Prospects for a Measurement of α_s via Scaling Violations of Fragmentation Functions in Deeply Inelastic Scattering

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Abstract: The prospects for a determination of the strong coupling constant α_s via scaling violations of fragmentation functions in deeply inelastic scattering are studied. The statistical error in the case of an integrated luminosity of $250\,\mathrm{pb}^{-1}$, and the theoretical errors due to the various parton density parametrizations and to the factorization scale dependence are estimated.

1 Introduction

The strong coupling constant α_s has been measured at HERA by means of the (2+1) jet rate [1]. This particular process has the advantage that α_s is, up to higher order corrections, directly proportional to the ratio of the measured cross-sections. Another route to a determination of α_s is given by scaling violations of phenomenological distribution functions. Perturbative QCD predicts the scale evolution of these quantities by means of renormalization group equations [2, 3]. As a consequence, again up to higher order corrections and resummation effects, the experimentally measurable quantities (the structure functions, and the fragmentation functions, which depend on factorization scale and scheme) are (symbolically) of the form $A + B\,\alpha_s \ln \mu^2$. Here μ is the factorization scale, which is to be identified, in deeply inelastic scattering, with a scale of the order of the photon virtuality Q, for lack of other hard scales related to the leading-order process. The distribution functions contain a μ-independent term A, and the α_s dependence is only logarithmic in the factorization scale. The scaling violations are therefore expected to be small, and will require large luminosity to be statistically significant. An α_s determination via scaling violations has the advantage that, in principle, no explicit model assumptions such as specific fragmentation models go into the measurement. In the case of scaling violations of structure functions[1] a completely inclusive quantity is measured, and the theoretical basis, namely the operator product expansion, is very transparent and can be derived rigorously from light cone dominance. In the case of one-particle-inclusive processes, where the operator product expansion is not available, the factorization theorem of perturbative QCD (see, for example, Ref. [4] and references therein) allows the separation of the hard

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[1] See, for example, the contributions of the working group on structure functions.

scattering process from the non-perturbative fragmentation process. The one-particle-inclusive cross-section is a convolution of a mass-factorized parton-level scattering cross-section, a parton density and a fragmentation function: $\sigma = \sigma_{\rm hard} \otimes f \otimes D$. A possible strategy for an α_s measurement at HERA is to perform a combined multiparameter fit of fragmentation functions and of the strong coupling constant to the x_F-distribution $\rho\,(x_F) = ({\rm d}\sigma/{\rm d}x_F)\,/\sigma_{\rm tot}$ (or to any other distribution sensitive to the fragmentation functions) of charged hadrons at two different scales Q. Here the variable x_F is defined to be $2h_L/W$, where h_L is the longitudinal momentum fraction of the observed charged hadron in the direction of the virtual photon in the hadronic centre-of-mass frame[2], and W is the total hadronic final state energy. In leading order, x_F is the momentum fraction of the final state current quark carried by the observed hadron. The total cross-section is denoted by $\sigma_{\rm tot}$. The strong coupling constant enters the expression for ρ in three places: (a) as an expansion parameter in the next-to-leading order expression for $\sigma_{\rm hard}$, and in the renormalization group equations of (b) the fragmentation functions and (c) the parton densities. The parton densities are an input to the analysis. Since they are obtained by a global fit, where a specific value of α_s is used, it is necessary to include this dependence as well as the variation due to the different parametrizations into the systematic error[3]. The next-to-leading-order one-particle-inclusive cross-section has been calculated in Ref. [6]. For our study, we use a recent recalculation and numerical implementation described in Ref. [7]. A comparison of the theoretical x_F-distribution with experimental data from the H1 and ZEUS Collaborations [8] has been done in Ref. [9]. It turns out that the theoretical description of the experimental data is quite satisfactory. The next-to-leading order result is always within one standard deviation of the experimental data points except for those at very large x_F, where the currently available fragmentation function parametrizations are not well constrained by e^+e^- data.

In the next section, we describe the estimate of the various errors[4] of the value of α_s. We also discuss the sensitivity to the strong coupling constant of various ranges in x_F. The paper closes with a short summary and conclusions.

