NUCLEAR GLUON DISTRIBUTION IN QCD

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Abstract: This letter is a digest of the result of Ref.[1], prepared for the DESY Workshop. The Glauber (Mueller) approach has been developed to calculate the gluon distributions in a nucleus. This approach includes shadowing corrections due to gluons rescatterings inside the nucleus. It is shown that the shadowing corrections are essential in the region of small x. As the second order corrections to the Glauber formula are sizable, we improve this description deriving a nonlinear evolution equation, and solving it in the semiclassical approach. The results are shown and discussed.

1 Introduction.

The gluon structure function is the observable that governs the physics of the high energy processes in QCD [2]. In this letter, we will show how the space-time picture [3] can be used to describe the interaction of a probe with the nucleus and obtain a nuclear gluon distribution function from Glauber (Mueller) approach. This approach was first suggested in Ref.[4] and extensively developed in Ref.[1]. In Glauber (Mueller) approach [5], a high energy virtual probe creates a GG pair with momentum Q^2, which penetrates the nucleus and suffers multiple interactions with the nucleons. These rescatterings of the gluon pair generate the shadowing corrections which could be taken into account in a closed formula, based on the GLAP evolution[7] at low x (high energy). We show that these corrections are relevant and modify the gluon distribution and the anomalous dimension in the region of small gluon x. We discuss also the kinematic region where this approach is valid and a possible improvement.

2 The Glauber approach to high energy in QCD

The main features and the kinematics of the Glauber approach to high energy processes have been discussed in Fig.1. The virtual probe with virtuality Q^2 decays in a GG pair with transverse separation $r_\perp$ and transverse momentum $\vec{k_\perp}$. This pair interacts with a nucleon due to exchange of a gluon ladder diagram. $l_\perp$ denotes the transverse moment of the gluon in the ladder attached to the GG pair. The Bjorken x is defined as $x_{Bj} \equiv \frac{Q^2}{s}$, where $\sqrt{s} = W$ is the

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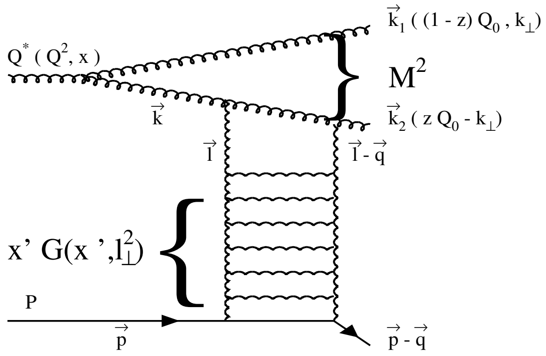

Figure 1: *Kinematics of the Glauber approach.*

c.m. energy of the incoming gluon. In DLA , the change in $r_\perp$ during the passage of the GG pair through the nucleus may be neglected. Indeed, $r_\perp$ can vary by an amount $\Delta r_\perp \propto R\frac{k_\perp}{E}$ where E denotes the energy of the pair in the target rest frame and R is the size of the target. The transverse momentum is $k_\perp \propto 1/r_\perp$. Therefore

$$\Delta\, r_\perp \,\propto\, R\,\frac{k_\perp}{E}\,\ll\, r_\perp\,, \tag{1}$$

which in terms of Bjorken x has the form:

$$x \,\ll\, \frac{1}{2\,m\,R}\,. \tag{2}$$

In Glauber (Mueller) approach, the nuclear gluon distribution is given by

$$xG_A(x,Q^2) = \frac{4}{\pi^2}\int_x^1 \frac{dx'}{x'}\int_{\frac{4}{Q^2}}^{\infty}\frac{d^2r_\perp}{\pi r_\perp^4}\int_0^{\infty}\frac{d^2b_\perp}{\pi}2\left\{1 - e^{-\frac{1}{2}\sigma_N^{GG}(x',r_\perp^2)S(b_\perp^2)}\right\} \tag{3}$$

The term in curly brackets is the total cross section of the interaction of the gluon pair with nucleus in the eikonal approach. This term is the solution of the s-channel unitarity constraint [6] if the elastic amplitude of the gluon nuclei scattering is purely imaginary at high energy and the structure of the final state is mostly the uniform distribution of the produced gluons.

