Rare B Decays in the Standard Model

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Abstract: We discuss the electromagnetic-penguin-dominated radiative B decays $B \to X_s + \gamma$, $B^{\pm(0)} \to K^{*\pm(0)} + \gamma$, $B_s \to \phi + \gamma$ and $\Lambda_b \to \Lambda + \gamma$ in the context of the standard model (SM) and their Cabibbo-Kobayashi-Maskawa (CKM)-suppressed counterparts, $B \to X_d + \gamma$, $B^\pm \to \rho^\pm + \gamma$, $B^0 \to (\rho^0, \omega) + \gamma$, $B_s \to K^{*0} + \gamma$, and $\Lambda \to n + \gamma$, using QCD sum rules for the exclusive decays. The importance of these decays in determining the parameters of the CKM matrix [1] is emphasized. The semileptonic decays $B \to X_s \ell^+ \ell^-$ are also discussed in the context of the SM and their role in determining the Wilson coefficients of the effective theory is stressed. Comparison with the existing measurements are made and SM-based predictions for a large number of rare B decays are presented.

1 Estimates of $\mathcal{B}(B \to X_s + \gamma)$ and $|V_{ts}|$ in the Standard Model

The Standard Model (SM) of particle physics does not admit Flavour-changing-neutral-current (FCNC) transitions in the Born approximation. However, they are induced through the exchange of $W^\pm$ bosons in loop diagrams. The short-distance contribution in rare decays is dominated by the (virtual) top quark contribution. Hence the decay characteristics provide quantitative information on the top quark mass and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{ti}; $i = d, s, b$ [1]. We shall discuss representative examples from several such transitions involving B decays, starting with the decay $B \to X_s + \gamma$, which has been measured by CLEO [2]. This was preceded by the measurement of the exclusive decay $B \to K^\star + \gamma$ by the same collaboration [3]. The present measurements give [4]:

$$\mathcal{B}(B \to X_s + \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \; , \tag{1}$$

$$\mathcal{B}(B \to K^\star + \gamma) = (4.2 \pm 0.8 \pm 0.6) \times 10^{-5} \; , \tag{2}$$

yielding an exclusive-to-inclusive ratio:

$$R_{K^*} = \frac{\Gamma(B \to K^\star + \gamma)}{\Gamma(B \to X_s + \gamma)} = (18.1 \pm 6.8)\% \; . \tag{3}$$

These decay rates determine the ratio of the CKM matrix elements $|V_{ts}|/|V_{cb}|$ and the quantity R_{K^*} provides information on the decay form factor in $B \to K^\star + \gamma$. In what follows we take up these points briefly.

The leading contribution to $b \to s+\gamma$ arises at one-loop from the so-called penguin diagrams. With the help of the unitarity of the CKM matrix, the decay matrix element in the lowest order can be written as:

$$\mathcal{M}(b \to s \ + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \lambda_t \left(F_2(x_t) - F_2(x_c) \right) q^\mu \epsilon^\nu \bar{s} \sigma_{\mu\nu} (m_b R \ + \ m_s L) b \ . \qquad (4)$$

where $x_i = \ m_i^2/m_W^2$, and q_μ and ϵ_μ are, respectively, the photon four-momentum and polarization vector. The GIM mechanism [5] is manifest in this amplitude and the CKM-matrix element dependence is factorized in $\lambda_t \equiv V_{tb}V_{ts}^*$. The (modified) Inami-Lim function $F_2(x_i)$ derived from the (1-loop) penguin diagrams is given by [6]:

$$F_2(x) = \frac{x}{24(x-1)^4} \ \left[6x(3x-2)\log x - (x-1)(8x^2+5x-7) \right] . \qquad (5)$$

The measurement of the branching ratio for $B \to X_s + \gamma$ can be readily interpreted in terms of the CKM-matrix element product $\lambda_t/|V_{cb}|$ or equivalently $|V_{ts}|/|V_{cb}|$. For a quantitative determination of $|V_{ts}|/|V_{cb}|$, however, QCD radiative corrections have to be computed and the contribution of the so-called long-distance effects estimated.

The appropriate framework to incorporate QCD corrections is that of an effective theory obtained by integrating out the heavy degrees of freedom, which in the present context are the top quark and $W^\pm$ bosons. The operator basis depends on the underlying theory and for the SM one has (keeping operators up to dimension 6),

$$\mathcal{H}_{eff}(b \to s + \gamma) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) \mathcal{O}_i(\mu), \qquad (6)$$

where the operator basis, the "matching conditions" $C_i(m_W)$, and the solutions of the renormalization group equations $C_i(\mu)$ can be seen in ref. [7]. The perturbative QCD corrections to the decay rate $\Gamma(B \to X_s + \gamma)$ have two distinct contributions:

- Corrections to the Wilson coefficients $C_i(\mu)$, calculated with the help of the renormalization group equation, whose solution requires the knowledge of the anomalous dimension matrix in a given order in α_s.
- Corrections to the matrix elements of the operators $\mathcal{O}_i$ entering through the effective Hamiltonian at the scale $\mu = O(m_b)$.

The anomalous dimension matrix is needed in order to sum up large logarithms, i.e., terms like $\alpha_s^n(m_W)\log^m(m_b/M)$, where $M = m_t$ or m_W and $m \leq n$ (with $n = 0, 1, 2, ...$). At present only the leading logarithmic corrections ($m = n$) have been calculated systematically and checked by several independent groups in the complete basis given in Eq. (6) [8]. First calculations of the NLO corrections to the anomalous dimension matrix have been recently reported by Misiak [9] and are found to be small. Next-to-leading order corrections to the matrix elements are now available completely. They are of two kinds:

- QCD Bremsstrahlung corrections $b \to s\gamma + g$, which are needed both to cancel the infrared divergences in the decay rate for $B \to X_s + \gamma$ and in obtaining a non-trivial QCD contribution to the photon energy spectrum in the inclusive decay $B \to X_s + \gamma$.

- Next-to-leading order virtual corrections to the matrix elements in the decay $b \to s + \gamma$.

The Bremsstrahlung corrections were calculated in [10, 11] in the truncated basis and last year also in the complete operator basis [12, 13]. The higher order matching conditions, i.e., $C_i(m_W)$, are known up to the desired accuracy, i.e., up to $O(\alpha_s(M_W))$ terms [14]. The next-to-leading order virtual corrections have also been calculated [15]. They reduce the scale-dependence of the inclusive decay width. The branching ratio $\mathcal{B}(B \to X_s + \gamma)$ can be expressed in terms of the semileptonic decay branching ratio

$$\mathcal{B}(B \to X_s\gamma) = [\frac{\Gamma(B \to \gamma + X_s)}{\Gamma_{SL}}]^{th} \mathcal{B}(B \to X \ell \nu_\ell) \qquad , \tag{7}$$

where the leading-order QCD corrected expression for Γ_{SL} can be seen in [7]. The leading order $(1/m_b)$ power corrections in the heavy quark expansion are identical in the inclusive decay rates for $B \to X_s + \gamma$ and $B \to X\ell\nu_\ell$, entering in the numerator and denominator in the square bracket, respectively, and hence drop out [16, 17].

In Ref. [7], the present theoretical errors on the branching ratio $\mathcal{B}(B \to X_s\gamma)$ are discussed, yielding:

$$\mathcal{B}(B \to X_s + \gamma) = (3.20 \pm 0.30 \pm 0.38 \pm 0.32) \times 10^{-4} \tag{8}$$

where the first error comes from the combined effect of Δm_t and $\Delta\mu$ (the scale dependence), the second error arises from the extrinsic sources (such as $\Delta(m_b)$, $\Delta(BR_{SL})$), and the third error is an estimate ($\pm 10\%$) of the NLO anomalous dimension piece in C_7^{eff}, the coefficient of the magnetic moment operator. Combining the theoretical errors in quadrature gives [7]:

$$\mathcal{B}(B \to X_s + \gamma) = (3.20 \pm 0.58) \times 10^{-4}, \tag{9}$$

which is compatible with the present measurement $\mathcal{B}(B \to X_s + \gamma) = (2.32 \pm 0.67) \times 10^{-4}$ [2]. Expressed in terms of the CKM matrix element ratio, one gets

$$\frac{|V_{ts}|}{|V_{cb}|} = 0.85 \pm 0.12(\mathrm{expt}) \pm 0.10(\mathrm{th}), \tag{10}$$

which is within errors consistent with unity, as expected from the unitarity of the CKM matrix.

