From HERA to LHC through the Color Glass Condensate

Raju Venugopalan
Physics Department
Brookhaven National Laboratory
Upton, NY 11973, USA.

Abstract

A classical effective field theory, the Color Glass Condensate (CGC), provides a unified treatment of high parton density effects in both DIS and hadron-hadron collisions at very high energies. The validity and limitations of $k_\perp$ factorization can be studied in this effective theory. Multi-parton correlations in the effective theory are described by universal dipole and multipole operators. The evolution of these operators with energy provides a sensitive test of multi-parton dynamics in QCD at high energies.

1 Introduction

In the Bjorken limit of QCD, $Q^2 \to \infty$, $s \to \infty$, $x_{\mathrm{Bj}} \approx Q^2/s = \text{fixed}$, we have a powerful framework to compute a large number of processes to high accuracy. Underlying this machinery is the Operator Product Expansion (OPE), where cross-sections are identified as a convolution of short distance "co-efficient functions" which are process dependent and long distance parton distribution functions which are universal. The evolution of the parton distribution functions with x and Q^2 is described by splitting functions, which determine the probability of "parent" partons to split into a pair of "daughter" partons. Both coefficient functions and splitting functions for DIS inclusive cross-sections are now available to Next-Next-Leading-Order (NNLO) accuracy [1].

While this is a tremendous achievement, the contribution of high Q^2 processes to the total cross-section is very small. The bulk of the cross-section can perhaps be better understood in the Regge asymptotic limit: $x_{\mathrm{Bj}} \to 0$, $s \to \infty$, $Q^2 = \text{fixed}$. The BFKL renormalization group equation [3] describes the leading $\alpha_S \ln(1/x)$ behavior of gluon distributions in this limit. The solutions of the BFKL equation predict that gluon distributions grow very rapidly with decreasing x. In the Regge asymptotics, since the transverse size of the partons is fixed, this growth of distributions will lead to the overlapping of partons in the transverse plane of the hadron. In this regime, contributions that were power suppressed in the BFKL scheme become important. These are recombination and screening effects which slow down the growth of gluon distributions leading ultimately to a saturation of these distributions [4, 5]. Such effects must appear at small x because the occupation number [1] of partons in QCD be at most of order $1/\alpha_S$.

Thus qualitatively, the competition between Bremsstrahlung and recombination/screening effects becomes of the same order when

$$\frac{1}{2\left(N_c^2 - 1\right)} \frac{x\, G(x, Q^2)}{\pi R^2 Q^2} \approx \frac{1}{\alpha_S(Q^2)}, \tag{1}$$

where R is the radius of the target. This relation is solved self-consistently when $Q \equiv Q_s(x)$. The scale $Q_s(x)$ is termed the saturation scale and it grows as one goes to smaller values of x. When $Q^2 \leq Q_s^2$, higher twist effects are important; at sufficiently small x, $Q_s^2 \gg \Lambda_{\mathrm{QCD}}^2$, which makes feasible a weak coupling analysis of these effects. At HERA, reasonable fits of small x inclusive and diffractive data

---

[1] This corresponds to the number of partons per unit transverse area, per unit transverse momentum, per unit rapidity, in light cone gauge. This condition has its gauge invariant counterpart in the requirement that the field strength squared not exceed $1/\alpha_S$.

for $x \leq 10^{-2}$ are obtained in saturation models with $Q_s^2(x) \approx Q_0^2 (x_0/x)^\lambda$, with $Q_0^2 = 1$ GeV^2 and $x_0 = 3 \cdot 10^{-4}$. Detailed estimates suggest that the saturation scale for gluons is $Q_s(x) \approx 1.4$ GeV at $x \approx 10^{-4}$ [7]. The applicability of weak coupling techniques at these scales is dubious. Nevertheless, they cannot be ruled out since the effective scale at which the coupling runs can be larger than the estimate. Leading twist evolution of "shadowed" distributions at the saturation scale can extend out to significantly large values of x. A hint of this possibility is suggested by the fact that geometrical scaling-the dependence of cross-sections on the dimensionless ratio Q^2/Q_s^2 alone-extends out to $Q^2 \approx 450$ GeV^2 at HERA [8].