In DLA perturbative QCD, the GG pair cross section with the nucleon $\sigma_{GG}(r_\perp^2)$ can be written (for $N_c = 3$)

$$\sigma_{GG}(r_\perp^2) = \frac{3\alpha_S(\frac{4}{r_\perp^2})}{4}\pi^2 {r_\perp}^2\left(xG_N^{GLAP}(x,\frac{4}{r_\perp^2})\right)\,. \tag{4}$$

This expression is valid in DLA since it was obtained considering $l_{i\perp} \ll k_\perp$ and neglecting the longitudinal part of the momentum l_i. The factor $1/r_\perp^4$ comes from the wave function of the two gluons in the high energy probe, after the integration over the fraction of energy of the probe carried by the gluon. Using the Gaussian parameterization for the nucleon profile function $S(b_\perp)$, we can take the integral over $b_\perp$ and obtain the answer ($N_c = N_f = 3$)

$$xG_A(x,Q^2) = \frac{2R_A^2}{\pi^2}\int_x^1 \frac{dx'}{x'}\int_{\frac{1}{Q^2}}^{\frac{1}{Q_0^2}} \frac{dr_\perp^2}{r_\perp^4}\left\{C + ln(\kappa_G(x',r_\perp^2)) + E_1(\kappa_G(x',r_\perp^2))\right\} \qquad (5)$$

where C is the Euler constant and E_1 is the exponential integral (see Ref.[9] Eq. **5.1.11**) and

$$\kappa_G(x',r_\perp^2) = \frac{3\alpha_S A\pi r_\perp^2}{2R_A^2} x' G_N^{GLAP}(x',\frac{1}{r_\perp^2}) \qquad (6)$$

where A is the number of the nucleons in a nucleus and R_A^2 is the mean nuclear radius. If eq. (5) is expanded for small κ, the first term will correspond to the usual GLAP equations in the small x region (Born approximation of Mueller formula), while the other terms will take into account the shadowing corrections (SC).

3 pQCD calculations from Mueller approach.

To calculate the nuclear gluon distribution $xG_A(x,Q^2)$ we use the GRV parameterization [10] for the nucleon gluon distribution. The GRV distribution is suited for our calculation because its initial virtuality is small and we can estimate the SC for large distance ($r_\perp^2 \propto 1/0.25\ GeV^{-2}$). In this distribution, the most essential contribution comes from the region where $\alpha_S ln(1/x) \approx 1$ and $\alpha_S ln Q^2 \approx 1$, which allows the use of DLA, where the Mueller formula is proven.

As the DLA does not work quite well in the accessible kinematic region ($Q^2 > 1$ GeV^2 and $x > 10^{-4}$) and the Mueller Formula is proven in DLA, we develop a more realistic approach subtracting the Born term of the Mueller formula and adding the GRV gluon distribution. This procedure gives

$$\begin{aligned} xG_A(x,Q^2) &= xG_A(x,Q^2)(\,eq.\ (3)\,) \ + \ AxG_N^{GRV}(x,Q^2) \\ &- \ A\,\frac{\alpha_S N_c}{\pi}\int_x^1\int_{Q_0^2}^{Q^2}\frac{dx'}{x'}\frac{dQ'^2}{Q'^2}\,x'G_N^{GRV}(x',Q'^2)\ . \end{aligned} \qquad (7)$$

The above equation includes also $AxG_N^{GRV}(x,Q_0^2)$ as the initial condition for the gluon distribution and gives $AxG_N^{GRV}(x,Q^2)$ as the first term of the expansion with respect to κ_G. Therefore, this equation is an attempt to include the full expression for the anomalous dimension for the scattering off each nucleon, while we use the DLA to take into account all SC

In order to investigate the general features of the nuclear gluon distribution given by eq. (7), we calculate the ratio

$$R_1 \ = \ \frac{xG_A(x,Q^2)(eq.\ (7))}{AxG_N^{GRV}(x,Q^2)}\ , \qquad (8)$$

which is shown in Fig.2. From this ratio we can see the general behavior of the SC as a function of $ln(1/x)$ and Q^2. The suppression due to the SC increases with $ln(1/x)$. For $A = 40$ (Ca) and $Q^2 = 10\,GeV^2$, the suppression varies from 4 % for $ln(1/x) = 3$ to 25 % for $ln(1/x) = 10$. For $A = 197$ (Au) the suppression is still bigger, going from 6% to 35% in the same kinematic region.