In order to get the complete amplitude for $B \to X_s + \gamma$ one has to include also the effects of the long-distance (LD) contributions, which arise from the matrix elements of the four-quark operators in $\mathcal{H}_{eff}$, $\langle X_s\gamma|\mathcal{O}_i|B\rangle$. For the exclusive radiative decays $B \to (\rho, \omega) + \gamma$, these contributions have been calculated using QCD sum rules [18, 19] and we shall discuss them later. Concerning the inclusive decays $B \to X_s + \gamma$, one has at present little choice other than invoking phenomenological models to estimate the LD-amplitude. They have been used in a number of papers [20], which typically give $\mathcal{M}(B \to X_s + \gamma)^{\mathrm{LD}}/\mathcal{M}(B \to X_s + \gamma)^{\mathrm{SD}} \leq O(0.1)$, based on which we shall ignore the LD-effects in the decays $B \to X_s + \gamma$ (likewise, in $B \to K^* + \gamma$).

2 Inclusive radiative decays $B \to X_d + \gamma$

The theoretical interest in studying the (CKM-suppressed) inclusive radiative decays $B \to X_d + \gamma$ lies in the first place in the possibility of determining the parameters of the CKM matrix. We shall use the Wolfenstein parametrization [21], in which case the matrix is determined in terms of the four parameters $A, \lambda = \sin\theta_C$, ρ and η. Since A and λ are well determined, and ρ and η are bounded from present measurements (in particular, from B_d^0 - $\overline{B_d^0}$ frequency x_d and $|\epsilon|$, the CP violation parameter in K decays) [22, 23], we shall concentrate on the improved determination of the last two parameters from radiative B decays. With that goal in view, one of the relevant quantities in the decays $B \to X_d + \gamma$ is the end-point photon energy spectrum, which has to be measured requiring that the hadronic system X_d recoiling against the photon does not contain strange hadrons to suppress the large-E_γ photons from the decay $B \to X_s + \gamma$. Assuming that this is feasible, one can determine from the ratio of the decay rates $\mathcal{B}(B \to X_d + \gamma)/\mathcal{B}(B \to X_s + \gamma)$ the CKM-Wolfenstein parameters ρ and η. This measurement was first proposed in [11], where the photon energy spectra were also worked out.

In close analogy with the $B \to X_s + \gamma$ case discussed earlier, the complete set of dimension-6 operators relevant for the processes $b \to d\gamma$ and $b \to d\gamma g$ can be written as:

$$\mathcal{H}_{eff}(b \to d) = -\frac{4G_F}{\sqrt{2}}\, \xi_t \sum_{j=1}^{8} C_j(\mu)\, \hat{O}_j(\mu), \tag{11}$$

where $\xi_j = V_{jb} V_{jd}^*$ for $j = t, c, u$. The operators $\hat{O}_j$, $j = 1, 2$, have implicit in them CKM factors. In the Wolfenstein parametrization [21], one can express these factors as :

$$\xi_u = A\,\lambda^3\,(\rho - i\eta), \quad \xi_c = -A\,\lambda^3, \quad \xi_t = -\xi_u - \xi_c. \tag{12}$$

We note that all three CKM-angle-dependent quantities ξ_j are of the same order of magnitude, $O(\lambda^3)$. This aspect can be taken into account by defining the operators $\hat{O}_1$ and $\hat{O}_2$ entering in $\mathcal{H}_{eff}(b \to d)$ as follows [11]:

$$\begin{aligned}
\hat{O}_1 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t}(\bar{u}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu u_{L\beta}), \\
\hat{O}_2 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t}(\bar{u}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu u_{L\beta}),
\end{aligned} \tag{13}$$

with the rest of the operators ($\hat{O}_j$; $j = 3...8$) defined like their counterparts O_j in $\mathcal{H}_{eff}(b \to s)$, with the obvious replacement $s \to d$. With this choice, the matching conditions $C_j(m_W)$ and the solutions of the RG equations yielding $C_j(\mu)$ become identical for the two operator bases O_j and $\hat{O}_j$. The essential difference between $\Gamma(B \to X_s + \gamma)$ and $\Gamma(B \to X_d + \gamma)$ lies in the matrix elements of the first two operators O_1 and O_2 (in $\mathcal{H}_{eff}(b \to s)$) and $\hat{O}_1$ and $\hat{O}_2$ (in $\mathcal{H}_{eff}(b \to d)$). The branching ratio $\mathcal{B}(B \to X_d + \gamma)$ in the SM can be written as:

$$\mathcal{B}(B \to X_d + \gamma) = D_1 \lambda^2 \{(1-\rho)^2 + \eta^2 - (1-\rho) D_2 - \eta D_3 + D_4\}, \tag{14}$$

where the functions D_i depend on the parameters m_t, m_b, m_c, μ, as well as the others we discussed in the context of $\mathcal{B}(B \to X_s + \gamma)$. These functions were first calculated in [11] in the leading logarithmic approximation. Recently, these estimates have been improved in [24], making use of the NLO calculations in [15]. For the central values of these parameters (m_t = 175 GeV, m_b = 4.8 GeV, $m_c/m_b = 0.29$, μ = 5 GeV, and $\alpha_s(m_b) = 0.212$), one gets $D_1 = 4.5 \times 10^{-4}$, $D_2 = 0.30$, $D_3 = 0.23$, and $D_4 = 0.06$ [24]. To get an estimate of the inclusive branching ratio, the CKM parameters ρ and η have to be constrained from the unitarity fits. Present data and theory restrict the parameters ρ and η to lie in the following range (at 95% C.L.) [22]:

$$\begin{aligned} 0.25 \leq \eta \leq 0.52, \\ -0.35 \leq \rho \leq 0.35 \ , \end{aligned} \tag{15}$$

which, on using the current lower bound from LEP on the B_s^0 - $\overline{B_s^0}$ mass difference $\Delta M_s > 9.2$ (ps)$^{(-1)}$ [25], is reduced to $-0.14 \leq \rho \leq 0.35$ using $\xi_s = 1.1$ where ξ_s is the $SU(3)$-breaking parameter $\xi_s = f_{B_s}\hat{B}_{B_s}/f_{B_d}\hat{B}_{B_d}$. The preferred CKM-fit values are

$$(\rho, \eta) = (0.05, 0.36) \quad (\text{with } \chi^2 = 6.6 \times 10^{-3}) \ , \tag{16}$$

Taking the preferred values of the fitted CKM parameters one gets [24]

$$\mathcal{B}(B \to X_d + \gamma) = 1.63 \times 10^{-5}, \tag{17}$$

whereas $\mathcal{B}(B \to X_d + \gamma) = 8.0 \times 10^{-6}$ and 2.8×10^{-5} for the other two extremes $\rho = 0.35$, $\eta = 0.50$ and $\rho = -\eta = -0.25$, respectively. In conclusion, we note that the functional dependence of $\mathcal{B}(B \to X_d + \gamma)$ on the Wolfenstein parameters (ρ, η) is mathematically different than that of ΔM_s. However, qualitatively they are very similar. From the experimental point of view, the situation $\rho < 0$ is favourable for both the measurements as in this case one expects (relatively) smaller values for ΔM_s and larger values for the branching ratio $\mathcal{B}(B \to X_d + \gamma)$, as compared to the $\rho > 0$ case which would yield larger ΔM_s and smaller $\mathcal{B}(B \to X_d + \gamma)$.

2.1 $\mathcal{B}(B \to V + \gamma)$ and constraints on the CKM parameters

Exclusive radiative B decays $B \to V + \gamma$, with $V = K^*, \rho, \omega$, are also potentially very interesting from the point of view of determining the CKM parameters [26]. The extraction of these parameters would, however, involve a trustworthy estimate of the SD- and LD-contributions in the decay amplitudes.