The possibility that weak coupling may apply at high energies is good news. Some of the remarkable regularities in high energy scattering data may be understood in a systematic way. The OPE, for instance, is no longer a good organizing principle since its usefulness is predicated on the twist expansion. In the next section, we will discuss an effective field theory approach which may provide a more efficient organizing principle at high parton densities.

2 The Color Glass Condensate

The physics of high parton densities can be formulated as a classical effective theory [6] because there is a Born-Oppenheimer separation between large x and small x modes [9] which are respectively the slow and fast modes in the effective theory. Large x partons are static sources of color charge for the dynamical wee (small x) parton fields. The generating functional of wee partons has the form

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+}[dA]\delta(A^+)e^{iS[A,\rho]-j\cdot A}}{\int^{\Lambda^+}[dA]\delta(A^+)e^{iS[A,\rho]}} \right\} \tag{2}$$

where the wee parton action has the form

$$S[A,\rho] = \frac{-1}{4} \int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c} \int d^2x_\perp dx^- \delta(x^-)\, \mathrm{Tr}\left(\rho(x_\perp)U_{-\infty,\infty}[A^-]\right). \tag{3}$$

In Eq. (2), ρ is a two dimensional classical color charge density and $W[\rho]$ is a weight functional of sources (which sits at momenta $k^+ > \Lambda^+$: note, $x = k^+/P_{\mathrm{hadron}}^+$). The sources are coupled to the dynamical wee gluon fields (which in turn sit at $k^+ < \Lambda^+$) via the gauge invariant term which is the second term on the RHS of Eq. (3). Here $U_{-\infty,\infty}$ denotes a path ordered exponential of the gauge field A^- in the x^+ direction. The first term in Eq. (3) is the QCD field strength tensor squared — thus the wee gluons are treated in full generality in this effective theory, which is formulated in the light cone gauge $A^+ = 0$. The source j is an external source — derivatives taken with respect to this source (with the source then put to zero) generate correlation functions in the usual fashion.

The argument for why the sources are classical is subtle and follows from a coarse graining of the effective action. The weight functional for a large nucleus is a Gaussian in the source density [6, 11], with a small correction for SU(N_c) coming from the $N_c - 2$ higher Casimir operators [10]. The variance of the Gaussian, the color charge squared per unit area μ_A^2, proportional to $A^{1/3}$, is a large scale — and is the only scale in the effective action [2]. Thus for $\mu_A^2 \gg \Lambda_{\mathrm{QCD}}^2$, $\alpha_S(\mu_A^2) \ll 1$, and one can compute the properties of the theory in Eq. (2) in weak coupling.

The saddle point of the action in Eq. (3) gives the classical distribution of gluons in the nucleus. The Yang-Mills equations can be solved analytically to obtain the classical field of the nucleus as a function of ρ: $A_{\mathrm{cl.}}(\rho)$ [6, 11, 12]. One can determine, for Gaussian sources, the occupation number $\phi = dN/\pi R^2/dk_\perp^2\, dy$ (the number of partons per unit transverse momentum, per unit rapidity y, where $y = \ln(1/x)$) of wee partons in the classical field of the nucleus. One finds for $k_\perp \gg Q_s^2$, the Weizsäcker-Williams spectrum $\phi \sim Q_s^2/k_\perp^2$; for $k_\perp \leq Q_s$, one obtains a complete resummation to all orders in $k_\perp$,

---

[2]μ_A^2 is simply related in the classical theory to the saturation scale Q_s^2 via the relation $Q_s^2 = \alpha_S N_c \mu_A^2 \ln(Q_s^2/\Lambda_{\mathrm{QCD}}^2)$

which gives $\phi \sim \frac{1}{\alpha_S} \ln(Q_s/k_\perp)$. (The behavior at low $k_\perp$ can, more accurately, be represented as $\frac{1}{\alpha_S}\Gamma(0,z)$ where Γ is the incomplete Gamma function and $z = k_\perp^2/Q_s^2$ [13]).

