Energy loss of high energy partons in a finite QCD medium

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Abstract: We shortly summarize results which we have recently obtained on the radiative energy loss of high energy quarks and gluons suffering multiple collisions in a QCD medium of finite length. These results are also generalized to the case of nuclear matter.

1 Introduction

Gluon radiation induced by multiple scatterings of a very energetic incident quark or gluon propagating through QCD matter becomes coherently suppressed compared to independent emission [1, 2, 3, 4]. In the case of QED the analogous suppression of photon radiation is known as the Landau-Pomeranchuk-Migdal (LPM) effect [5, 6].

In order to calculate the soft gluon radiation intensity static and screened scattering centres are assumed [7]. This leads to a probability distribution in terms of the transverse scattering momentum $\vec{q}_\perp$,

$$V(\vec{q}_\perp^{\,2})d^2q_\perp \equiv \frac{\mu^2}{\pi(\vec{q}_\perp^{\,2}+\mu^2)^2}d^2q_\perp. \tag{1}$$

It is reasonable to assume that successive scatterings are independent. As a consequence the mean free path λ of the parton has to be large compared to the characteristic range μ^{-1} of the "Coulomb potential" (1), i.e. $\lambda \gg \mu^{-1}$.

We may actually generalize this treatment to the case of cold nuclear matter and define a probability distribution $V(\vec{q}_\perp^{\,2})$ related to the gluon distribution function [4].

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2 Gluon radiation spectrum

For a fast quark of energy E, which e.g. is produced through deep inelastic scattering and which propagates through a hot medium of length L, the induced gluon intensity (per unit length of propagation) reads in leading order of the strong coupling α_s,

$$\frac{\omega dI}{d\omega dz} \simeq \frac{3\alpha_s N_c}{\pi L} \ln \left| \frac{\sin\left(\frac{\omega_0}{\lambda}L\right)}{\frac{\omega_0}{\lambda}L} \right|, \tag{2}$$

where ω denotes the energy of the soft gluon, $\omega \ll E$, and λ is the mean free path of the quark. Here the number of colours N_c is assumed to be large (for a more detailed discussion see [1, 3]). The quantity ω_0 charcterizes the spectrum, $\omega_0 \equiv \sqrt{i\kappa\tilde{v}(1/\sqrt{\kappa})}$. The value of the dimensionless parameter $\kappa = \frac{\mu^2\lambda}{2\omega}$ determines the shape of the radiation intensity. $\tilde{v}$, related to the Fourier transform of the potential $V(\vec{q}_\perp^{\,2})$ in the impact parameter space $\vec{B}$, is given by

$$\tilde{v}(B^2) \equiv 4\frac{1 - V(\vec{B}^2)}{\vec{B}^2} \simeq \begin{cases} \ln 1/B^2 \text{ for the Coulomb case (1)} \\ \langle \vec{q}_\perp^{\,2} \rangle / \mu^2 \text{ otherwise} \end{cases} \tag{3}$$

for $B^2 \to 0$. The parameter κ may be viewed as the typical phase difference between neighbouring scattering centres. The interesting coherent phenomena for which the medium properties are important show up for $\kappa \ll 1$, because then a group of $N_{coh} \simeq 1/\sqrt{\kappa}$ centres radiate coherently.

For large L the spectrum (2) approaches [1, 2, 3]

$$\frac{\omega dI}{d\omega dz} \simeq \frac{3\alpha_s N_c}{2\pi} \sqrt{\frac{\mu^2}{\lambda\omega} \ln(1/\sqrt{\kappa})}, \tag{4}$$

when putting $\tilde{v}(1/\sqrt{\kappa}) \simeq \ln 1/\sqrt{\kappa}$ for $\kappa \ll 1$. Modulo logarithms (4) is valid for

$$O(\lambda\mu^2) < \omega < \omega_{\text{fact}} \mathrel{\hat{=}} O\left(\frac{\mu^2}{\lambda}L^2\right) < E, \tag{5}$$

as long as $L \leq L_{cr} \mathrel{\hat{=}} O(\sqrt{E\lambda/\mu^2})$ for $E \to \infty$. For $\omega \geq \omega_{\text{fact}}$ the radiation is totally suppressed. It is important to state that the spectra (2) and (4), respectively, do not depend on the parton's energy E.

