High Q^2 and Low x

Boris Kopeliovich[ab] and Bogdan Povh[a]

[a] Max-Planck Institut für Kernphysik, Postfach 103980, 69029 Heidelberg, Germany
[b] Joint Institute for Nuclear Research, Dubna, 141980 Moscow Region, Russia

Abstract: Nuclear shadowing corrections are dominated by soft interaction and grow as function of $1/x$ more slowly than the single scattering term, which has an essential contribution from hard interaction. Therefore we predict vanishing nuclear shadowing at very low x provided that Q^2 is high and fixed. At the same time, at medium and low Q^2, nuclear shadowing grows with $1/x$ as is well known for soft hadronic interactions.

Experimental observation [1, 2] of nuclear shadowing in deep-inelastic scattering at low x was probably the first signal that this process is substantially contaminated by soft physics even at high Q^2. Since nuclear shadowing is closely related to diffraction [3], it is not surprising that recent measurements at HERA found diffraction to be a large fraction of the total cross section.

The structure function $F_2(x,Q^2)$ is proportional to the total cross section of interaction of a virtual photon with the target. This invites one to consider deep-inelastic lepton scattering in the rest frame of the target, where the virtual photon demonstrates its hadronic properties. Namely, the hadronic fluctuations of the photon interact strongly with the target [4]. Such a process looks quite different from the partonic interpretation of deep-inelastic scattering. The observables are Lorentz-invariant, but the space-time interpretation depends on the reference frame.

The observed virtual photoabsorption cross section on a nucleon is an average of total interaction cross sections σ_{tot}^{hN} of hadronic fluctuations weighted by probabilities $W_h^{\gamma^*}$,

$$\sigma_{tot}^{\gamma^* N}(x,Q^2) = \sum_h W_h^{\gamma^*}(x,Q^2)\ \sigma_{tot}^{hN} \equiv \langle\sigma_{tot}^{hN}\rangle\ . \tag{1}$$

In the case of a nuclear target the same procedure leads to [5],

$$\frac{\sigma_{tot}^{\gamma^* A}(x,Q^2)}{\sigma_{tot}^{\gamma^* N}(x,Q^2)} = 1 - \frac{1}{4}\frac{\langle(\sigma_{tot}^{hN})^2\rangle}{\langle\sigma_{tot}^{hN}\rangle}\ \langle T\rangle\ F_A^2(q_L) + ... \tag{2}$$

Here $T(b) = \int_{-\infty}^{\infty} dz\ \rho_A(b,z)$ is the nuclear thickness at impact parameter b and $\langle T\rangle = (1/A)\ \int d^2b\ T^2(b)$ is its mean value. $\rho_A(b,z)$ is the nuclear density dependent on b and longitudinal coordinate z. The longitudinal nuclear formfactor

$$F_A^2(q_L) = \frac{1}{A\langle T\rangle}\ \int d^2b\ \left|\int\limits_{-\infty}^{\infty} dz\ \rho_A(b,z)\ \exp(iq_L z)\right|^2 \tag{3}$$

takes into account the effects of the finite lifetime $t_c \approx 1/q_L$ (the coherence time) of hadronic fluctuations of the photon, where, $q_L = (m_h^2+Q^2)/2m_N\nu$ is the longitudinal momentum transfer in $\gamma^* N \to hN$. At large $q_L > 1/R_A$, the nuclear formfactor (3) vanishes and suppresses the shadowing term (2). This is easily interpreted: for large q_L the fluctuation lifetime and its path in nuclear medium are shorter, and shadowing is reduced. For further estimations we assume that the mean mass squared of a photon fluctuation is Q^2, leading to $q_L = 2m_N x$.