A high energy hadron is a Color Glass Condensate for the following reasons [2]. The 'color' is obvious since the parton degrees of freedom are colored. It is a glass because the sources, static on time scales much larger than time scales characteristic of the system, induce a stochastic (space-time dependent) coupling between the partons under quantum evolution — this is analogous to a spin glass. Finally, the matter is a condensate because the wee partons have large occupation numbers (of order $1/\alpha_S$) and have momenta peaked about Q_s. These properties are enhanced by quantum evolution in x. The classical field retains its structure — while the saturation scale grows: $Q_s(x') > Q_s(x)$ for $x' < x$.

Small fluctuations about the effective action in Eq. (3) give large corrections of order $\alpha_S \ln(1/x)$ (see Ref. [14]). The Gaussian weight functional is thus fragile under quantum evolution of the sources. A Wilsonian renormalization group (RG) approach systematically treats these corrections [15]. In particular, the change of the weight functional $W[\rho]$ with x is described by the JIMWLK- non-linear RG equations [15]. These equations form an infinite hierarchy of ordinary differential equations for the gluon correlators $\langle A_1 A_2 \cdots A_n \rangle_Y$, where $Y = \ln(1/x)$ is the rapidity. The JIMWLK equation for an arbitrary operator $\langle O \rangle$ is

$$\frac{\partial \langle \mathcal{O}[\alpha] \rangle_Y}{\partial Y} = \left\langle \frac{1}{2} \int_{x_\perp, y_\perp} \frac{\delta}{\delta \alpha_Y^a(x_\perp)} \chi_{x_\perp, y_\perp}^{ab}[\alpha] \frac{\delta}{\delta \alpha_Y^b(y_\perp)} \mathcal{O}[\alpha] \right\rangle_Y , \qquad (4)$$

where $\alpha = (\nabla_\perp^2)^{-1}\rho$. Here χ is a non-local object expressed in terms of path ordered (in rapidity) Wilson lines of α [2]. This equation is analogous to a (generalized) functional Fokker-Planck equation, where Y is the "time" and χ is a generalized diffusion coefficient. It illustrates the stochastic properties of operators in the space of gauge fields at high energies. For the gluon density, which is proportional to a two-point function $\langle \alpha^a(x_\perp)\alpha^b(y_\perp) \rangle$, one recovers the BFKL equation in the limit of low parton densities.

3 Dipoles in the CGC

In the limit of large N_c and large A ($\alpha_S^2 A^{1/3} \gg 1$), the JIMWLK hierarchy closes for the two point correlator of Wilson lines because the expectation value of the product of traces of Wilson lines factorizes into the product of the expectation values of the traces:

$$\langle \mathrm{Tr}(V_x V_z^\dagger) \mathrm{Tr}(V_z V_y^\dagger) \rangle \longrightarrow \langle \mathrm{Tr}(V_x V_z^\dagger) \rangle \langle \mathrm{Tr}(V_z V_y^\dagger) \rangle , \qquad (5)$$

where $V_x = \mathcal{P} \exp\left(\int dz^- \alpha^a(z^-, x_\perp) T^a\right)$. Here $\mathcal{P}$ denotes path ordering in x^- and T^a is an adjoint SU(3) generator. In Mueller's dipole picture, the cross-section for a dipole scattering off a target can be expressed in terms of these 2-point dipole operators as [16, 17]

$$\sigma_{q\bar{q}N}(x, r_\perp) = 2 \int d^2 b \, \mathcal{N}_Y(x, r_\perp, b) , \qquad (6)$$

where $\mathcal{N}_Y = 1 - \frac{1}{N_c}\langle \mathrm{Tr}(V_x V_y^\dagger) \rangle_Y$, the imaginary part of the forward scattering amplitude. Note that the size of the dipole, $\vec{r}_\perp = \vec{x}_\perp - \vec{y}_\perp$, and the impact parameter, $\vec{b} = (\vec{x}_\perp + \vec{y}_\perp)/2$. The JIMWLK equation for the two point Wilson correlator is identical in the large A, large N_c mean field limit to an equation derived independently by Balitsky and Kovchegov — the Balitsky-Kovchegov equation [18], which has the operator form

$$\frac{\partial \mathcal{N}_Y}{\partial Y} = \frac{\alpha_S N_c}{\pi} \mathcal{K}_{\mathrm{BFKL}} \otimes \left\{ \mathcal{N}_Y - \mathcal{N}_Y^2 \right\} . \qquad (7)$$