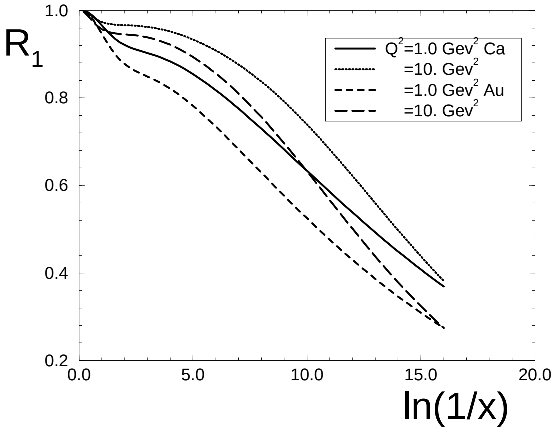

Figure 2: R_1 *as a function of* $ln(1/x)$ *and* Q^2

In the semiclassical approach (see [2]), the nucleon structure function is supposed to have Q^2 and x dependence as

$$xG_N(x,Q^2) \;\propto\; \{Q^2\}^{<\gamma>}\; \{\frac{1}{x}\}^{<\omega>}\;. \tag{9}$$

where $<\omega>$ is the average value of effective power of xG_A for $x \to 0$ and $<\gamma>$ is the average value of the anomalous dimension . We can calculate both exponents using the definitions

$$<\omega> = \frac{\partial \ln(xG_A(x,Q^2))}{\partial \ln(1/x)}\;. \tag{10}$$

$$<\gamma> = \frac{\partial \ln(xG_A(x,Q^2))}{\partial \ln(Q^2/Q_0^2)}\;; \tag{11}$$

Fig.3 shows the calculation of $<\omega>$ as a function of x and Q^2 for GRV distribution and for Au. We can see that the SC give rise to a flattening of the nuclear distribution as A grows in small x region.

Fig.4 shows the results for $<\gamma>$ as a function of $ln(1/x)$ and Q^2 for GRV distribution and Au. As x decreases, the anomalous dimension presents a sizeable reduction and goes to zero for $ln(1/x) > 15$, unlike the GLAP evolution.

We can estimate what distances work in the SC calculation from the average value of the anomalous dimension. Let us consider the expansion of eq. (5) in respect to κ (see eq. (6)). From semiclassical approach, κ may be written $\kappa \propto (Q^2)^\gamma/Q^2$ and if $\gamma \leq 1$, the integral over $r_\perp$ in eq. (5) becomes divergent. If $1 \leq \gamma \leq 1/2$ only the second term of the expansion (first SC term) is concentrated at small distances. From Fig.4, we see that $1 \leq \gamma \leq 1/2$ for $Q^2 > 1GeV^2$ and even for $Q^2 = 1GeV^2$ at very small values of x.

4 The generalization of Glauber Approach.

The Mueller formula (5) is not an evolution equation but an analogue of the Glauber formula which gives the possibility to calculate $xG_A(x,Q^2)$ and the SC for nucleus from the solution of

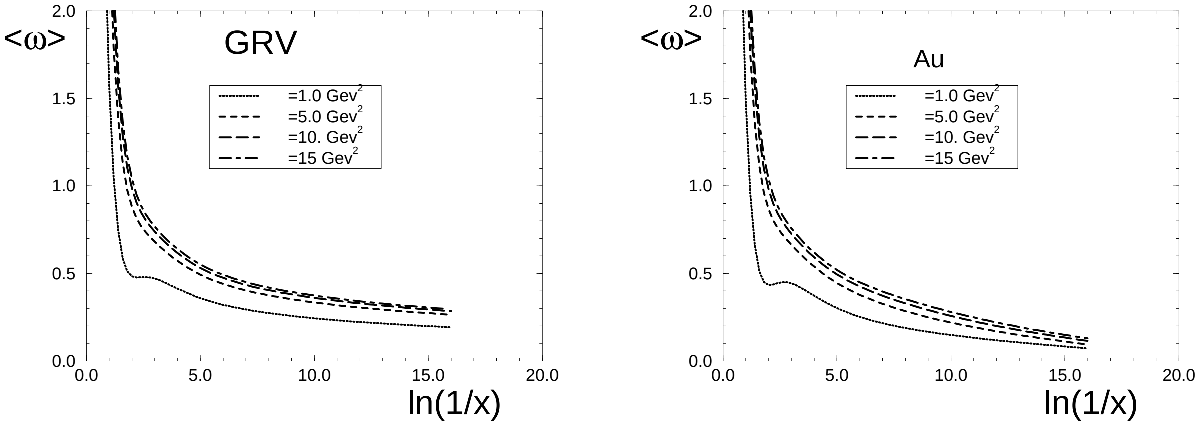

Figure 3: $<\omega>$ *as a function of* x *and* Q^2 *for GRV distribution a) and Au b)*