Thus, all the factors in the first-order shadowing term (2) are known, except $\langle(\sigma_{tot}^{hN})^2\rangle$. First, let us analyse the Q^2-behaviour of this factor. It is known that $\langle\sigma_{tot}^{hN}\rangle \propto 1/Q^2$ according to Bjorken scaling. In QCD this is usually interpreted as a consequence of color screening: the higher the value of Q^2, the smaller the mean transverse size squared $\langle\rho^2\rangle \sim 1/Q^2$ of its hadronic fluctuation. Due to color screening the cross section of interaction of such a fluctuation with external gluonic fields vanishes as $\sim 1/Q^2$. However, the situation is more complicated, as a finite admixture of soft fluctuations having large size is unavoidable [6, 7, 8]. We classify in a simplified way the hard and soft mechanisms of deep-inelastic scattering in Table 1.

Table 1. Contributions of soft and hard fluctuations of a virtual photon to the DIS cross section and to nuclear shadowing

Fluctuation	$W_h^{\gamma*}$	σ_{tot}^{hN}	$W_h^{\gamma*}\sigma_{tot}^{hN}$	$W_h^{\gamma*}(\sigma_{tot}^{hN})^2$
Hard	~ 1	$\sim 1/Q^2$	$\sim 1/Q^2$	$\sim 1/Q^4$
Soft	$\sim \mu^2/Q^2$	$\sim 1/\mu^2$	$\sim 1/Q^2$	$\sim 1/\mu^2 Q^2$

As previously stated, the mean fluctuation of a highly virtual photon is hard and has a small transverse size $\sim 1/Q^2$. This is why we assign to it a weight close to 1 and a small $\sim 1/Q^2$ cross section in Table 1. On the contrary, a soft fluctuation having a large size $\sim 1/\mu^2$, where μ is a soft parameter of the order of Λ_{QCD}, is expected to be quite rare in the photon, suppressed by factor $\sim \mu^2/Q^2$.[1] On the other hand, such a soft fluctuation has a large $\sim 1/\mu^2$ cross section. Therefore, the soft contribution to $\langle\sigma_{tot}^{hN}\rangle$ has the same leading twist behaviour $\sim 1/Q^2$ as the hard one. Thus, according to Table 1 one cannot say that Bjorken scaling results from the smallness of the interaction cross section of hard photon fluctuations, that also arises from the rareness of the soft components of a virtual photon.

The last column of Table 1 summarizes the Q^2-dependence of hard and soft contributions to $(\langle\sigma_{tot}^{hN})^2\rangle$, which sets the size of nuclear shadowing and diffraction effects. In this case the hard component turns out to be a higher twist effect, and the leading contribution comes from soft interaction. This is why the applicability of pure perturbative calculations to nuclear shadowing or diffraction is questionable.

Since $(\langle\sigma_{tot}^{hN})^2\rangle$ is dominated by soft interactions, we can parameterize it as [5],

$$\frac{(\langle\sigma_{tot}^{hN})^2\rangle}{\langle\sigma_{tot}^{hN}\rangle} = \frac{N}{F_2^p(x,Q^2)}\left(\frac{1}{x}\right)^{2\Delta_P(\mu^2)} \tag{4}$$

[1]This is shown to be true for a transverse photon by perturbative calculations [8], but in a longitudinal photon soft fluctuations have an extra suppression $\sim 1/Q^4$ [8]. Therefore shadowing is a higher twist effect.

We are interested in the behaviour of this factor at very low x and assume for the numerator dominance of the soft Pomeron with intercept $\alpha_P(0) = 1 + \Delta(\mu^2)$, where $\Delta(\mu^2) \approx 0.1$ is known from Regge phenomenology of soft hadronic interactions. This explains particularly why a soft Pomeron intercept was found in diffraction at high Q^2 [9], while the intercept describing x-dependence of $F_2^p(x, Q^2)$ at high Q^2 is much larger, $\Delta_P(Q^2) \approx 0.3 \div 0.4$ [5].

The proton structure function was fixed in [5] by fitting available data. The only unknown parameter N is universal for all nuclei and is fixed by the fit at $N = 3\ GeV^{-2}$ [5].