Here $\mathcal{K}_{\mathrm{BFKL}}$ is the well known BFKL kernel. When $\mathcal{N} \ll 1$, the quadratic term is negligible and one has BFKL growth of the number of dipoles; when $\mathcal{N}$ is close to unity, the growth saturates. The approach to

unity can be computed analytically [19]. The B-K equation is the simplest equation including both the Bremsstrahlung responsible for the rapid growth of amplitudes at small x as well as the repulsive many body effects that lead to a saturation of this growth.

A saturation condition which fixes the amplitude at which this change in behavior is significant, say $\mathcal{N} = 1/2$, determines the saturation scale. One obtains $Q_s^2 = Q_0^2 \exp(\lambda Y)$, where $\lambda = c\alpha_S$ with $c \approx 4.8$. The saturation condition affects the overall normalization of this scale but does not affect the power λ. In fixed coupling, the power λ is large and there are large pre-asymptotic corrections to this relation–which die off only slowly as a function of Y. BFKL running coupling effects change the behavior of the saturation scale completely–one goes smoothly at large Y to $Q_s^2 = Q_0^2 \exp(\sqrt{2b_0 c(Y + Y_0)})$ where b_0 is the coefficient of the one-loop QCD β-function. The state of the art computation of Q_s is the work of Triantafyllopoulos, who obtained Q_s by solving NLO-resummed BFKL in the presence of an absorptive boundary (which corresponds to the CGC) [20]. The pre-asymptotic effects are much smaller in this case and the coefficient $\lambda \approx 0.25$ is very close to the value extracted from saturation model fits to the HERA data [21]. Fits of CGC inspired models to the HERA data have been discussed elsewhere [22] and will not be discussed here.

4 Hadronic scattering and $k_\perp$ factorization in the CGC

Collinear factorization is the pQCD mechanism to compute hard scattering. At collider energies, a new window opens up where $\Lambda_{\mathrm{QCD}}^2 \ll M^2 \ll s$, where M is the invariant mass of the final state. In principle, cross-sections in this window can be computed in the collinear factorization language–however, one needs to sum up large logarithmic corrections in s/M^2. An alternative formalism is that of $k_\perp$-factorization [23, 24], where one has a convolution of $k_\perp$ dependent "un-integrated" gluon distributions from the two hadrons with the hard scattering matrix. In this case, the in-coming partons from the wavefunctions have non-zero $k_\perp$. Levin et al. [25] suggested that at high energies the typical $k_\perp$ is the saturation scale Q_s. The rapidity dependence of the unintegrated distributions is given by the BFKL or BK equations. However, unlike the structure functions, it has not been proven that these unintegrated distributions are universal functions.

At small x, both the collinear factorization and $k_\perp$ factorization limits can be understood in a systematic way in the framework of the Color Glass Condensate. The expectation value of an operator $\mathcal{O}$ can be computed as

$$\langle \mathcal{O} \rangle_Y = \int [d\rho_1] [d\rho_2] W_{x_1}[\rho_1] W_{x_2}[\rho_2] \mathcal{O}(\rho_1, \rho_2) \,, \qquad (8)$$

where $Y = \ln(1/x_F)$ and $x_F = x_1 - x_2$. Quantum information, to leading logarithms in x, is contained in the source functionals $W_{x_1(x_2)}[\rho_1(\rho_2)]$ of the two hadrons. The operator $\mathcal{O}$ corresponding to the final state is expressed in terms of gauge fields $A^\mu[\rho_1, \rho_2](x)$. Inclusive gluon production in the CGC is computed by solving the Yang-Mills equations $[D_\mu, F^{\mu\nu}]^a = J^{\nu,a}$ for $A^\mu[\rho_1, \rho_2]$, where the current is given by $J^\nu = \rho_1 \, \delta(x^-)\delta^{\nu+} + \rho_2 \, \delta(x^+)\delta^{\nu-}$ with initial conditions determined by the Yang-Mills fields of the two hadrons before the collision. These are obtained self-consistently by matching the solutions of the Yang-Mills equations on the light cone [26]. Since we have argued in Section 2 that we can *compute* the Yang-Mills fields in the nuclei before the collision, the classical problem is in principle completely solvable. Quantum corrections not enhanced by powers of $\alpha_S \ln(1/x)$ can be computed systematically. Those terms enhanced by powers of $\alpha_S \ln(1/x)$ are absorbed into the weight functionals $W[\rho_{1,2}]$.