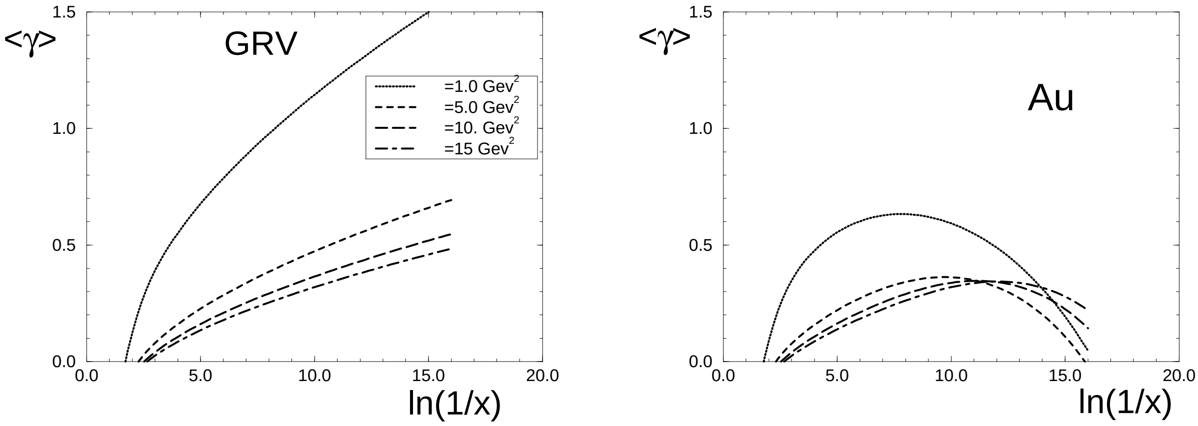

Figure 4: $<\gamma>$ *as a function of* x *and* Q^2 *for a) GRV distribution and b) Au*

GLAP equations. In order to investigate the corrections to this approach the second iteration of the Mueller formula must be calculated. The second iteration takes into account the rescattering of the next to the fastest gluon in the gluon-nucleon process. This task was done in ref [1] and the main conclusion is that the second iteration gives a sizable contribution for $x < 10^{-2}$ and becomes of the order of the first iteration for $x < 10^{-3}$. It occurs because the second iteration gives correction of the order of $\alpha_{\rm S} ln(Q^2/Q_0^2) ln(1/x) \approx 1$ for the DLA.

In order to improve the calculation of the nuclear distribution $xG_A(x,Q^2)$ we suggest to derive the Mueller formula in respect to $y = ln(1/x)$ and $\xi = ln(Q^2)$, and write it as an equation for κ. The result reads

$$\frac{\partial^2 \kappa(y,\xi)}{\partial y \partial \xi} + \frac{\partial \kappa(y,\xi)}{\partial y} = \frac{N_c\,\alpha_{\rm S}}{\pi}\,\{\, C \,+\, \ln \kappa(y,\xi) \,+\, E_1(\kappa(y,\xi))\,\} \,\equiv\, F(\kappa)\,. \qquad (12)$$

where κ is given by

$$\kappa \;=\; \frac{N_c \alpha_{\rm S} \pi}{2 Q^2 R_A^2}\, xG_A(x,Q^2) \qquad (13)$$

This is a generalized nonlinear evolution equation for nuclear gluon distribution. This equation sums all contributions of the order $(\,\alpha_{\rm S}\, y\, \xi\,)^n$ absorbing them in $xG_A(y,\xi)$, as well as all contributions of the order of κ^n. This equation provides the correct matching both with the GLR equation and with the Glauber (Mueller) formula in the kinematic region where $\alpha_{\rm S} y \xi \,\leq\, 1$.

