NLO QCD calculations in DIS at HERA based on the dipole formalism

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Abstract: We briefly describe a new general algorithm for carrying out QCD calculations to next-to-leading order in perturbation theory. The algorithm can be used for computing arbitrary jet cross sections in arbitrary processes and can be straightforwardly implemented in general-purpose Monte Carlo programs. We show numerical results for the specific case of jet cross sections in deep inelastic scattering at HERA energies.

1 Introduction

In order to make quantitative predictions in perturbative QCD, it is essential to work to (at least) next-to-leading order (NLO). However, this is far from straightforward because for all but the simplest quantities, the necessary phase-space integrals are too difficult to do analytically, making numerical methods essential. But the individual integrals are divergent, and only after they have been regularized and combined is the result finite. The usual prescription, dimensional regularization, involves working in a fractional number of dimensions, making analytical methods essential.

To avoid this dilemma, one must somehow set up the calculation such that the singular parts can be treated analytically, while the full complexity of the integrals can be treated numerically. Efficient techniques have been set up to do this, at least to NLO, during the last few years.

A new general algorithm was recently presented [1], which can be used to compute arbitrary jet cross sections in arbitrary processes. It is based on two key ingredients: the *subtraction method* for cancelling the divergences between different contributions; and the *dipole factorization theorems* for the universal (process-independent) analytical treatment of individual divergent terms. These are sufficient to write a general-purpose Monte Carlo program in which any jet quantity can be calculated simply by making the appropriate histogram in a user routine.

In this contribution we give a brief summary of these two ingredients (more details and references to other general methods can be found in Refs. [1]–[3]) and show numerical results for the specific case of jets in deep-inelastic lepton-hadron scattering (DIS) at HERA energies.

A Monte Carlo program for jet calculations in DIS based on a different method [4] is presented in Ref. [5]. Previous calculations [6, 7] were limited to a specific jet definition and used approximated methods.

2 The Dipole Formalism

2.1 The subtraction method

Considering the case of DIS (full details for other processes are given in Ref. [1]), the general structure of a QCD cross section in NLO is:

$$\sigma = \sum_a f_a \times (\sigma_a^{LO} + \sigma_a^{NLO}) \tag{1}$$

where the symbol $\times$ denotes the convolution with the density f_a of the parton a in the incoming hadron and the leading-order (LO) cross section σ_a^{LO} is obtained by integrating the fully exclusive Born cross section $d\sigma_a^B$ over the phase space for the corresponding jet quantity. We suppose that this LO calculation involves m final-state partons, and write:

$$\sigma_a^{LO} = \int_m d\sigma_a^B \ . \tag{2}$$

At NLO, we receive contributions from real and virtual processes (we assume that the ultraviolet divergences of the virtual term are already renormalized) and from a collinear counterterm that is necessary to define the scale-dependent parton densities:

$$\sigma_a^{NLO} = \int_{m+1} d\sigma_a^R + \int_m \left[d\sigma_a^V + d\sigma_a^C \right] \ . \tag{3}$$

As is well known, each of these is separately divergent, although their sum is finite. These divergences are regulated by working in $d = 4 - 2\epsilon$ dimensions, where they are replaced by singularities in $1/\epsilon$. Their cancellation only becomes manifest once the separate phase space integrals have been performed.

The essence of the subtraction method [8] is to use the *exact* identity

$$\sigma_a^{NLO} = \int_{m+1} \left[d\sigma_a^R - d\sigma_a^A \right] + \int_{m+1} d\sigma_a^A + \int_m \left[d\sigma_a^V + d\sigma_a^C \right] \ , \tag{4}$$

which is obtained by subtracting and adding back the 'approximate' (or 'fake') cross section contribution $d\sigma_a^A$, which has to fulfil two main properties. Firstly, it must exactly match the singular behaviour (in d dimensions) of $d\sigma_a^R$ itself. Thus it acts as a *local* counterterm for $d\sigma_a^R$ and one can safely perform the limit $\epsilon \to 0$ under the integral sign in the first term on the right-hand side of Eq. (4). Secondly, $d\sigma_a^A$ must be analytically integrable (in d dimensions) over the one-parton subspace leading to the divergences. Thus we can rewrite the integral in the last term of Eq. (4), to obtain

$$\sigma_a^{NLO} = \int_{m+1} \left[\left(d\sigma_a^R \right)_{\epsilon=0} - \left(d\sigma_a^A \right)_{\epsilon=0} \right] + \int_m \left[d\sigma_a^V + d\sigma_a^C + \int_1 d\sigma_a^A \right]_{\epsilon=0} \ . \tag{5}$$

Performing the analytic integration $\int_1 d\sigma_a^A$, one obtains ϵ-pole contributions that can be combined with those in $d\sigma_a^V$ and $d\sigma_a^C$, thus cancelling all the divergences. Equation (5) can be easily implemented in a 'partonic Monte Carlo' program that generates appropriately weighted partonic events with $m+1$ final-state partons and events with m partons.