The SD-contribution in the exclusive decays $(B^\pm, B^0) \to (K^{*\pm}, K^{*0}) + \gamma$, $(B^\pm, B^0) \to (\rho^\pm, \rho^0) + \gamma$, $B^0 \to \omega + \gamma$ and the corresponding B_s decays, $B_s \to \phi + \gamma$, and $B_s \to K^{*0} + \gamma$, involve the magnetic moment operator $\mathcal{O}_7$ and the related one obtained by the obvious change $s \to d$, $\hat{O}_7$. The transition form factors governing these decays can be generically defined as:

$$\langle V, \lambda | \frac{1}{2} \bar{\psi} \sigma_{\mu\nu} q^\nu b | B \rangle = i \epsilon_{\mu\nu\rho\sigma} e_\nu^{(\lambda)} p_B^\rho p_V^\sigma F_S^{B \to V}(0). \tag{18}$$

Here V is a vector meson with the polarization vector $e^{(\lambda)}$, $V = \rho, \omega, K^*$ or ϕ; B is a generic B-meson $B^\pm, B^0$ or B_s, and ψ stands for the field of a light u, d or s quark. The vectors p_B, p_V

and $q = p_B - p_V$ correspond to the 4-momenta of the initial B-meson and the outgoing vector meson and photon, respectively. Keeping only the SD-contribution leads to obvious relations among the exclusive decay rates, exemplified here by the decay rates for $(B^\pm, B^0) \to (\rho^\pm, \rho^0) + \gamma$ and $(B^\pm, B^0) \to (K^{*\pm}, K^{*0}) + \gamma$:

$$\frac{\Gamma((B^\pm, B^0) \to (\rho^\pm, \rho^0) + \gamma)}{\Gamma((B^\pm, B^0) \to (K^{*\pm}, K^{*0}) + \gamma)} = \frac{|\xi_t|^2}{|\lambda_t|^2} \frac{|F_S^{B\to\rho}(0)|^2}{|F_S^{B\to K^*}(0)|^2} \Phi_{u,d} \simeq \kappa_{u,d} \left[\frac{|V_{td}|}{|V_{ts}|}\right]^2 , \tag{19}$$

where $\Phi_{u,d}$ is a phase-space factor which in all cases is close to 1 and $\kappa_i \equiv [F_S(B_i \to \rho\gamma)/F_S(B_i \to K^*\gamma)]^2$. The transition form factors F_S are model dependent. Estimates of F_S in the QCD sum rule approach in the normalization of Eq. (18) range between $F_S(B \to K^*\gamma) = 0.31$ (Narison in [27]) to $F_S(B \to K^*\gamma) = 0.37$ (Ball in [27]), with a typical error of $\pm 15\%$, and hence are all consistent with each other. This, for example, gives $R_{K^*} = 0.16 \pm 0.05$, using the result from [26], which is in good agreement with data. The ratios of the form factors, i.e. κ_i, should therefore be reliably calculable as they depend essentially only on the SU(3)-breaking effects.

If the SD-amplitudes were the only contributions, the measurements of the CKM-suppressed radiative decays $(B^\pm, B^0) \to (\rho^\pm, \rho^0) + \gamma$, $B^0 \to \omega + \gamma$ and $B_s \to K^* + \gamma$ could be used in conjunction with the decays $(B^\pm, B^0) \to (K^{*\pm}, K^{*0}) + \gamma$ to determine the CKM parameters. The present experimental upper limits on the CKM ratio $|V_{td}|/|V_{ts}|$ from radiative B decays are indeed based on this assumption. The 90% C.L. limits on some of the decay modes are given in Table 1, which yield at 90% C.L.[4]:

$$\left|\frac{V_{td}}{V_{ts}}\right| \leq 0.45 - 0.56 , \tag{20}$$

depending on the models used for the $SU(3)$ breaking effects in the form factors [26, 27].

The possibility of significant LD-contributions in radiative B decays from the light quark intermediate states has been raised in a number of papers [20]. Their amplitudes necessarily involve other CKM matrix elements and hence the simple factorization of the decay rates in terms of the CKM factors involving $|V_{td}|$ and $|V_{ts}|$ no longer holds thereby invalidating the relation (19) given above. As we already discussed, the LD-contributions are small in the exclusive decays $B \to K^* + \gamma$ and so this issue hinges sensitively upon the LD-contributions in the CKM-suppressed charged and neutral exclusive B-decays, $B^\pm \to \rho^\pm\gamma$ and $B \to (\rho^0, \omega)\gamma$.

The LD-contributions in $B \to V + \gamma$ are induced by the matrix elements of the four-Fermion operators $\hat{O}_1$ and $\hat{O}_2$ (likewise O_1 and O_2). Estimates of these contributions require non-perturbative methods. This problem has been investigated in [18, 19] using a technique [28] which treats the photon emission from the light quarks in a theoretically consistent and model-independent way. This has been combined with the light-cone QCD sum rule approach to calculate both the SD and LD — parity conserving and parity violating — amplitudes in the decays $(B^\pm, B^0) \to (\rho^\pm, \rho/\omega) + \gamma$. To illustrate this, we concentrate on the $B^\pm$ decays, $B^\pm \to \rho^\pm + \gamma$ and take up the neutral B decays $B^0 \to \rho(\omega) + \gamma$ at the end. The LD-amplitude of the four-Fermion operators $\hat{O}_1$, $\hat{O}_2$ is dominated by the contribution of the weak annihilation of valence quarks in the B meson and it is color-allowed for the decays of charged $B^\pm$ mesons. Using factorization, the LD-amplitude in the decay $B^\pm \to \rho^\pm + \gamma$ can be written in terms of the form factors F_1^L and F_2^L,

$$\mathcal{A}_{long} = -\frac{e\,G_F}{\sqrt{2}} V_{ub} V_{ud}^* \left(C_2 + \frac{1}{N_c} C_1 \right) m_\rho \varepsilon_\mu^{(\gamma)} \varepsilon_\nu^{(\rho)} \times \left\{ -i \left[g^{\mu\nu} (q \cdot p) - p^\mu q^\nu \right] \cdot 2F_1^L(q^2) + \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \cdot 2F_2^L(q^2) \right\} . \qquad (21)$$

Again, one has to invoke a model to calculate the form factors. Estimates from the light-cone QCD sum rules give [19]:

$$F_1^L/F_S = 0.0125 \pm 0.0010\,, \quad F_2^L/F_S = 0.0155 \pm 0.0010\ , \qquad (22)$$

where the errors correspond to the variation of the Borel parameter in the QCD sum rules. Including other possible uncertainties, one expects an accuracy of the ratios in (22) of order 20%. The parity-conserving and parity-violating amplitudes turn out to be numerically close to each other in the QCD sum rule appraoch, $F_1^L \simeq F_2^L \equiv F_L$, hence the ratio of the LD- and the SD- contributions reduces to a number [19]

$$\mathcal{A}_{long}/\mathcal{A}_{short} = R_{L/S}^{B^\pm \to \rho^\pm \gamma} \cdot \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*}\ . \qquad (23)$$

Using $C_2 = 1.10$, $C_1 = -0.235$, $C_7^{eff} = -0.306$ from Ref. [7] (corresponding to the scale $\mu = 5$ GeV) gives:

$$R_{L/S}^{B^\pm \to \rho^\pm \gamma} \equiv \frac{4\pi^2 m_\rho (C_2 + C_1/N_c)}{m_b C_7^{eff}} \cdot \frac{F_L^{B^\pm \to \rho^\pm \gamma}}{F_S^{B^\pm \to \rho^\pm \gamma}} = -0.30 \pm 0.07\ , \qquad (24)$$

which is not small. To get a ball-park estimate of the ratio $\mathcal{A}_{long}/\mathcal{A}_{short}$, we take the central value from the CKM fits, yielding $|V_{ub}|/|V_{td}| \simeq 0.33$ [22],

$$|\mathcal{A}_{long}/\mathcal{A}_{short}|^{B^\pm \to \rho^\pm \gamma} = |R_{L/S}^{B^\pm \to \rho^\pm \gamma}| \frac{|V_{ub} V_{ud}|}{|V_{td} V_{tb}|} \simeq 10\%\ . \qquad (25)$$

Thus, the CKM factors suppress the LD-contributions.