We solve eq. (12) in semiclassical approach for $\alpha_{\rm S}$ constant using the method of characteristics (see ref [1] for a detailed discussion). In semiclassical approach, we write κ in the form

$$\kappa = e^S \qquad (14)$$

where the partial derivatives $\partial S/\partial y = \omega$ and $\partial S/\partial \xi = \gamma$ are supposed to be smooth functions of y and ξ. The initial condition for the characteristics is given by

$$\begin{aligned} S_0 &= ln\, \kappa_{in}(y_0,\xi_0) \\ \gamma_0 &= \left.\frac{\partial ln \kappa_{in}(y_0,\xi)}{\partial \xi}\right|_{\xi=\xi_0} = < \gamma >_{Glauber} -1\,. \end{aligned} \qquad (15)$$

where κ_{in} is

$$\kappa \;\rightarrow\; \kappa_{in} \;=\; \frac{N_c\,\alpha_{\rm S}\,\pi}{2\,Q^2\,R_A^2}\, xG_A(x,Q^2)\;, \qquad (16)$$

and xG_A is given by the Mueller formula. The GLAP equation for κ is obtained taking the term in the curly in r.h.s of eq. (12) equal to κ.

Fig.5 shows the general features of the solution, calculated for $x_0 = 10^{-2}$ and several values of Q^2 close to $1\, GeV^2$; Fig.5a shows the characteristic curves in the y v.s. ξ plane; Fig.5b shows the evolution of the γ values with y. When the value of γ goes to zero, the nonlinear effects play an important role and the solution goes to an asymptotic solution ($\kappa_{asymp}(y)$) which is ξ independent and satisfies the equation: $\frac{d\kappa_{asymp}(y)}{dy} = F(\kappa)$ (see [1] for details). When γ goes to a constant value, the nonlinear effects disappear, and the solution tends to the usual GLAP evolution. This figure also shows the lines with definite value of the ratio $R = \frac{xG(x,Q^2)(generalized\ equation)}{xG(x,Q^2)(GLAP)}$ (horizontal lines). These lines give a way to estimate how big are the SC, which are rather big.

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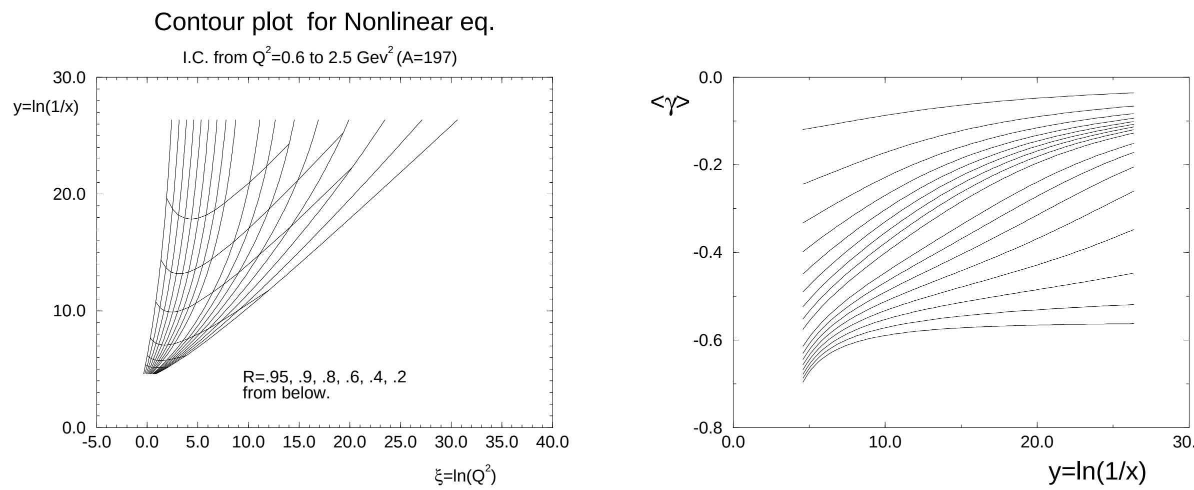

Figure 5: *a) The characteristics curves for solution of eq. (12); b) the* γ *evolution.* $R = \frac{xG(x,Q^2)(generalized\ equation)}{xG(x,Q^2)(GLAP)}$.