# The Ratio of Gluon Distributions in Sn and C

T. Gousset[a1] and H.J. Pirner[a]

[a] Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 19, D-69120 Heidelberg, Germany

The investigation of nuclei with deep inelastic lepton scattering has produced very accurate data on nuclear structure functions $F_2^A(x)$. Especially the measurement of the ratio of tin to carbon structure functions has been carried out recently with very high statistics [1]. It shows shadowing for $x \leq 0.05$. Antishadowing ($\leq 2\%$) is visible around $x = 0.1$. In the interval $0.25 \leq x \leq 0.8$ the nuclear valence quark density is reduced. By measuring three different $\mu$-beam energies (120, 200 and 280 GeV) NMC has also been able to determine the weak $\ln Q^2$ dependence of the structure function ratio. Using perturbative QCD evolution equations one can extract from this dependence the little known underlying gluon densities [2]. Measurements of the gluon distribution in nuclei give experimental windows viewing the partonic structure of nuclear binding. Currently there is a discussion to use nuclear beams in HERA. The study of the gluon distribution in nuclei at small $x$ would be one of the outstanding new opportunities to investigate nuclei on the parton level. Also the forthcoming heavy ion experiments at RHIC and LHC need a good understanding of the nuclear parton distributions to calculate the cross sections for hard processes initiated by nuclear collisions.

At Leading-Log (LL), the structure function per nucleon is the sum over active flavors of quark and antiquark densities weighted by the corresponding charge squared $F_2(x, Q^2) = x \sum_i e_i^2 q_i(x, Q^2)$. As usual, we define the Bjorken variable relative to nucleon kinematics $x = Q^2/2M_N\nu$, with $M_N$ the nucleon mass, $\nu$ the virtual photon energy in the rest-frame of the nucleon or nucleus, and $Q^2 = -q^2$ its virtuality. The LL $Q^2$-evolution of $F_2$ reads

$$\frac{\partial F_2}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} x \left( \sum_i e_i^2 P_{qG} * G + P_{qq} * \frac{F_2}{x} \right), \tag{1}$$

where $G$ is the gluon density in the nucleon or in the nucleus. The splitting functions are denoted by $P_{qq}$ and $P_{qG}$. It is understood that distributions, structure functions, as well as $\alpha_S$ are $Q^2$ dependent.

For $x$ sufficiently small, the second term in the evolution equation of $F_2$ is small in comparison with the first. Furthermore, the first term can be approximated in a simple way to easily extract the gluon distribution from the evolution of $F_2$. One has

$$\frac{\partial F_2}{\partial \ln Q^2} \approx \frac{\alpha_S}{2\pi} \left( \frac{2}{3} \frac{\sum_i e_i^2}{2} 2xG(2x, Q^2) + \Delta(x, Q^2) \right). \tag{2}$$

[1] Supported by the Federal Ministry of Education, Science, Research and Technology (BMBF) under grant no. 06 HD 742

The second term, $\Delta(x, Q^2)$, is a certain integral over $F_2$ [2] which can be computed from $F_2$ data. Due to the steep rise of $G$, $\Delta$ becomes a small correction as soon as $x \leq 0.1$, so that only an approximate knowledge of $F_2$ is needed to estimate $\Delta$. In the following, we shall take into account both contributions to the evolution equation.

We can now turn to the study of the new NMC data on tin to carbon ratio of structure functions. Namely, measurements of

$$f_1(x) = \frac{F_2^{\rm Sn}}{F_2^{\rm C}}, \qquad f_2(x) = \frac{\partial}{\partial \ln Q^2} \frac{F_2^{\rm Sn}}{F_2^{\rm C}}, \tag{3}$$

have been performed [1]. Using Eq. (2), it is easy to relate the tin to carbon ratio of gluon distributions

$$r(x, Q^2) = \left. \frac{G^{\rm Sn}}{G^{\rm C}} \right|_{x,Q^2},$$

to $f_1$ and $f_2$, and one finds

$$r(2x, \langle Q^2 \rangle) = f_1 + \left. \frac{f_2 - \frac{\alpha_S}{2\pi} \frac{\Delta^{\rm Sn} - f_1 \Delta^{\rm C}}{F_2^{\rm C}}}{f_3 - \frac{\alpha_S}{2\pi} \frac{\Delta^{\rm C}}{F_2^{\rm C}}} \right|_{x, \langle Q^2 \rangle}, \tag{4}$$

where

$$f_3 = \frac{\partial \ln F_2^{\rm C}}{\partial \ln Q^2} = \frac{\partial \ln F_2^d}{\partial \ln Q^2} + \frac{\partial}{\partial \ln Q^2} \ln \frac{F_2^{\rm C}}{F_2^d}. \tag{5}$$

In this analysis, we use the fact that the ratio of carbon to deuteron structure functions shows practically no $Q^2$ dependence, so that we can drop the second term in the right hand side of Eq. (5), and use for $f_3$ the deuteron data.