2.2 Dipole factorization and universal subtraction term

The fake cross section $d\sigma_a^A$ can be constructed in a fully process-independent way, by using the factorizing properties of gauge theories. Specifically, in the soft and collinear limits, which give rise to the divergences, the factorization theorems can be used to write the cross section as the contraction of the Born cross section with universal soft and collinear factors (provided that colour and spin correlations are retained). However, these theorems are only valid in the exactly singular limits, and great care should be used in extrapolating them away from these limits. In particular, a careful treatment of momentum conservation is required. Care has also to be taken in order to avoid double counting the soft and collinear divergences in their overlapping region (e.g. when a gluon is both soft and collinear to another parton). The use of the dipole factorization theorem introduced in Ref. [2] allows one to overcome these difficulties in a straightforward way.

The dipole factorization formulae relate the singular behaviour of $\mathcal{M}_{m+1}$, the tree-level matrix element with $m+1$ partons, to $\mathcal{M}_m$. They have the following symbolic structure:

$$|\mathcal{M}_{m+1}(p_1,...,p_{m+1})|^2 = |\mathcal{M}_m(\tilde{p}_1,...,\tilde{p}_m)|^2 \otimes \boldsymbol{V}_{ij} + \dots \ . \tag{6}$$

The dots on the right-hand side stand for contributions that are not singular when $p_i \cdot p_j \to 0$. The dipole splitting functions $\boldsymbol{V}_{ij}$ are universal (process-independent) singular factors that depend on the momenta and quantum numbers of the m partons in the tree-level matrix element $|\mathcal{M}_m|^2$. Colour and helicity correlations are denoted by the symbol $\otimes$. The set $\tilde{p}_1,...,\tilde{p}_m$ of modified momenta on the right-hand side of Eq. (6) is defined starting from the original $m+1$ parton momenta in such a way that the m partons in $|\mathcal{M}_m|^2$ are physical, that is, they are on-shell and energy-momentum conservation is implemented exactly. The detailed expressions for these parton momenta and for the dipole splitting functions are given in Ref. [1].

Equation (6) provides a *single* formula that approximates the real matrix element $|\mathcal{M}_{m+1}|^2$ for an arbitrary process, in *all* of its singular limits. These limits are approached smoothly, avoiding double counting of overlapping soft and collinear singularities. Furthermore, the precise definition of the m modified momenta allows an *exact* factorization of the $m+1$-parton phase space, so that the universal dipole splitting function can be integrated once and for all.

This factorization, which is valid for the total phase space, is not sufficient to provide a universal fake cross section however, as its phase space should depend on the particular jet observable being considered. The fact that the m parton momenta are physical provides a simple way to implement this dependence. We construct $d\sigma_a^A$ by adding the dipole contributions on the right-hand side of Eq. (6) and for each contribution we calculate the jet observable not from the original $m+1$ parton momenta, but from the corresponding m parton momenta, $\tilde{p}_1,...,\tilde{p}_m$. Since these are fixed during the analytical integration, it can be performed without any knowledge of the jet observable.

3 Final Results

Refering to Eq. (5), the final procedure is then straightforward. The calculation of any jet quantity to NLO consists of an $m+1$-parton integral and an m-parton integral. These can be performed separately using standard Monte Carlo methods.

For the $m+1$-parton integral, a phase-space point is generated and the corresponding real matrix element in $d\sigma_a^R$ is calculated. These are passed to a user routine, which can analyse the event in any way and histogram any quantities of interest. Next, for each dipole term (there are at most 10 of them in the calculation of (2+1)-jet observables in DIS) in $d\sigma_a^A$, the set of m parton momenta is derived from the same phase-space point and the corresponding dipole contribution is calculated. These are also given to the user routine. They are such that for any singular $m+1$-parton configuration, one or more of the m-parton configurations becomes indistinguishable from it, so that they fall in the same bin of any histogram. Simultaneously, the real matrix element and dipole term will have equal and opposite weights, so that the total contribution to that histogram bin is finite. Thus the first integral of Eq. (5) is finite.