Now we are in a position to predict nuclear shadowing down to low x. The fact that $2\Delta_P(\mu^2) < \Delta_P(Q^2)$ at high Q^2 leads to the unusual prediction of vanishing nuclear shadowing at very low x. That is, the first shadowing correction in (2) decreases with $1/x$ provided that the nuclear formfactor saturates, $F_A^2(x) \to 1$. This is demonstrated in Figs. 1-2, where we have plotted our predictions for carbon and tin versus x at fixed values of Q^2. Note that formula (2) does not include the small (a few percent) effect of nuclear antishadowing. We have renormalized all curves by factor 1.03 in order to incorporate this effect, which we assume to be A-independent for simplicity.

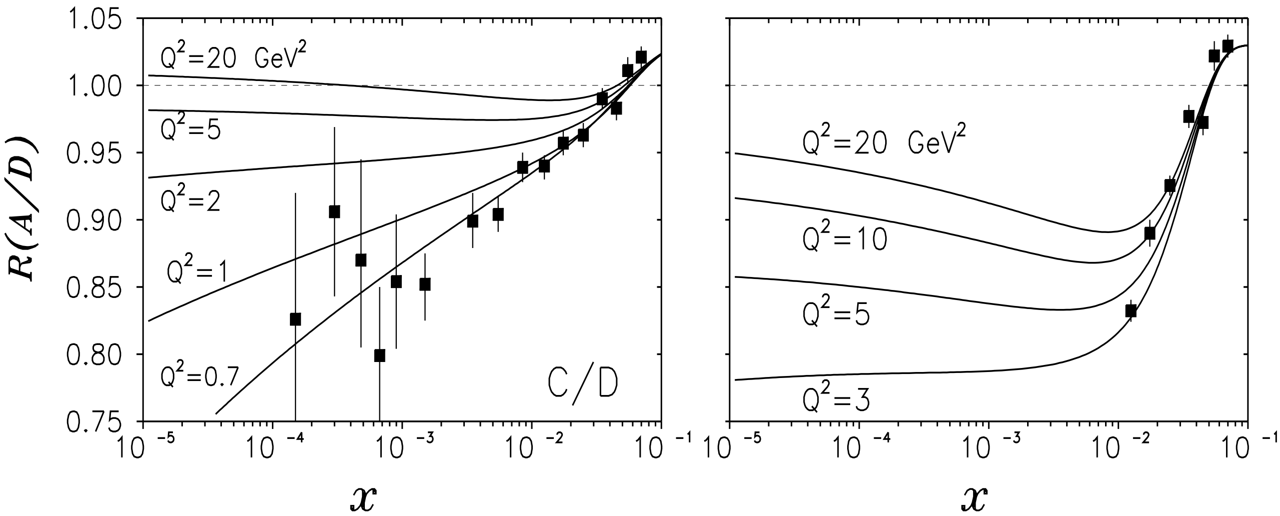


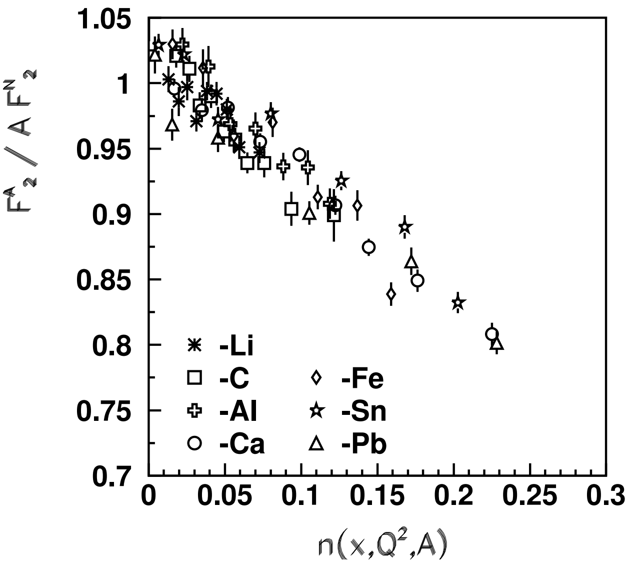
Figure 1: *Normalized ratio $F_2^A(x, Q^2)/F_2^D(x, Q^2)$ calculated for carbon using (2). The data points are from [1].*

