

Diffraction Higgs production: theory

Jeff Forshaw

Particle Physics Group, School of Physics & Astronomy,
University of Manchester, Manchester, M13 9PL. United Kingdom.

Abstract

We review the calculation for Higgs production via the exclusive reaction $pp \rightarrow p + H + p$. In the first part we review in some detail the calculation of the Durham group and emphasise the main areas of uncertainty. Afterwards, we comment upon other calculations.

1 Introduction

Our aim is to compute the cross-section for the process $pp \rightarrow p + H + p$. We shall only be interested in the kinematic situation where all three final state particles are very far apart in rapidity with the Higgs boson the most central. In this “diffractive” situation the scattering protons lose only a very small fraction of their energy, but nevertheless enough to produce the Higgs boson. Consequently, we are in the limit where the incoming protons have energy E much greater than the Higgs mass m_H and so we will always neglect terms suppressed by powers of m_H/E . In the diffractive limit cross-sections do not fall as the beam energy increases as a result of gluonic (spin-1) exchanges in the t -channel.

Given the possibility of instrumenting the LHC to detect protons scattered through tiny angles with a high resolution [1–4], diffractive production of any central system X via $pp \rightarrow p + X + p$ is immediately of interest if the production rate is large enough. Even if X is as routine as a pair of high p_T jets we can learn a great deal about QCD in a new regime [2, 3, 5, 6]. But no doubt the greatest interest arises if X contains “new physics” [7–19]. The possibility arises to measure the new physics in a way that is not possible using the LHC general purpose detectors alone. For example, its invariant mass may be measured most accurately, and the spin and CP properties of the system may be explored in a manner more akin to methods hitherto thought possible only at a future linear collider. Our focus here is on the production of a Standard Model Higgs boson [7, 8, 13, 18, 19]. Since the production of the central system X effectively factorizes, our calculation will be seen to be of more general utility.

Most of the time will be spent presenting what we shall call the “Durham Model” of central exclusive production [7, 8]. It is based in perturbative QCD and is ultimately to be justified a posteriori by checking that there is not a large contribution arising from physics below 1 GeV. A little time will also be spent explaining the non-perturbative model presented by the Saclay group [13] and inspired by the original paper of Bialas and Landshoff [20]. Even less time will be devoted to other approaches which can be viewed, more-or-less, as hybrids of the other two [18, 19].

Apart from the exclusive process we study here, there is also the possibility to produce the new physics in conjunction with other centrally produced particles, e.g. $pp \rightarrow p + H + X + p$. This more inclusive channel typically has a much higher rate but does not benefit from the various advantages of exclusive production. Nevertheless, it must be taken into account in any serious phenomenological investigation into the physics potential of central exclusive production [21, 22]

2 The Durham Model

The calculation starts from the easier to compute parton level process $qq \rightarrow q + H + q$ shown in Figure 1. The Higgs is produced via a top quark loop and a minimum of two gluons need to be exchanged in order that no colour be transferred between the incoming and outgoing quarks. Quark exchange in the t -channel leads to contributions which are suppressed by an inverse power of the beam energy and so the diagram in Figure 1 is the lowest order one. Our strategy will be to compute only the imaginary part

of the amplitude and we shall make use of the Cutkosky rules to do that – the relevant cut is indicated by the vertical dotted line in Figure 1. There is of course a second relevant diagram corresponding to the Higgs being emitted from the left-hand gluon. We shall assume that the real part of the amplitude is negligible, as it will be in the limit of asymptotically high centre-of-mass energy when the quarks are scattered through small angles and the Higgs is produced centrally.

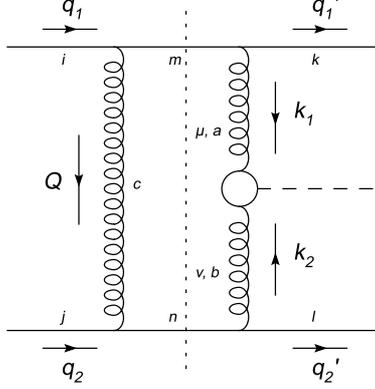


Fig. 1: The relevant Feynman graph for $qq \rightarrow q + H + q$.