The analogous LD-contributions to the neutral B decays $B^0 \to \rho\gamma$ and $B^0 \to \omega\gamma$ are expected to be much smaller, a point that has also been noted in the context of the VMD and quark model based estimates [20]. The corresponding form factors for the decays $B^0 \to \rho^0(\omega)\gamma$ are obtained from the ones for the decay $B^\pm \to \rho^\pm\gamma$ discussed above by the replacement of the light quark charges $e_u \to e_d$, which gives the factor $-1/2$; in addition, and more importantly, the LD-contribution to the neutral B decays is colour-suppressed, which reflects itself through the replacement of the factor a_1 by a_2. This yields for the ratio

$$\frac{R_{L/S}^{B^0 \to \rho\gamma}}{R_{L/S}^{B^\pm \to \rho^\pm \gamma}} = \frac{e_d a_2}{e_u a_1} \simeq -0.13 \pm 0.05, \qquad (26)$$

where the numbers are based on using $a_2/a_1 = 0.27 \pm 0.10$ [29]. This would then yield at most $R_{L/S}^{B^0 \to \rho\gamma} \simeq R_{L/S}^{B^0 \to \omega\gamma} = 0.05$, which in turn gives

$$\frac{\mathcal{A}_{long}^{B^0 \to \rho\gamma}}{\mathcal{A}_{short}^{B^0 \to \rho\gamma}} \leq 0.02. \qquad (27)$$

Even if this underestimates the LD-contribution by a factor 2, due to the approximations made in [18, 19], one can safely neglect the LD-contribution in the neutral B-meson radiative decays.

Restricting to the colour-allowed LD-contributions, the relations, which obtains ignoring such contributions (and isospin invariance),

$$\Gamma(B^\pm \to \rho^\pm\gamma) = 2\ \Gamma(B^0 \to \rho^0\gamma) = 2\ \Gamma(B^0 \to \omega\gamma)\ , \tag{28}$$

get modified to

$$\begin{aligned}\frac{\Gamma(B^\pm \to \rho^\pm\gamma)}{2\Gamma(B^0 \to \rho\gamma)} &= \frac{\Gamma(B^\pm \to \rho^\pm\gamma)}{2\Gamma(B^0 \to \omega\gamma)} = \left|1 + R_{L/S}^{B^\pm\to\rho^\pm\gamma}\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right|^2 = \\ &= 1 + 2\cdot R_{L/S}V_{ud}\frac{\rho(1-\rho)-\eta^2}{(1-\rho)^2+\eta^2} + (R_{L/S})^2V_{ud}^2\frac{\rho^2+\eta^2}{(1-\rho)^2+\eta^2}\,.\end{aligned} \tag{29}$$

where $R_{L/S} \equiv R_{L/S}^{B^\pm\to\rho^\pm\gamma}$. The ratio $\Gamma(B^\pm \to \rho^\pm\gamma)/2\Gamma(B^0 \to \rho\gamma)(= \Gamma(B^\pm \to \rho^\pm\gamma)/2\Gamma(B^0 \to \omega\gamma))$ is shown in Fig. 1 as a function of the parameter ρ, with $\eta = 0.2$, 0.3 and 0.4. This suggests that a measurement of this ratio would constrain the Wolfenstein parameters (ρ, η), with the dependence on ρ more marked than on η. In particular, a negative value of ρ leads to a constructive interference in $B^\pm \to \rho^\pm\gamma$ decays, while large positive values of ρ give a destructive interference.

The ratio of the CKM-suppressed and CKM-allowed decay rates for charged B mesons gets modified due to the LD contributions. Following [20], we ignore the LD-contributions in $\Gamma(B \to K^*\gamma)$. The ratio of the decay rates in question can therefore be written as:

$$\begin{aligned}\frac{\Gamma(B^\pm \to \rho^\pm\gamma)}{\Gamma(B^\pm \to K^{*\pm}\gamma)} &= \kappa_u\lambda^2[(1-\rho)^2+\eta^2] \\ &\times\left\{1 + 2\cdot R_{L/S}V_{ud}\frac{\rho(1-\rho)-\eta^2}{(1-\rho)^2+\eta^2} + (R_{L/S})^2V_{ud}^2\frac{\rho^2+\eta^2}{(1-\rho)^2+\eta^2}\right\},\end{aligned} \tag{30}$$

Using the central value from the estimates of the ratio of the form factors squared $\kappa_u = 0.59 \pm 0.08$ [26], we show the ratio (30) in Fig. 2 as a function of ρ for $\eta = 0.2, 0.3$, and 0.4. It is seen that the dependence of this ratio is rather weak on η but it depends on ρ rather sensitively. The effect of the LD-contributions is modest but not negligible, introducing an uncertainty comparable to the $\sim 15\%$ uncertainty in the overall normalization due to the $SU(3)$-breaking effects in the quantity κ_u.

Neutral B-meson radiative decays are less-prone to the LD-effects, as argued above, and hence one expects that to a good approximation the ratio of the decay rates for neutral B meson obtained in the approximation of SD-dominance remains valid [26]:

$$\frac{\Gamma(B^0 \to \rho^0\gamma, \omega\gamma)}{\Gamma(B \to K^*\gamma)} = \kappa_d\lambda^2[(1-\rho)^2+\eta^2]\ , \tag{31}$$

where this relation holds for each of the two decay modes separately.

Finally, combining the estimates for the LD- and SD-form factors in [19] and [26], respectively, and restricting the Wolfenstein parameters in the range $-0.25 \leq \rho \leq 0.35$ and

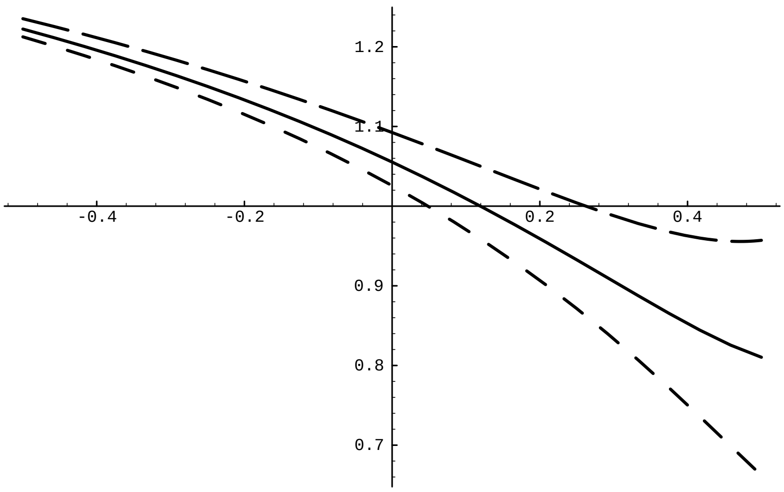

Figure 1: Ratio of the neutral and charged B-decay rates $\Gamma(B^\pm \to \rho^\pm\gamma)/2\Gamma(B^0 \to \rho\gamma)$ as a function of the Wolfenstein parameter ρ, with $\eta = 0.2$ (short-dashed curve), $\eta = 0.3$ (solid curve), and $\eta = 0.4$ (long-dashed curve). (Figure taken from [19].)

$0.2 \leq \eta \leq 0.4$, as suggested by the CKM-fits [22], we give the following ranges for the absolute branching ratios:

$$\begin{aligned} \mathcal{B}(B^\pm \to \rho^\pm\gamma) &= (1.5 \pm 1.1) \times 10^{-6} \;, \\ \mathcal{B}(B^0 \to \rho\gamma) \simeq \mathcal{B}(B^0 \to \omega\gamma) &= (0.65 \pm 0.35) \times 10^{-6} \;, \end{aligned} \tag{32}$$

where we have used the experimental value for the branching ratio $\mathcal{B}(B \to K^* + \gamma)$ [3], adding the errors in quadrature. The large error reflects the poor knowledge of the CKM matrix elements and hence experimental determination of these branching ratios will put rather stringent constraints on the Wolfenstein parameter ρ.