We note that the matter properties enter in the spectra (2) and (4) via the ratio $\hat{q} \equiv \mu^2\tilde{v}/\lambda$. Effectively $\hat{q}$ plays the role of the transport coefficient [6],

$$\hat{q} \equiv \mu^2\tilde{v}/\lambda \mathrel{\hat{=}} \frac{\langle \vec{q}_\perp^{\,2} \rangle}{\lambda} = \rho \int d^2q_\perp \vec{q}_\perp^{\,2} \frac{d\sigma}{d^2q_\perp}, \tag{6}$$

with ρ the density of the scattering centres and $d\sigma/d^2q_\perp$ the "parton-centre" cross-section, where $\lambda \equiv 1/\rho\sigma$. Within logarithmic accuracy the jet broadening is therefore given by

$$\langle \vec{q}_\perp^{\,2} \rangle_L \equiv \hat{q}L \simeq \frac{\mu^2}{\lambda}\tilde{v}(\lambda/L)L, \tag{7}$$

after the parton's passage through matter of length L, i.e. (7) is effectively a random walk result.

3 Energy loss in finite length media

In [3] the radiative energy loss in a hot medium of finite extent L, $L \leq L_{cr}$ is derived. For a quark jet the result is

$$- dE/dz \simeq \frac{\alpha_s N_c}{8} \hat{q} L, \tag{8}$$

which amounts to the total loss of $-\Delta E(L) \simeq \frac{\alpha_s N_c}{8} \hat{q} L^2$, i.e. it is increasing quadratically with the length L [8] and independent of E for $E \to \infty$. This remarkable dependence on the length L can also be heuristically obtained by integrating the spectrum given by (4) over ω and z, with $0 \leq \omega \leq \omega_{\rm fact}$ and $0 \leq z \leq L$. This result may actually be generalized to cold nuclear matter [4]: the relation (8) between the energy loss and the jet broadening (7) is shown to be valid in general and indeed does not depend on the parameters of the model.

The energy loss per unit distance (8) may be compared to the one for an infinite medium $L \to \infty$, with E large but fixed, which is obtained in [1] by

$$- dE/dz \simeq \alpha_s N_c \sqrt{\hat{q} E}, \tag{9}$$

i.e. depending on the incident quark energy.

4 Phenomenology

In order to estimate the energy loss e.g. of a very energetic quark in cold matter of finite extent [9] one has to know the coefficient $\hat{q}$ in (8). One possibility [4, 11] is to use jet broadening (7), which e.g. is measured by the momentum imbalance of jets in deep inelastic production of dijets [10]. The analysis by Luo et al. [10] determines $\hat{q} \simeq \frac{4}{3}\pi^2 \alpha_s (A^{1/3}/L) \lambda^2_{LQS}$, with $L \simeq \frac{3}{4} R$ expressed in terms of the nuclear radius $R \simeq 1.2 A^{1/3} fm$. With $\lambda^2_{LQS} \simeq 0.05 - 0.1 \mathrm{GeV}^2$ [10], this gives the estimate [11]

$$\hat{q} \simeq (3.7 - 7.4)\ \alpha_s\, \mathrm{GeV}/fm^2. \tag{10}$$

According to (8) this gives $-dE/dz \simeq (1.2 - 2.5)$ GeV$/fm$ for $L \simeq 10 fm$ and $\alpha_s \simeq 0.3$, which is a rather large loss per unit distance.

An independent estimate may be obtained by relating $\hat{q}$ to the proton's gluon structure function $xG(x, Q^2)$ [4, 11, 12]. This uses (6) in terms of the (transport) quark-proton cross section

$$\int_0^{\mu^2/B^2} d^2 q_\perp \vec{q}_\perp^{\,2} \frac{d\sigma^{q-\mathrm{proton}}}{d^2 q_\perp} \simeq \frac{\alpha_s C_F \pi^2}{2} xG(x, \mu^2/B^2 \simeq \mu^2/\sqrt{\kappa}), \tag{11}$$

where x is small and estimated to be $x \simeq O(1/(\lambda m_{\rm proton})) \simeq O(0.1)$.

Depending on $xG(x) \simeq 1 - 5$ in the region of interest of small x, and with the nuclear density $\rho \simeq 0.17 fm^{-3}$, one finds [4, 11]

$$\hat{q} \simeq (0.2 - 1.0)\ \alpha_s\, \mathrm{GeV}/fm^2, \tag{12}$$

which gives an energy loss $-dE/dz < 0.35$ GeV$/fm$, i.e. a significantly smaller value than the one quoted above!

In case of hot matter a corresponding estimate yields $-dE/dz \sim 3$ GeV$/fm$ [1].

Not only because of these uncertainties careful measurements of the energy loss of partons at HERA's future experiments using different nuclei are indeed an important challenge. It is especially crucial to look for possible differences of the parton propagation in cold versus hot matter, which are expected on the basis of the above (still crude, because asymptotic) estimates [1, 3, 4, 11].

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