Hadronic scattering in the CGC can therefore be studied through a systematic power counting in the density of sources in powers of $\rho_{1,2}/k_{\perp;1,2}^2$. This power counting is more relevant at high energies than whether the incoming projectile is a hadron or a nucleus. In addition, one can study the applicability of collinear and $k_\perp$ factorization at small x in this approach.

The power counting is applicable as well to a proton at small x. The relevant quantity here is Q_s, which, as one may recall, is enhanced both for large A and small x. As long as $k_\perp \gg Q_s \gg \Lambda_{\mathrm{QCD}}$,

one can consider the proton or nucleus as being dilute. To lowest order in $\rho_{p1}/k_\perp^2$ and $\rho_{p2}/k_\perp^2$, one can compute inclusive gluon production analytically [26]. At large transverse momenta, $Q_s \ll k_\perp$, the scattering can be expressed in a $k_\perp$-factorized form. The inclusive cross-section is expressed as the product of two unintegrated ($k_\perp$ dependent) distributions times the matrix element for the scattering. The comparison of this result to the collinear pQCD $gg \to gg$ process and the $k_\perp$ factorized $gg \to g$ was performed in Ref. [27]. At this order, the result is equivalent to the pQCD result first derived by Gunion and Bertsch [28]. This result for gluon production is substantially modified, as we shall discuss shortly, by high parton density effects either because the target is a large nucleus or because small values of x are being probed in the hadron (as in forward pp scattering).

$k_\perp$ factorization is a good assumption at large momenta for quark pair-production. This was worked out in the CGC approach by François Gelis and myself [29]. The result for inclusive quark pair production can be expressed in $k_\perp$ factorized form as

$$\frac{d\sigma_1}{dy_p dy_q d^2 p_\perp d^2 q_\perp} \propto \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \frac{d^2 k_{2\perp}}{(2\pi)^2} \delta(k_{1\perp} + k_{2\perp} - p_\perp - q_\perp)$$

$$\times \phi_1(k_{1\perp}) \phi_2(k_{2\perp}) \frac{\text{Tr}\left(\left|m_{ab}^{-+}(k_1, k_2; q, p)\right|^2\right)}{k_{1\perp}^2 k_{2\perp}^2} , \tag{9}$$

where ϕ_1 and ϕ_2 are the unintegrated gluon distributions in the projectile and target respectively (with the gluon distribution defined as $xG(x, Q^2) = \int_0^{Q^2} d(k_\perp^2)\, \phi(x, k_\perp)$).

The matrix element $\text{Tr}\left(\left|m_{ab}^{-+}(k_1, k_2; q, p)\right|^2\right)$ is identical to the result derived in the $k_\perp$–factorization approach [23, 24]. In the limit $|\vec{k_{1\perp}}|, |\vec{k_{2\perp}}| \to 0$, $\text{Tr}\left(\left|m_{ab}^{-+}(k_1, k_2; q, p)\right|^2\right) / (k_{1\perp}^2 k_{2\perp}^2)$ is well defined–after integration over the azimuthal angles in Eq. (9), one obtains the usual matrix element $|\mathcal{M}|^2_{gg \to q\bar{q}}$, recovering the lowest order pQCD collinear factorization result.