The calculation can be further simplified by making use of the eikonal approximation for those vertices which couple the gluons to the external quarks. The gluons are very soft and so, modulo corrections which are suppressed by the inverse of the beam energy, we can approximate the qqg vertices by $2g\tau_{ij}^a q_{1,2}\delta_{\lambda,\lambda'}$, where τ^a is a Gell-Mann matrix, g is the QCD coupling and the Kronecker delta tells us that the quark does not change its helicity. The calculation of the amplitude is now pretty straightforward:

$$\begin{aligned} \text{Im}A_{jl}^{ik} &= \frac{1}{2} \times 2 \int d(P_S)_2 \delta((q_1 - Q)^2)\delta((q_2 + Q)^2) \\ &\quad \frac{2gq_1^\alpha}{Q^2} \frac{2gq_{2\alpha}}{k_1^2} \frac{2gq_1^\mu}{k_1^2} \frac{2gq_2^\nu}{k_2^2} V_{\mu\nu}^{ab} \tau_{im}^c \tau_{jn}^c \tau_{mk}^a \tau_{nl}^b. \end{aligned} \quad (1)$$

The factor of 1/2 is from the cutting rules and the factor of 2 takes into account that there are two diagrams. The phase-space factor is

$$d(P_S)_2 = \frac{s}{2} \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2} d\alpha d\beta \quad (2)$$

where we have introduced the Sudakov variables via $Q = \alpha q_1 + \beta q_2 + Q_T$. The delta functions fix the cut quark lines to be on-shell, which means that $\alpha \approx -\beta \approx \mathbf{Q}_T^2/s \ll 1$ and $Q^2 \approx Q_T^2 \equiv -\mathbf{Q}_T^2$. As always, we are neglecting terms which are energy suppressed such as the product $\alpha\beta$. For the Higgs production vertex we take the Standard Model result:

$$V_{\mu\nu}^{ab} = \delta^{ab} \left(g_{\mu\nu} - \frac{k_{2\mu}k_{1\nu}}{k_1 \cdot k_2} \right) V \quad (3)$$

where $V = m_H^2 \alpha_s / (4\pi v) F(m_H^2/m_t^2)$ and $F \approx 2/3$ provided the Higgs is not too heavy. The Durham group also include a NLO K-factor correction to this vertex. After averaging over colours we have

$$\tau_{im}^c \tau_{jn}^c \tau_{mk}^a \tau_{nl}^b \rightarrow \frac{\delta^{ab}}{4N_c^2}.$$

We can compute the contraction $q_1^\mu V_{\mu\nu}^{ab} q_2^\nu$ either directly or by utilising gauge invariance which requires that $k_1^\mu V_{\mu\nu}^{ab} = k_2^\nu V_{\mu\nu}^{ab} = 0$. Writing¹ $k_i = x_i q_i + k_{iT}$ yields

$$q_1^\mu V_{\mu\nu}^{ab} q_2^\nu \approx \frac{k_{1T}^\mu}{x_1} \frac{k_{2T}^\nu}{x_2} V_{\mu\nu}^{ab} \approx \frac{s}{m_H^2} k_{1T}^\mu k_{2T}^\nu V_{\mu\nu}^{ab} \quad (4)$$

since $2k_1 \cdot k_2 \approx x_1 x_2 s \approx m_H^2$. Note that it is as if the gluons which fuse to produce the Higgs are transversely polarized, $\epsilon_i \sim k_{iT}$. Moreover, in the limiting case that the outgoing quarks carry no transverse momentum $Q_T = -k_{1T} = k_{2T}$ and so $\epsilon_1 = -\epsilon_2$. This is an important result; it clearly generalizes to the statement that the centrally produced system should have a vanishing z -component of angular momentum in the limit that the protons scatter through zero angle (i.e. $q_{iT}^2 \ll Q_T^2$). Since we are experimentally interested in very small angle scattering this selection rule is effective. One immediate consequence is that the Higgs decay to b -quarks may now be viable. This is because, for massless quarks, the lowest order $q\bar{q}$ background vanishes identically (it does not vanish at NLO). The leading order $b\bar{b}$ background is therefore suppressed by a factor $\sim m_b^2/m_H^2$. Beyond leading order, one also needs to worry about the $b\bar{b}g$ final state.