In addition to studying the radiative penguin decays of the $B^\pm$ and B^0 mesons discussed above, hadron machines such as HERA-B will be in a position to study the corresponding decays of the B_s^0 meson and Λ_b baryon, such as $B_s^0 \to \phi + \gamma$ and $\Lambda_b \to \Lambda + \gamma$, which have not been measured so far. We list below the branching ratios in a number of interesting decay modes calculated in the QCD sum rule approach in [26].

$$\begin{aligned} \mathcal{B}(B_s \to \phi\gamma) &= \mathcal{B}(B^0 \to K^{*0}\gamma) = (4.2 \pm 2.0) \times 10^{-5} \;, \\ \frac{\mathcal{B}(B_s \to K^*\gamma)}{\mathcal{B}(B^0 \to K^{*0}\gamma)} &\simeq (0.36 \pm 0.14)|\frac{|V_{td}|}{|V_{ts}|}|^2 \\ &\Longrightarrow \mathcal{B}(B_s \to K^*\gamma) = (0.75 \pm 0.5) \times 10^{-6} \;. \end{aligned} \tag{33}$$

The branching ratio for the radiative decay $\Lambda_b \to \Lambda\gamma$ can be estimated as follows. Assuming the electromagnetic penguin dominance, one expects $\Gamma(\Lambda_b \to Y_s + \gamma) \simeq \Gamma(B \to X_s + \gamma)$, where

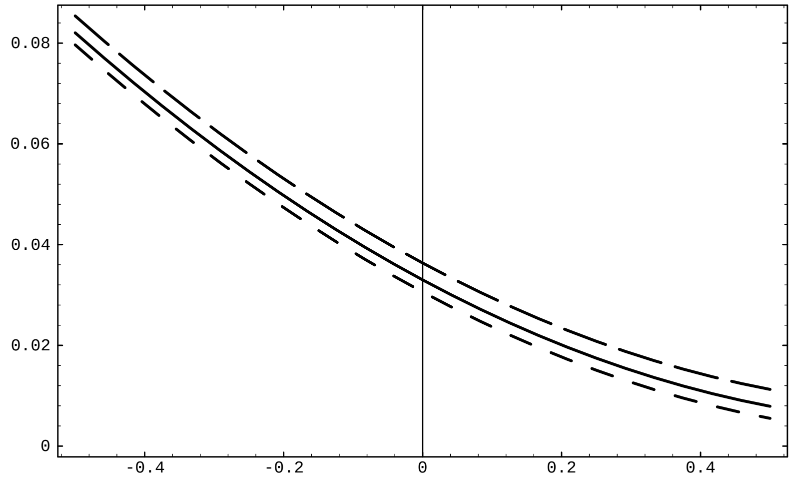

Figure 2: Ratio of the CKM-suppressed and CKM-allowed radiative B-decay rates $\Gamma(B^\pm \to \rho^\pm\gamma)/\Gamma(B \to K^*\gamma)$ (with $B = B^\pm$ or B^0) as a function of the Wolfenstein parameter ρ, a) with $\eta = 0.2$ (short-dashed curve), $\eta = 0.3$ (solid curve), and $\eta = 0.4$ (long-dashed curve). (Figure taken from [19].)

Y_s represents a baryonic system having an overall net strangeness quantum number. The exclusive decay rate can be calculated in terms of the exclusive-to-inclusive function $R_\Lambda \equiv \Gamma(\Lambda_b \to \Lambda + \gamma)/\Gamma(\Lambda_b \to Y_s + \gamma)$, analogous to the ratio R_{K^*}, defined for the B meson decays earlier. This would then give:

$$\begin{aligned} \mathcal{B}(\Lambda_b \to \Lambda\gamma) &= R_\Lambda \mathcal{B}(\Lambda_b \to Y_s\gamma) \simeq R_{K^*}\mathcal{B}(B_d \to X_s\gamma)\left[\frac{\tau(B_d)}{\tau(\Lambda_b)}\right] \\ &= (3.8 \pm 1.5) \times 10^{-5} \ , \end{aligned} \qquad (34)$$

where we have used the SM estimate $\mathcal{B}(B_d \to X_s\gamma) = (3.2 \pm 0.6) \times 10^{-4}$ [7], assumed $R_\Lambda = R_{K^*}$, with $R_{K^*} = 0.15 \pm 0.05$ [26] and $\tau(B_d)/\tau(\Lambda_b) = 0.78 \pm 0.04$ [30]. The CKM-suppressed decay $\Lambda_b^0 \to n + \gamma$ is related to the decay $\Lambda_b^0 \to \Lambda + \gamma$ by the CKM ratio $|V_{td}|/|V_{ts}|^2$ in the $SU(3)$ limit. Using the central value $|V_{td}|/|V_{ts}|^2 = 0.058$, this gives $\mathcal{B}(\Lambda_b \to n + \gamma) = (2.2 \pm 1.0) \times 10^{-6}$. The estimated branching ratios in a number of inclusive and exclusive radiative B decay modes are given in Table 1, where we have also listed the branching ratios for $B_s \to \gamma\gamma$ and $B_d \to \gamma\gamma$.

2.2 Inclusive rare decays $B \to X_s \ell^+\ell^-$ in the SM

The decays $B \to X_s\ell^+\ell^-$, with $\ell = e, \mu, \tau$, provide a more sensitive search strategy for finding new physics in rare B decays than for example the decay $B \to X_s\gamma$, which constrains the magnitude of C_7^{eff}. The sign of C_7^{eff}, which depends on the underlying physics, is not determined by the measurement of $\mathcal{B}(B \to X_s + \gamma)$. This sign, which in our convention is negative in the SM, is in general model dependent. It is known (see for example [31]) that in supersymmetric (SUSY) models, both the negative and positive signs are allowed as one scans over the allowed SUSY parameter space. The determination of the sign of C_7^{eff} is an important matter as this will impose further constraints on the parameters of many models. We recall that for low dilepton masses, the differential decay rate for $B \to X_s \ell^+ \ell^-$ is dominated by the contribution of the virtual photon to the charged lepton pair, which in turn depends on the effective Wilson coefficient C_7^{eff}. However, as is well known, the $B \to X_s \ell^+ \ell^-$ amplitude in the standard model has two additional terms, arising from the two FCNC four-Fermi operators, [1] which are not constrained by the $B \to X_s + \gamma$ data. Calling their coefficients C_9 and C_{10}, it has been argued in [31] that the signs and magnitudes of all three coefficients C_7^{eff}, C_9 and C_{10} can, in principle, be determined from the decays $B \to X_s + \gamma$ and $B \to X_s \ell^+ \ell^-$.

The SM-based rates for the decay $b \to s \ell^+ \ell^-$, calculated in the free quark decay approximation, have been known in the LO approximation for some time [32]. The LO calculations have the unpleasant feature that the decay distributions and rates are scheme-dependent. The required NLO calculation is in the meanwhile available, which reduces the scheme-dependence of the LO effects in these decays [33]. In addition, long-distance (LD) effects, which are expected to be very important in the decay $B \to X_s \ell^+ \ell^-$ [34], have also been estimated from data on the assumption that they arise dominantly due to the charmonium resonances J/ψ and ψ' through the decay chains $B \to X_s J/\psi(\psi', ...) \to X_s \ell^+ \ell^-$. The effect of these resonances persists even far away from the resonant masses deforming the short-distance based distributions appreciably [34]. Likewise, the leading $(1/{m_b}^2)$ power corrections to the partonic decay rate and the dilepton invariant mass distribution have been calculated with the help of the operator product expansion in the effective heavy quark theory [35]. The results of [35] have, however, not been confirmed in a recent independent calculation [36], which finds that the power corrections in the branching ratio $\mathcal{B}(B \to X_s \ell^+ \ell^-)$ are negligible. Moreover, the end-point dilepton invariant mass spectrum is not calculable in the heavy quark expansion and will have to be modelled. In what follows, we shall ignore the power corrections.

The amplitude for $B \to X_s \ell^+ \ell^-$ is calculated in the effective theory approach, which we have discussed earlier, by extending the operator basis of the effective Hamiltonian introduced in Eq. (6):

$$\mathcal{H}_{eff}(b \to s + \gamma; b \to s + \ell^+ \ell^-) = \mathcal{H}_{eff}(b \to s + \gamma) - \frac{4 G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[C_9 \mathcal{O}_9 + C_{10} \mathcal{O}_{10} \right], \qquad (35)$$

where the two additional operators are:

$$\begin{aligned} \mathcal{O}_9 &= \frac{\alpha}{4\pi} \bar{s}_\alpha \gamma^\mu P_L b_\alpha \bar{\ell} \gamma_\mu \ell, \\ \mathcal{O}_{10} &= \frac{\alpha}{4\pi} \bar{s}_\alpha \gamma^\mu P_L b_\alpha \bar{\ell} \gamma_\mu \gamma_5 \ell \; . \end{aligned} \qquad (36)$$

[1]This also holds for a large class of models such as MSSM and the two-Higgs doublet models but not for all SM-extensions. In LR symmetric models, for example, there are additional FCNC four-Fermi operators involved [37].