The m-parton integral in Eq. (5) has a simpler structure: it is identical to the LO integration in Eq. (2), but with the Born term replaced by the finite sum of the virtual matrix element in $d\sigma_a^V$, the collinear counterterm $d\sigma_a^C$ and the analytical integral of the dipole contributions in $d\sigma_a^A$ (to be precise, the second term on the right-hand side of Eq. (5) involves an additional one-dimensional convolution [1], which is a finite remainder that is left after the cancellation of the singularities in $d\sigma_a^C$).

Note that our algorithm does not require the convolution with the parton densities f_a to be made during Monte Carlo integration. One is free to choose either to calculate the hadron-level cross section in Eq. (1), thus including the convolution, or the parton-level cross section $\sigma_a = \sigma_a^{LO} + \sigma_a^{NLO}$ as a function of the partonic momentum fraction. The latter can then be convoluted with the parton densities after Monte Carlo integration. This can be extremely useful in many respects. For instance, one can produce cross sections with a wide variety of parton densities, or study the scheme- and scale-dependence of the results without having to reintegrate for each new scheme or scale.

For the specific case of jet observables in DIS, we have implemented the algorithm as a Monte Carlo program, which can be obtained from `http://suryall.cern.ch/users/seymour/nlo/`. The program uses the matrix elements evaluated by the Leiden group [9]. In Fig. 1a we show as an example the differential jet rate as a function of jet resolution parameter, f_{cut}, using the $k_\perp$ jet algorithm [10]. We see that the NLO corrections are generally small and positive, except at very small f_{cut} (where large logarithmic terms, $-\alpha_s \log^2 f_{cut}$ arise at each higher order). In Fig. 1b, we show the variation of the jet rate at a fixed f_{cut} with factorization and renormalization scales. The scale dependence is considerably smaller at NLO.

4 Conclusion

The subtraction method provides an *exact* way to calculate arbitrary quantities in a given process using a general purpose Monte Carlo program. The dipole formalism provides a way to construct such a program from process-independent components. Recent applications have included jets in DIS.

We have constructed a Monte Carlo program that can be used to carry out NLO QCD calculations for any infrared- and collinear-safe observable (jet cross sections using different jet algorithms, event shapes, energy correlations and so forth) in $(2+1)$-jet configurations in DIS. Possible applications to future physics at HERA include determinations of the strong coupling constant $\alpha_S(Q)$ and extraction of the parton densities. More details of the program, and its results, will be given elsewhere.

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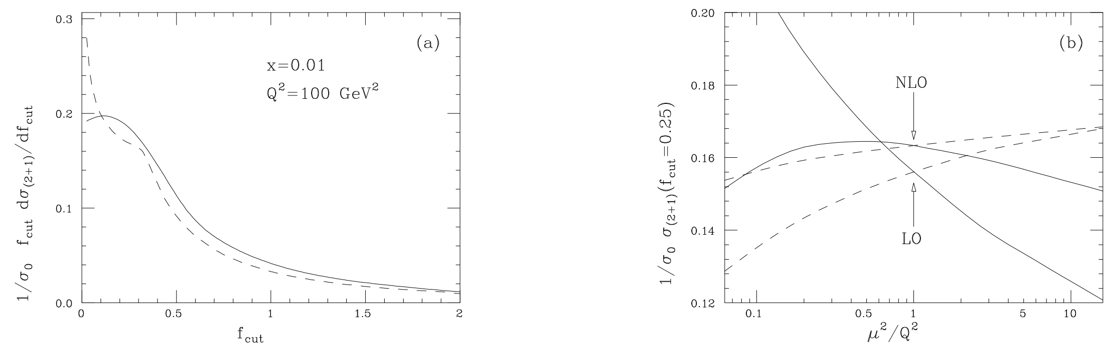

Figure 1: Jet cross sections in ep collisions at HERA energies ($\sqrt{s}$ = 300 GeV). (a) The distribution of resolution parameter f_{cut} at which DIS events are resolved into $(2+1)$ jets according to the $k_\perp$ jet algorithm. Curves are LO (dashed) and NLO (solid) using factorization and renormalization scales equal to Q^2, and the MRS D$-'$ distribution functions. Both curves are normalized to the LO cross section. (b) The rate of events with exactly $(2+1)$ jets at f_{cut} = 0.25 with variation of renormalization (solid) and factorization (dashed) scales. Normalization is again the LO cross section with fixed factorization scale.

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