4.1 Gluon and quark production in forward pp and pA collisions

Many analytical results are available when one of the hadrons is dilute and the other is dense. This may correspond to either pA collisions or forward pp collisions. One solves the Yang–Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$ with the light cone sources $J^{\nu,a} = \delta^{\nu+} \delta(x^-) \rho_p^a(x_\perp) + \delta^{\nu-} \delta(x^+) \rho_A^a(x_\perp)$, to determine the gluon field produced–to lowest order in the source density of one projectile ($\rho_p/k_\perp^2 \ll 1$) and to all orders ($\rho_A/k_\perp^2 \sim 1$) in the source density of the other. The inclusive gluon production cross-section, in this framework, was first computed by Kovchegov and Mueller [30] and shown to be $k_\perp$ factorizable in Ref. [31, 34]. The "unintegrated" gluon distribution in the dense system however is here replaced by the gluon "dipole" distribution $\mathcal{N}_Y$ we discussed previously. It is no longer a leading twist object but includes all twists enhanced by high parton density effects. The well known "Cronin" effect observed in Deuteron-Gold collisions at RHIC is obtained in this formalism and can be simply understood in terms of the multiple scattering of a parton from the projectile with those in the target. The energy evolution of the dipole distribution is given by the BK equation, leading to a suppression of the Cronin effect at high densities due to the shadowing of nuclear distributions. This prediction appears to be confirmed by the RHIC data. The "dipole" operators extracted from DIS can therefore be used to predict inclusive hadron production in pp and pA collisions. One can similarly compute Drell-Yan and photon production in forward pp and pA collisions [33, 35].

Unlike gluon production, neither quark pair-production nor single quark production is strictly $k_\perp$ factorizable. The pair production cross-section can however still be written in $k_\perp$ factorized form as a product of the unintegrated gluon distribution in the proton times a sum of terms with three unintegrated distributions, $\phi_{g,g}$, $\phi_{q\bar{q},g}$ and $\phi_{q\bar{q},q\bar{q}}$. These are respectively proportional to 2-point (dipole), 3-point and 4-point correlators of the Wilson lines we discussed previously. Again, these operators include all twist contributions. For instance, the distribution $\phi_{q\bar{q},g}$ is the product of fundamental Wilson lines coupled to

a $q\bar{q}$ pair in the amplitude and adjoint Wilson lines coupled to a gluon in the complex conjugate amplitude. For large transverse momenta or large-mass pairs, the 3-point and 4-point distributions collapse to the unintegrated gluon distribution, and we recover the previously discussed $k_\perp$-factorized result for pair production in the dilute/pp-limit. Single quark distributions are straightforwardly obtained and depend only on the 2-point quark and gluon correlators and the 3-point correlators. For Gaussian sources, as in the McLerran-Venugopalan-model, these 2-,3- and 4-point functions can be computed exactly as discussed in Ref. [32].

The situation gets complicated when one enters a regime where both projectiles are dense–as defined in our power counting. $k_\perp$ factorization breaks down decisively and analytical approaches are likely not possible. Nevertheless, numerical techniques have been developed, which allow the computation of final states, at least to leading logs in x [38].

The results for gluon and quark production in forward pp and pA or dA collisions (for a review, see Ref. [37]), coupled with the previous results for inclusive and diffractive [33–36] distributions in DIS, suggest an important new paradigm. *At small x in DIS and hadron colliders, previously interesting observables such as quark and gluon densities are no longer the only observables to capture the relevant physics. Instead, they should be complemented by dipole and multipole correlators of Wilson lines that seem ubiquitous in all high energy processes and are similarly gauge invariant and process independent. The renormalization group running of these operators may be a powerful and sensitive harbinger of new physics.*

References

[1] A. Vogt, S. Moch and J. A. M. Vermaseren, Nucl. Phys. B **691**, 129 (2004); S. A. Larin, P. Nogueira, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. B **492**, 338 (1997).

[2] E. Iancu and R. Venugopalan, hep-ph/0303204, in QGP3, Ed. R. Hwa and X.-N. Wang, World Scientific (2004).

[3] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977) [Zh. Eksp. Teor. Fiz. **72**, 377 (1977)]; I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978) [Yad. Fiz. **28**, 1597 (1978)].

[4] L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Rept. **100**, 1 (1983).

[5] A.H. Mueller, J-W. Qiu, Nucl. Phys. **B 268**, 427 (1986).

[6] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); *ibid.*, 3352, (1994); *ibid.*, **50**, 2225 (1994).

[7] H. Kowalski and D. Teaney, Phys. Rev. D **68**, 114005 (2003).

[8] A. M. Stasto, K. Golec-Biernat and J. Kwiecinski, Phys. Rev. Lett. **86**, 596 (2001).