The analytic expressions for $C_9(m_W)$ and $C_{10}(m_W)$ can be seen in [33] and will not be given here. We recall that the coefficient C_9 in LO is scheme-dependent. However, this is compensated by an additional scheme-dependent part in the (one loop) matrix element of $\mathcal{O}_9$. We call the sum C_9^{eff}, which is scheme-independent and enters in the physical decay amplitude given below,

$$\begin{aligned}\mathcal{M}(b \to s + \ell^+\ell^-) &= \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{\alpha}{\pi} \\ &\times \left[C_9^{eff} \bar{s}\gamma^\mu P_L b \bar{\ell}\gamma_\mu \ell + C_{10} \bar{s}\gamma^\mu P_L b \bar{\ell}\gamma_\mu\gamma_5 \ell - 2C_7^{eff} \bar{s} i\sigma_{\mu\nu} \frac{q^\nu}{q^2}(m_b P_R + m_s P_L) b \bar{\ell}\gamma^\mu \ell \right],\end{aligned} \tag{37}$$

with

$$C_9^{eff}(\hat{s}) \equiv C_9 \eta(\hat{s}) + Y(\hat{s}). \tag{38}$$

The function $Y(\hat{s})$ is the one-loop matrix element of $\mathcal{O}_9$ and can be seen in literature [33, 7]. The differential decay rates in $B \to X_s \ell^+\ell^-$ (ignoring lepton masses) are,

$$\begin{aligned}\frac{\mathrm{d}\mathcal{B}(\hat{s})}{\mathrm{d}\hat{s}} &= \mathcal{B}_{sl} \frac{\alpha^2}{4\pi^2} \frac{\lambda_t^2}{|V_{cb}|^2} \frac{1}{f(\hat{m}_c)\kappa(\hat{m}_c)} u(\hat{s}) \left[\left(|C_9^{eff}(\hat{s})|^2 + C_{10}^2 \right) \alpha_1(\hat{s}, \hat{m}_s) \right. \\ &\left. + \frac{4}{\hat{s}} (C_7^{eff})^2 \alpha_2(\hat{s}, \hat{m}_s) + 12 \alpha_3(\hat{s}, \hat{m}_s) C_7^{eff} \Re(C_9^{eff}(\hat{s})) \right],\end{aligned} \tag{39}$$

with $u(\hat{s}) = \sqrt{[\hat{s} - (1+\hat{m}_s)^2][\hat{s} - (1-\hat{m}_s)^2]}$, $f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z$, $\kappa(z) = 1 - 2\alpha_s(\mu)/3\pi\, [(\pi^2 - 31/4)(1-z)^2 + 3/2]$, and

$$\alpha_1(\hat{s}, \hat{m}_s) = -2\hat{s}^2 + \hat{s}(1+\hat{m}_s^2) + (1-\hat{m}_s^2)^2, \tag{40}$$

$$\alpha_2(\hat{s}, \hat{m}_s) = -(1+\hat{m}_s^2)\hat{s}^2 - (1 + 14\hat{m}_s^2 + \hat{m}_s^4)\hat{s} + 2(1+\hat{m}_s^2)(1-\hat{m}_s^2)^2, \tag{41}$$

$$\alpha_3(\hat{s}, \hat{m}_s) = (1-\hat{m}_s^2)^2 - (1+\hat{m}_s^2)\hat{s}. \tag{42}$$

Here $\Re(C_7^{eff})$ represents the real part of C_7^{eff}. A useful quantity is the differential FB asymmetry in the c.m.s. of the dilepton defined in refs. [38]:

$$\frac{d\mathcal{A}(\hat{s})}{d\hat{s}} = \int_0^1 \frac{d\mathcal{B}}{dz} - \int_0^{-1} \frac{d\mathcal{B}}{dz}, \tag{43}$$

where $z = \cos\theta$, which can be expressed as:

$$\frac{\mathrm{d}\mathcal{A}(\hat{s})}{\mathrm{d}\hat{s}} = -\mathcal{B}_{sl} \frac{3\alpha^2}{4\pi^2} \frac{1}{f(\hat{m}_c)} u^2(\hat{s}) C_{10} \left[\hat{s}\Re(C_9^{eff}(\hat{s})) + 2C_7^{eff}(1+\hat{m}_s^2) \right]. \tag{44}$$

The Wilson coefficients C_7^{eff}, C_9^{eff} and C_{10} appearing in the above equations can be determined from data by solving the partial branching ratio $\mathcal{B}(\Delta\hat{s})$ and partial FB asymmetry $\mathcal{A}(\Delta\hat{s})$, where $\Delta\hat{s}$ defines an interval in the dilepton invariant mass [31].

There are other quantities which one can measure in the decays $B \to X_s \ell^+\ell^-$ to disentangle the underlying dynamics. We mention here the longitudinal polarization of the lepton in $B \to X_s \ell^+\ell^-$, in particular in $B \to X_s \tau^+\tau^-$, proposed by Hewett [39]. In a recent paper, Krüger and

Sehgal [40] have stressed that complementary information is contained in the two orthogonal components of polarization (P_T, the component in the decay plane, and P_N, the component normal to the decay plane), both of which are proportional to m_ℓ/m_b, and therefore significant for the $\tau^+\tau^-$ channel. A third quantity, called energy asymmetry, proposed by Cho, Misiak and Wyler [41], defined as

$$\mathcal{A} = \frac{N(E_{\ell^-} > E_{\ell^+}) - N(E_{\ell^+} > E_{\ell^-})}{N(E_{\ell^-} > E_{\ell^+}) + N(E_{\ell^+} > E_{\ell^-})} , \tag{45}$$

where $N(E_{\ell^-} > E_{\ell^+})$ denotes the number of lepton pairs where ℓ^+ is more energetic than ℓ^- in the B-rest frame, is, however, not an independent measure, as it is directly proportional to the FB asymmetry discussed above. The relation is [36]:

$$\int \mathcal{A}(\hat{s}) = \mathcal{B} \times A . \tag{46}$$

This is easy to notice if one writes the Mandelstam variable $u(\hat{s})$ in the dilepton c.m. and the B-hadron rest systems.

Next, we discuss the effects of LD contributions in the processes $B \to X_s \ell^+\ell^-$. Note that the LD contributions due to the vector mesons such as J/ψ and ψ', as well as the continuum $c\bar{c}$ contribution already discussed, appear as an effective $(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\ell)$ interaction term only, i.e. in the operator $\mathcal{O}_9$. This implies that the LD-contributions should change C_9 effectively, C_7 as discussed earlier is dominated by the SD-contribution, and C_{10} has no LD-contribution. In accordance with this, the function $Y(\hat{s})$ is replaced by,

$$Y(\hat{s}) \to Y'(\hat{s}) \equiv Y(\hat{s}) + Y_{\rm res}(\hat{s}), \tag{47}$$

where $Y_{\rm res}(\hat{s})$ is given as [38],

$$Y_{\rm res}(\hat{s}) = \frac{3}{\alpha^2}\kappa\,(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \sum_{V_i = J/\psi,\psi',\ldots} \frac{\pi\Gamma(V_i \to l^+l^-)M_{V_i}}{M_{V_i}^2 - \hat{s}m_b^2 - iM_{V_i}\Gamma_{V_i}}, \tag{48}$$

where κ is a fudge factor, which appears due to the inadequacy of the factorization framework in describing data on $B \to J/\psi X_s$. Here we use $\kappa\,(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) = +0.88$ for the numerical calculation, which reproduces (in average) the measured branching ratios for $B \to J/\psi X_s$ and $B \to \psi' X_s$, after the contributions from the χ_c states have been subtracted. The long-distance effects lead to significant interference effects in the dilepton invariant mass distribution and the FB asymmetry in $B \to X_s \ell^+\ell^-$ shown in Figs. 3 and 4, respectively. This can be used to test the SM, as the signs of the Wilson coefficients in general are model dependent.