Figure 2: *The same as in Fig. 1 but for tin. The data points are from [10].*

Note that comparison with data [1] in Fig. 1 is marginal, since Q^2 substantially varies from point to point. To make the comparison with data more sensible it was suggested in [5] to plot the data against a new variable,

$$n(x, Q^2, A) = \frac{1}{4} \frac{N}{F_2^p(x, Q^2)} \langle T(b) \rangle F_A^2(q_L) \left(\frac{1}{x}\right)^{2\Delta_P(\mu^2)} . \tag{5}$$

One may expect according to (2) - (4) that nuclear shadowing is A-, x- and Q^2-independent at fixed $n(x, Q^2, A)$. Data from the NMC experiment plotted against $n(x, Q^2, A)$ in Fig. 3 nicely confirm such a scaling. Note that the data points for different nuclei may differ within a few percent due to the antishadowing effect, if it is A-dependent.



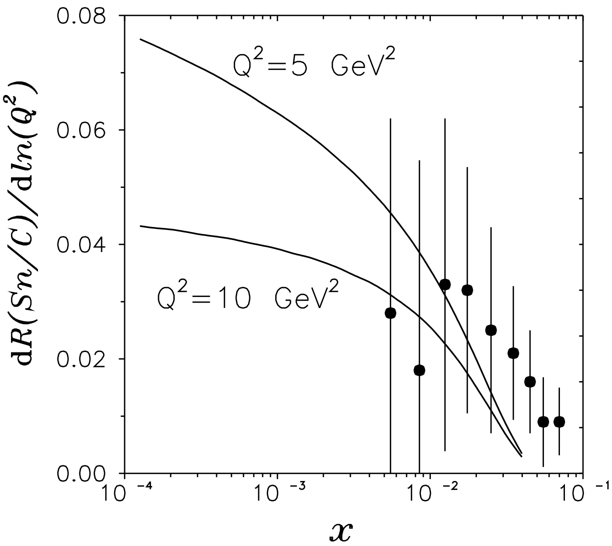


Figure 3: *Nuclear shadowing versus scaling variable $n(x, Q^2, A)$ (see text). The data for Li, C and Ca are from [1, 2], other data points are from [10].*

Figure 4: *Logarithmic Q^2-derivative of the ratio of structure functions for tin to carbon. The data are from [11].*

The results depicted in Figs. 1-2 demonstrate a substantial variation of nuclear shadowing with Q^2, especially at low x. Q^2-dependence of shadowing was observed recently by the NMC experiment [11]. Their data are plotted in Fig. 3 together with our predictions, which reproduce well the order of magnitude of the effect. We cannot claim a precise description, since the data represent the results of averaging over large interval of Q^2 down to quite low values, where our approximation may not work. It is important that all Q^2-dependence of shadowing originates in formula (2) only from the proton structure function in the denominator. Consequently, this effect in (2) has no relation to shadowing of gluons in nuclei.

To conclude, we would like to comment on the approximations used.

First of all, in saying that $\langle\sigma_{tot}^{hN}\rangle$ is dominated by soft interaction we neglected the higher twist corrections $\sim 1/Q^2$ presented in Table 1. Thus, one should be cautious using this approximation at low Q^2.

Although we use a double-leading-log type parameterization for $F_2^p(x, Q^2)$, which provides a vanishing effective Δ_P at $x \to 0$, it is almost a constant in the range of x under discussion, i.e. is compatible with the BFKL solution [12].