2 Error Estimates

To get a quantitative estimate of the dependence of the scale evolution of fragmentation functions on the employed value of α_s, we fix the fragmentation functions at a scale of $\mu_0 = 2\,{\rm GeV}$ as the leading-order parametrization of Ref. [10]. We then evolve this input with two different values for $\Lambda^{(4)}_{\rm QCD}$ of $0.1\,{\rm GeV}$ $\{{\rm a}\}$ and $0.2\,{\rm GeV}$ $\{{\rm b}\}$. The corresponding x_F-distributions $\rho^{\{a\}}$ and $\rho^{\{b\}}$ are determined for these two sets of fragmentation functions. We now assume that the x_F-distributions ρ are measured in two different bins i, j of the factorization scale Q. The ratios $\lambda^{\{ij\}} = \rho^{\{i\}}/\rho^{\{j\}}$ for an arbitrary coupling constant α_s (taken at the mass of the Z boson) are expanded in a power series in α_s, where only the linear term is kept:

$$\lambda^{\{ij\}} = \lambda^{\{aij\}} + \frac{\lambda^{\{bij\}} - \lambda^{\{aij\}}}{\alpha_s^{\{b\}} - \alpha_s^{\{a\}}} \left(\alpha_s - \alpha_s^{\{a\}}\right). \tag{1}$$

[2]It might be possible to reduce the dependence on the parton densities and on the not yet well understood physics of the forward direction by performing an analysis in the Breit frame. I thank N. Brook and T. Doyle for remarks concerning this issue.

[3]Alternatively, parton density parametrizations with varying values of the strong coupling constant [5] can be employed.

[4]We do not consider experimental systematic errors.

Based on this formula, a quantitative estimate of the statistical error of α_s is possible. Moreover, by varying $\lambda^{\{ij\}}$, for example by using various parton density distributions or by modifying the factorization scale, the impact of systematic effects can be studied.

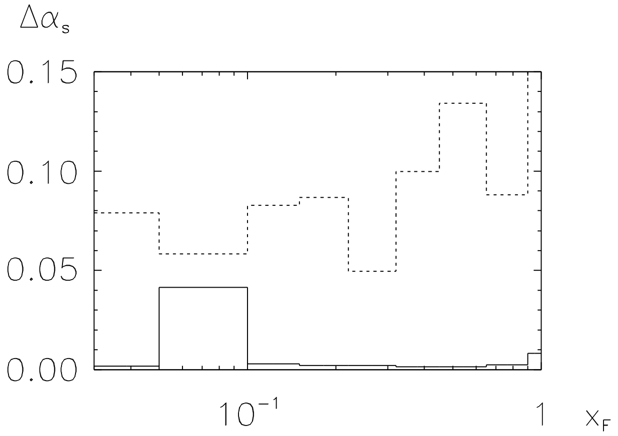

Figure 1: *Statistical errors of α_s for the individual x_F-bins, for the Q-bins $\{1\}$, $\{2\}$ [——] and $\{2\}$, $\{3\}$ [- - -].*

To be more specific, we assume the phase space cuts and x_F-bins of the ZEUS analysis, except for the range in the photon virtuality Q, where we consider three bins: $[3.16, 12.6]$ GeV $\{1\}$, $[12.6, 100]$ GeV $\{2\}$ and $[100, 150]$ GeV $\{3\}$. To obtain explicit numerical values, we use the CTEQ 3L parametrization [11] for the parton densities (for simplicity, we work in leading order). The integrated luminosity is assumed to be $250\,\mathrm{pb}^{-1}$. Under the assumption of Gaussian statistical errors we arrive at a statistical error of $\alpha_s(M_Z^2)$ of $\Delta\alpha_s^{\mathrm{stat}} = \pm 0.0007$ for an analysis based on bins $\{1\}$ and $\{2\}$ and of ± 0.027 for an analysis based on bins $\{2\}$ and $\{3\}$. The individual errors of α_s for the various x_F-bins are shown in Fig. 1. The large error around $x_F \sim 0.08$ for the large-Q bins comes from the fact that the evolution of the fragmentation functions around this value of the momentum fraction is quite small (for smaller values, the fragmentation functions increase with increasing factorization scale, and for larger values, they decrease). The sensitivity of the cross-section to a variation of α_s is largest at large x_F, but this region also suffers from small statistics of the data sample. It turns out that the full x_F-range is about equally important.