Taking into account the spread in the values of the input parameters, μ, Λ, m_t, and $\mathcal{B}_{SL}$ discussed in the previous section in the context of $\mathcal{B}(B \to X_s + \gamma)$, we estimate the following branching ratios for the SD-piece only (i.e., from the intermediate top quark contribution only) [36]:

$$\begin{aligned}
\mathcal{B}(B \to X_s e^+e^-) &= (8.4 \pm 2.2) \times 10^{-6}, \\
\mathcal{B}(B \to X_s \mu^+\mu^-) &= (5.7 \pm 1.2) \times 10^{-6}, \\
\mathcal{B}(B \to X_s \tau^+\tau^-) &= (2.6 \pm 0.5) \times 10^{-7},
\end{aligned} \tag{49}$$

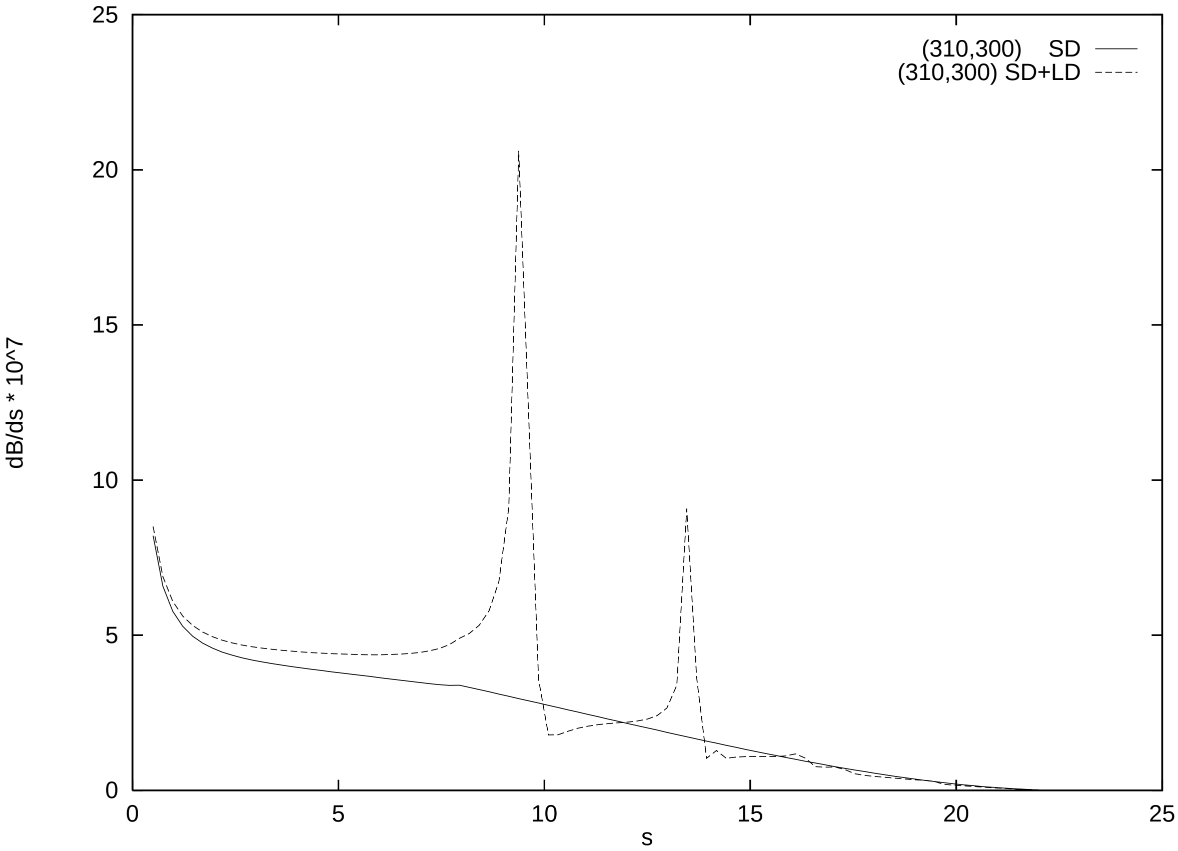

Figure 3: Differential branching ratio $d\mathcal{B}/ds$ for $B \to X_s \ell^+ \ell^-$ calculated in the SM using the next-to-leading order QCD corrections and Fermi motion effect (solid curve), and including the LD-contributions (dashed curve). The Fermi motion model parameters (P_F, m_q) are displayed in the figure. (Figure taken from [36].)

where theoretical errors and the error on $\mathcal{B}_{SL}$ have been added in quadrature. The present experimental limit for the inclusive branching ratio in $B \to X_s \ell^+ \ell^-$ is actually still the one set by the UA1 collaboration some time ago [42], namely $\mathcal{B}(B \to X_s \mu^+ \mu^-) > 5.0 \times 10^{-5}$. As far as we know, there are no interesting limits on the other two modes, involving $X_s e^+ e^-$ and $X_s \tau^+ \tau^-$.

It is obvious from Fig. 3 that only in the dilepton mass region far away from the resonances is there a hope of extracting the Wilson coefficients governing the short-distance physics. The region below the J/ψ resonance is well suited for that purpose as the dilepton invariant mass distribution here is dominated by the SD-piece. Including the LD-contributions, following branching ratio has been estimated for the dilepton mass range 2.1 GeV$^2 \leq s \leq$ 2.9 GeV^2 in [36]:

$$\mathcal{B}(B \to X_s \mu^+ \mu^-) = (1.3 \pm 0.3) \times 10^{-6}, \tag{50}$$

with $\mathcal{B}(B \to X_s e^+ e^-) \simeq \mathcal{B}(B \to X_s \mu^+ \mu^-)$. The FB-asymmetry is estimated to be in the range 10% - 27%, as can be seen in Fig. 4. These branching ratios and the FB asymmetry are expected to be measured within the next several years at BABAR, BELLE, CLEO, CDF, D0, and HERA-B. In the high invariant mass region, the short-distance contribution dominates. However, the rates are down by roughly an order of magnitude compared to the region below the J/ψ-mass. Estimates of the branching ratios are of $O(10^{-7})$, which should be accessible at the LHC.