[9] L. Susskind, Phys. Rev. **165**, 1535, (1968).

[10] S. Jeon and R. Venugopalan, Phys. Rev. D **70**, 105012 (2004).

[11] Yu.V. Kovchegov, Phys. Rev. **D 54**, 5463 (1996); *ibid.*, **55**, 5445 (1997).

[12] J. Jalilian-Marian, A. Kovner, L.D. McLerran, H. Weigert, Phys. Rev. **D 55**, 5414 (1997).

[13] D. Triantafyllopoulos, hep-ph/0502114.

[14] A. Ayala, J. Jalilian-Marian, L.D. McLerran, R. Venugopalan, Phys. Rev. **D 52**, 2935 (1995); *ibid.*, **53**, 458 (1996).

[15] J. Jalilian-Marian, A. Kovner, A. Leonidov, H. Weigert, Phys. Rev. **D 59**, 014014 (1999); E. Iancu, A. Leonidov, L.D. McLerran, Nucl. Phys. **A 692**, 583 (2001).

[16] A. H. Mueller, Nucl. Phys. B **415**, 373 (1994).

[17] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C**49**, (1991) 607.

[18] I. Balitsky, Nucl. Phys. **B 463**, 99 (1996); Yu.V. Kovchegov, Phys. Rev. **D 61**, 074018 (2000).

[19] E.M. Levin, K. Tuchin, Nucl. Phys. **B 573**, 833 (2000).

[20] D. N. Triantafyllopoulos, Nucl. Phys. B **648**, 293 (2003).

[21] K. Golec-Biernat, M. Wüsthoff, Phys. Rev. **D 59**, 014017 (1999).

[22] R. Venugopalan, arXiv:hep-ph/0412396.

[23] S. Catani, M. Ciafaloni, F. Hautmann, Nucl. Phys. B **366**, 135 (1991).

[24] J. C. Collins and R. K. Ellis, Nucl. Phys. B **360**, 3 (1991).

[25] E. M. Levin, M. G. Ryskin, Y. M. Shabelski and A. G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657 (1991).

[26] A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **52**, 3809 (1995); Y. V. Kovchegov and D. H. Rischke, Phys. Rev. C **56**, 1084 (1997).

[27] M. Gyulassy and L. D. McLerran, Phys. Rev. C **56**, 2219 (1997).

[28] J. F. Gunion and G. Bertsch, Phys. Rev. D **25**, 746 (1982).

[29] F. Gelis and R. Venugopalan, Phys. Rev. D **69**, 014019 (2004).

[30] Y. V. Kovchegov and A. H. Mueller, Nucl. Phys. B **529**, 451 (1998).

[31] J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A **743**, 13 (2004).

[32] J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A **743**, 57 (2004); H. Fujii, F. Gelis and R. Venugopalan, hep-ph/0504047.

[33] F. Gelis and J. Jalilian-Marian, Phys. Rev. D **66**, 094014 (2002); Phys. Rev. D **66**, 014021 (2002); A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett. **89**, 022301 (2002).

[34] Y. V. Kovchegov and K. Tuchin, Phys. Rev. D **65**, 074026 (2002); D. Kharzeev, Yu. Kovchegov, K. Tuchin, Phys. Rev. D **68**, 094013 (2003).

[35] B. Z. Kopeliovich, J. Raufeisen, A. V. Tarasov and M. B. Johnson, Phys. Rev. C **67**, 014903 (2003).

[36] A. Kovner and U. A. Wiedemann, Phys. Rev. D **64**, 114002 (2001).

[37] J. Jalilian-Marian and Y. V. Kovchegov, arXiv:hep-ph/0505052.

[38] A. Krasnitz and R. Venugopalan, Nucl. Phys. B **557**, 237 (1999); Phys. Rev. Lett. **84**, 4309 (2000); A. Krasnitz, Y. Nara and R. Venugopalan, Phys. Rev. Lett. **87**, 192302 (2001); Nucl. Phys. A **717**, 268 (2003); Nucl. Phys. A **727**, 427 (2003); T. Lappi, Phys. Rev. C **67**, 054903 (2003).