As briefly mentioned already in the introduction, an input parton density has to be chosen. To estimate the size of this effect, we determine the spread of the results for $\alpha_s(M_Z^2)$ depending on the next-to-leading-order parton densities from Refs. [11, 12]. For the bins $\{1\}$ and $\{2\}$, the spread is $\Delta\alpha_s^{\mathrm{PDF}} = \pm 0.017$, and for the bins $\{2\}$ and $\{3\}$, it is ± 0.005. Future global fits of parton densities including improved HERA data should reduce this systematic uncertainty.

Perturbative QCD allows for some freedom in the choice of the factorization scale μ of the fragmentation functions $D(z, \mu^2)$. This brings out the inherent uncertainty in the theoretical prediction, and can be interpreted as an effect of unknown higher order contributions. To obtain an estimate of this uncertainty, the ratios λ are determined for the three choices $Q/2$, Q and $2Q$ of this scale. The change of cross-section has for consequence a variation in the extracted $\alpha_s(M_Z^2)$ value of $\Delta\alpha_s^{\mathrm{scale}} = \pm 0.013$ and ± 0.011 for the combinations of the bins $\{1\}$, $\{2\}$ and $\{2\}$, $\{3\}$, respectively.

3 Summary and Conclusions

We have studied the prospects of a measurement of the strong coupling constant in deeply inelastic scattering at HERA by means of scaling violations of fragmentation functions. The combinations of the obtained values for $\Delta\alpha_s^{\rm stat}$, $\Delta\alpha_s^{\rm PDF}$ and $\Delta\alpha_s^{\rm scale}$ are large compared with the present error $\Delta\alpha_s = 0.006$ of the world average. It is therefore likely that a measurement of this kind will not be competitive, concerning the size of the error. Nevertheless, it is worth doing as an independent quantitative test of QCD and, more important, because it complements the other (potential) HERA measurements based on (2+1) jet rates and scaling violations of structure functions.

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References

[1] T. Ahmed et al. (H1 Collaboration), Phys. Lett. B346 (1995) 415; M. Derrick et al. (ZEUS Collaboration), Phys. Lett. B363 (1995) 201.

[2] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.

[3] R. Baier and K. Fey, Z. Phys. C2 (1979) 339.

[4] J.C. Collins, D.E. Soper and G. Sterman, in: *Perturbative Quantum Chromodynamics,* ed. A.H. Mueller (World Scientific, Singapore, 1989).

[5] A. Vogt, Phys. Lett. B354 (1995) 145; A.D. Martin, W.J. Stirling and R.G. Roberts, Rutherford Appleton Lab. preprint RAL-TR-95-013 (June 1995).

[6] G. Altarelli, R.K. Ellis, G. Martinelli and S.Y. Pi, Nucl. Phys. B160 (1979) 301.

[7] D. Graudenz, *Heavy-Quark Production in the Target Fragmentation Region,* preprint CERN–TH/96–52, in preparation.

[8] I. Abt et al. (H1 Collaboration) Z. Phys. C63 (1994) 377; M. Derrick et al. (ZEUS Collaboration), Z. Phys. C70 (1996) 1.

[9] D. Graudenz, *Charged-Meson Production and Scaling Violations of Fragmentation Functions in Deeply Inelastic Scattering at HERA,* preprint CERN–TH/96–155 (June 1996).

[10] J. Binnewies, B.A. Kniehl and G. Kramer, Z. Phys. C65 (1995) 471; Phys. Rev. D52 (1995) 4947.

[11] CTEQ Collaboration, Phys. Rev. D51 (1995) 4763.

[12] M. Glück, E. Reya and A. Vogt, Z. Phys. C67 (1995) 433; A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Rev. D51 (1995) 4756.