It is easy to show that the higher-order shadowing corrections omitted in (2) are soft as well. However, the x-dependence of the n-fold correction is governed by the power $n\Delta_P(\mu^2) - \Delta_P(Q^2)$ which may be positive for $n = 3$ or 4 and so on. A question arises, whether the growth of higher-order shadowing corrections can change our conclusion about the shadowing decreasing with $1/x$. We think it cannot. Indeed, let us consider an eikonal shadowing where the first term correspond to the hard Pomeron with a large $\Delta_P(Q^2)$, but all other terms correspond to the soft $\Delta_P(\mu^2)$. Eikonalization of formula (2) leads to the full shadowing correction, which reads

$$1 - \frac{\sigma_{tot}^{\gamma^* A}(x,Q^2)}{A\ \sigma_{tot}^{\gamma^* N}(x,Q^2)} = \frac{1}{\langle\sigma_{tot}^{hN}\rangle}\left\{\sqrt{\langle(\sigma_{tot}^{hN})^2\rangle} - \frac{2}{A}\int d^2b\left[1-\exp\left(-\frac{1}{2}\sqrt{\langle(\sigma_{tot}^{hN})^2\rangle}T(b)\right)\right]\right\}\ , \quad (6)$$

where we assume x small and fix $F_A^2 = 1$. The first term in curly brackets is bigger than the second one and both grow with $1/x$. For this reason, the right hand side of (6) decreases with $1/x$ more steeply than $(1/x)^{\Delta_P(\mu^2)-\Delta_P(Q^2)}$. Thus, addition of higher order shadowing corrections makes vanishing of the shadowing for $x \to 0$ even stronger.

In order to estimate q_L in (2)-(3) we assumed $\langle m_h^2\rangle \approx Q^2$. This may not be a good approximation for so called triple-Pomeron term in shadowing, which provides a mass distribution in diffractive dissociation $\propto 1/m_h^2$, not steep enough to neglect the high-mass tail. The nuclear formfactor in Gaussian form, convoluted with this mass distribution, results in a modified formfactor

$$\widetilde{F}_A^2(x) = \frac{Ei\,(-q_{max}^2 R_A^2/3) - Ei\,(-q_{min}^2 R_A^2/3)}{2\ln(q_{max}/q_{min})}\ , \quad (7)$$

where Ei is the integral exponential function, $q_{min} = m_N(x + m_{min}^2/2m_N\nu)$ and $q_{max} = m_N(x + m_{max}^2/2m_N\nu)$. The limit of integration over m_h are m_{min} and m_{max}. In contrast to formfactor (3), the modified one (7) grows logarithmically with $1/x$. However, with a reasonable choice of the mass interval, this growth does not stop the power decrease (4) of the shadowing correction, even if the triple Pomeron contribution (i.e. gluon fusion) dominates nuclear shadowing.

Summarising, we predict the unusual phenomenon of vanishing nuclear shadowing for $x \to 0$ at fixed large Q^2.

References

[1] CERN NMC, P. Amaudruz et al., Z. Phys. **C51** (1991) 387

[2] CERN NMC, M. Arneodo et al., Nucl. Phys. **B441** (1995)

[3] V.N. Gribov, Sov. Phys. JETP **29** (1969) 483

[4] T.H. Bauer et al., Rev. Mod. Phys. **50** (1978) 261

[5] B.Z. Kopeliovich and B. Povh, Phys. Lett. **B367** (1996) 329

[6] J.D. Bjorken and J. Kogut, Phys. Rev. **D8**, (1973) 1341

[7] L.L. Frankfurt and M.I. Strikman, Phys. Rept. **160** (1988) 235

[8] N.N. Nikolaev and B.G. Zakharov, Z. Phys. **C49** (1991)

[9] DESY H1, T. Ahmed et al., Phys. Lett. **B348** (1995) 681

[10] The NM Collaboration, preliminary data, A. Witzmann, talk at the Conf. on High-Energy Physics, Brussels, 1995

[11] A. Mücklich, Ph.D. thesis, Universität Heidelberg, 1995; A. Mücklich, talk given at the Workshop on Deep Inelastic scattering and QCD, Paris, 1995.

[12] L.N. Lipatov, Sov. Phys. JETP **63** (1986) 904