We plot the ratio $r(x)$ in Fig. 1 together with $f_1(x)$. One observes a signal for a stronger antishadowing in the gluon case (≈8%) than in the quark case (≈1%). In the shadowing region, there is no evidence for a difference between quarks and gluons. It is, however, presently impossible to draw a firm conclusion in this region.

There are theoretical approximations. Neglect of NLL corrections together with the use of Eq. (2) lead to an approximation presumably better than 20% for an absolute extraction of gluon density. Because the ratio $r(x)$ remains close to 1 in the whole range of $x$ being considered, they are hopefully even far better for the ratio we are evaluating.

For the discussion of the accuracy reachable with HERA using nuclear beams there are two scenarios. If the present H1 systematic errors are added independently for each nucleus, the ratios $f_1 = F_2^{A1}/F_2^{A2}$ of nuclear structure functions would have prohibitively large errors (10%) of the same size as the EMC-effect. Drastically reduced errors, however, can be obtained if two nuclei can be stored and/or measured simultaneously. Then the leading error on the ratio of nuclear gluon densities would be the error in $f_2 = d(F_2^{A1}/F_2^{A2})/d\ln Q^2$. The situation would be similar to the NMC measurements, but with a much extended range in $Q^2$ and $x$. Note in Fig. 1 only the statistical errors have been included, a consideration of systematic errors ( at small $x$ mostly from radiative corrections) would increase our uncertainty by 50%.

The LL approximation becomes insufficient at very small $x$. An extension to NLL has been proposed for the proton case and can be carried out for the nuclear case as well. In the

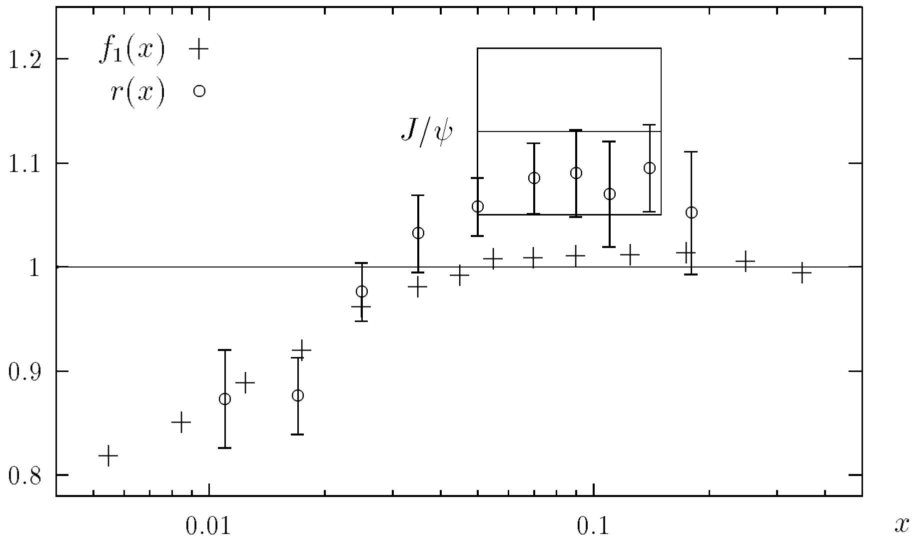


Figure 1: The ratio $r(x) = G^{\rm Sn}(x)/G^{\rm C}(x)$ of tin to carbon gluon density as a function of $x$ together with the ratio of structure function, $f_1(x) = F_2^{\rm Sn}(x)/F_2^{\rm C}(x)$. The statistical error on $f_1$ is less than 1% in the whole range of $x$. We also show the extraction of $r$, $\langle r\rangle = 1.13\pm0.08$, from $J/\psi$ electroproduction data as a rectangle the sides of which are the $x$-range of the data and the errorbar in $\langle r\rangle$.

nuclear case, additionnal merging terms are expected to play an important role in the evolution equation with the increase of the gluon density. These non-linear effects are expected to appear at very small $x$ ($< 10^{-2}$). With such possible contributions, it is necessary to carry out the measure of the gluon density in other channels. In this respect, the ratio of longitudinal to transverse virtual photon cross sections at small $x$ provides a direct information on $G(x)$. It is also important to extend the inclusive measurements of $F_2$ and $F_L$ to jet, open charm and charmonium productions.

Apart from its own interest, the gluon density is a basic input in heavy ion collision observables. The range of $x$ values accessible via the NMC $F_2$-measurements $0.01 \leq x \leq 0.2$ is sufficient to estimate the importance of shadowing for the minijet production rate at RHIC. One needs, however, much smaller $x$ information, down to $x \approx 10^{-4}$, for heavy ion collision at LHC. To cover this range, HERA with nuclear beams would be very helpful.

# References


[1] A. Műcklich, Ph.D. thesis, Universităt Heidelberg, 1995; A. Műcklich, talk given at the Workshop on Deep Inelastic scattering and QCD, Paris, 1995.

[2] T. Gousset and H.J. Pirner, Phys. Lett. B 375 (1996) 349.