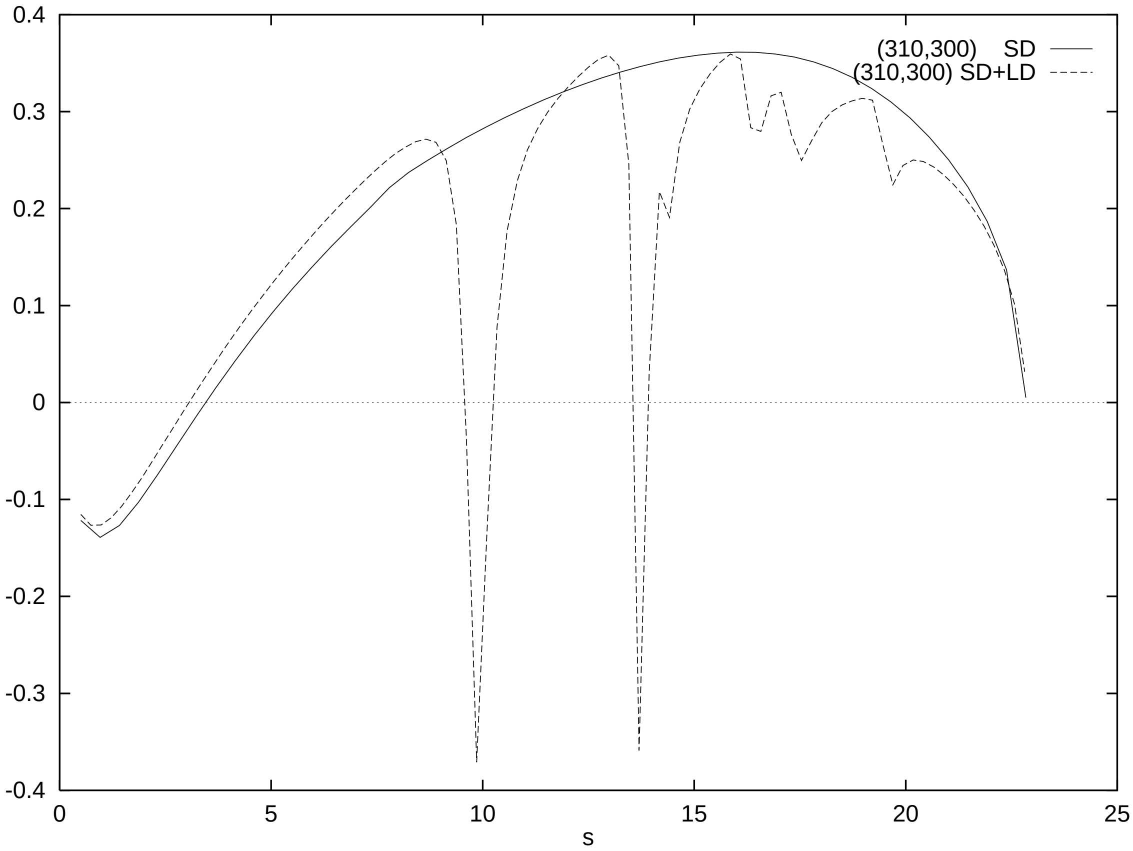

Figure 4: Normalized FB asymmetry $\mathcal{A}(s)$ for $B \to X_s \ell^+ \ell^-$ calculated in the SM using the next-to-leading order QCD corrections and Fermi motion effect (solid curve), and including the LD-contributions (dashed curve). The Fermi motion model parameters (P_F, m_q) are displayed in the figure. (Figure taken from [36].)

The experimental limits on the decay rates of the exclusive decays $B \to (K, K^*)\ell^+\ell^-$ [29, 43], while arguably closer to the SM-based estimates, can only be interpreted in specific models of form factors, which hinders somewhat their transcription in terms of the information on the underlying Wilson coefficients. However, many of these form factors can be related to the known ones using ideas from heavy quark effective theory augmented with QCD sum rule estimates in studying the heavy to light form factors in B decays. We will not take up such exclusive decays, important as they are, and content ourselves with presenting the expected branching ratios for some of the experimentally interesting decay modes. Using the exclusive-to-inclusive ratios $R_{K\ell\ell} \equiv \Gamma(B \to K\ell^+\ell^-)/\Gamma(B \to X_s\ell^+\ell^-) = 0.07 \pm 0.02$ and $R_{K^*\ell\ell} \equiv \Gamma(B \to K^*\ell^+\ell^-)/\Gamma(B \to X_s\ell^+\ell^-) = 0.27 \pm 0.0.07$, which were estimated in [45], the results are presented in Table 1.

In conclusion, the semileptonic FCNC decays $B \to X_s\ell^+\ell^-$ (and also the exclusive decays) will provide very precise tests of the SM, as they will determine the signs and magnitudes of the three Wilson coefficients, C_7, C_9^{eff} and C_{10}. This, perhaps, may also reveal physics beyond-the-SM if it is associated with not too high a scale. The MSSM model is a good case study where measurable deviations from the SM are anticipated and worked out [31, 41].

Decay Modes	$\mathcal{B}$(SM)	Measurements and Upper Limits (90% C.L.)
$(B^\pm, B^0) \to X_s\gamma$	$(3.2 \pm 0.58) \times 10^{-4}$	$(2.32 \pm 0.67) \times 10^{-4}$ CLEO [2]
$(B^\pm, B^0) \to K^*\gamma$	$(4.0 \pm 2.0) \times 10^{-5}$	$(4.2 \pm 0.8 \pm 0.6) \times 10^{-5}$ CLEO [4]
$(B^\pm, B^0) \to X_d\gamma$	$(1.6 \pm 1.2) \times 10^{-5}$	–
$B^\pm \to \rho^\pm + \gamma$	$(1.5 \pm 1.1) \times 10^{-6}$	$< 1.1 \times 10^{-5}$ CLEO [4]
$B^0 \to \rho^0 + \gamma$	$(0.65 \pm 0.35) \times 10^{-6}$	$< 3.9 \times 10^{-5}$ CLEO [4]
$B^0 \to \omega + \gamma$	$(0.65 \pm 0.35) \times 10^{-6}$	$< 1.3 \times 10^{-5}$ CLEO [4]
$B_s \to \phi + \gamma$	$(4.2 \pm 2.0) \times 10^{-5}$	–
$B_s \to K^* + \gamma$	$(0.8 \pm 0.5) \times 10^{-6}$	–
$\Lambda_b \to \Lambda + \gamma$	$(3.8 \pm 1.5) \times 10^{-5}$	–
$\Lambda_b \to n + \gamma$	$(2.2 \pm 1.0) \times 10^{-6}$	–
$(B_d, B_u) \to X_s e^+ e^-$	$(8.4 \pm 2.2) \times 10^{-6}$	–
$(B_d, B_u) \to X_d e^+ e^-$	$(4.9 \pm 2.9) \times 10^{-7}$	–
$(B_d, B_u) \to X_s \mu^+ \mu^-$	$(5.7 \pm 1.2) \times 10^{-6}$	$< 3.6 \times 10^{-5}$ D0 [47]
$(B_d, B_u) \to X_d \mu^+ \mu^-$	$(3.3 \pm 1.9) \times 10^{-7}$	–
$(B_d, B_u) \to X_s \tau^+ \tau^-$	$(2.6 \pm 0.5) \times 10^{-7}$	–
$(B_d, B_u) \to X_d \tau^+ \tau^-$	$(1.5 \pm 0.8) \times 10^{-8}$	–
$(B_d, B_u) \to K e^+ e^-$	$(5.9 \pm 2.3) \times 10^{-7}$	$< 1.2 \times 10^{-5}$ CLEO [43]
$(B_d, B_u) \to K \mu^+ \mu^-$	$(4.0 \pm 1.5) \times 10^{-7}$	$< 0.9 \times 10^{-5}$ CLEO [43]
$(B_d, B_u) \to K \mu^+ \mu^-$	$(4.0 \pm 1.5) \times 10^{-7}$	$< 0.9 \times 10^{-5}$ CLEO [43]
$(B_d, B_u) \to K^* e^+ e^-$	$(2.3 \pm 0.9) \times 10^{-6}$	$< 1.6 \times 10^{-5}$ CLEO [43]
$(B_d, B_u) \to K^* \mu^+ \mu^-$	$(1.5 \pm 0.6) \times 10^{-6}$	$< 2.5 \times 10^{-5}$ CDF [44]
$B_s \to \gamma\gamma$	$(3.0 \pm 1.0) \times 10^{-7}$	$< 1.1 \times 10^{-4}$ L3 [48]
$B_d \to \gamma\gamma$	$(1.2 \pm 0.8) \times 10^{-8}$	$< 3.8 \times 10^{-5}$ L3 [48]
$B_s \to \tau^+ \tau^-$	$(7.4 \pm 1.9) \times 10^{-7}$	–
$B_d \to \tau^+ \tau^-$	$(3.1 \pm 1.9) \times 10^{-8}$	–
$B_s \to \mu^+ \mu^-$	$(3.5 \pm 1.0) \times 10^{-9}$	$< 8.4 \times 10^{-6}$ CDF [44]
$B_d \to \mu^+ \mu^-$	$(1.5 \pm 0.9) \times 10^{-10}$	$< 1.6 \times 10^{-6}$ CDF [44]
$B_s \to e^+ e^-$	$(8.0 \pm 3.5) \times 10^{-14}$	–
$B_d \to e^+ e^-$	$(3.4 \pm 2.3) \times 10^{-15}$	–

Table 1: Estimates of the branching fractions for FCNC B-decays in the standard model taking into account the uncertainties in the input parameters as discussed in [7]. The entries in the second column correspond to the short-distance contributions only except for the radiative decays $B^\pm \to \rho^\pm + \gamma$ and $B^0 \to (\rho^0, \omega) + \gamma$, where long-distance effects have also been included. For the two-body branching ratios, we have used $f_{B_d} = 200$ MeV and $f_{B_s}/f_{B_d} = 1.16$. Experimental measurements and upper limits are also listed. (Updated from [7].)

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