Returning to the task in hand, we can write the colour averaged amplitude as

$$\frac{\text{Im}A}{s} \approx \frac{N_c^2 - 1}{N_c^2} \times 4\alpha_s^2 \int \frac{d^2\mathbf{Q}_T}{\mathbf{Q}_T^2 \mathbf{k}_{1T}^2 \mathbf{k}_{2T}^2} \frac{-\mathbf{k}_{1T} \cdot \mathbf{k}_{2T}}{m_H^2} V. \quad (5)$$

Using $d^3\mathbf{q}_1' d^3\mathbf{q}_2' d^3\mathbf{q}_H \delta^{(4)}(q_1 + q_2 - q_1' - q_2' - q_H) = d^2\mathbf{q}_{1T}' d^2\mathbf{q}_{2T}' dy E_H$ (y is the rapidity of the Higgs) the cross-section is therefore

$$\frac{d\sigma}{d^2\mathbf{q}_{1T}' d^2\mathbf{q}_{2T}' dy} \approx \left(\frac{N_c^2 - 1}{N_c^2} \right)^2 \frac{\alpha_s^6}{(2\pi)^5} \frac{G_F}{\sqrt{2}} \left[\int \frac{d^2\mathbf{Q}_T}{2\pi} \frac{\mathbf{k}_{1T} \cdot \mathbf{k}_{2T}}{\mathbf{Q}_T^2 \mathbf{k}_{1T}^2 \mathbf{k}_{2T}^2} \frac{2}{3} \right]^2 \quad (6)$$

and for simplicity here we have taken the large top mass limit of V (i.e. $m_t \gg m_H$). We are mainly interested in the forward scattering limit whence

$$\frac{\mathbf{k}_{1T} \cdot \mathbf{k}_{2T}}{\mathbf{Q}_T^2 \mathbf{k}_{1T}^2 \mathbf{k}_{2T}^2} \approx -\frac{1}{\mathbf{Q}_T^4}.$$

As it stands, the integral over Q_T diverges. Let us not worry about that for now and instead turn our attention to how to convert this parton level cross-section into the hadron level cross-section we need.²

What we really want is the hadronic matrix element which represents the coupling of two gluons into a proton, and this is really an off-diagonal parton distribution function [23]. At present we don't have much knowledge of these distributions, however we do know the diagonal gluon distribution function. Figure 2 illustrates the Durham prescription for coupling the two gluons into a proton rather than a quark. The factor K would equal unity if $x' = x$ and $k_T = 0$ which is the diagonal limit. That we should, in the amplitude, replace a factor of $\alpha_s C_F/\pi$ by $\partial G(x, Q_T)/\partial \ln Q_T^2$ can be easily derived starting from the DGLAP equation for evolution off an initial quark distribution given by $q(x) = \delta(1-x)$. The Durham approach makes use of a result derived in [24] which states that in the case $x' \ll x$ and $k_T^2 \ll Q_T^2$ the off-diagonality can be approximated by a multiplicative factor, K . Assuming a Gaussian form factor suppression for the k_T -dependence they estimate that

$$K \approx e^{-bk_T^2/2} \frac{2^{2\lambda+3} \Gamma(\lambda + 5/2)}{\sqrt{\pi} \Gamma(\lambda + 4)} \quad (7)$$

¹We can do this because $x_i \sim m_H/\sqrt{s}$ whilst the other Sudakov components are $\sim Q_T^2/s$.

²We note that (6) was first derived by Bialas and Landshoff, except that they made a factor of 2 error in the Higgs width to gluons.

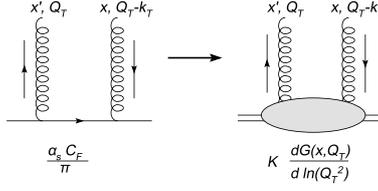


Fig. 2: The recipe for replacing the quark line (left) by a proton line (right).

and this result is obtained assuming a simple power-law behaviour of the gluon density, i.e. $G(x, Q) \sim x^{-\lambda}$. For the production of a 120 GeV Higgs boson at the LHC, $K \sim 1.2 \times e^{-bk_T^2/2}$. In the cross-section, the off-diagonality therefore provides an enhancement of $(1.2)^4 \approx 2$. Clearly the current lack of knowledge of the off-diagonal gluon is one source of uncertainty in the calculation. We also do not really know what to take for the slope parameter b . It should perhaps have some dependence upon Q_T and for $Q_T \sim 1.5$ GeV, which it will turn out is typical for a 120 GeV scalar Higgs, one might anticipate the same k_T -dependence as for diffractive J/ψ production which is well measured, i.e. $b \approx 4 \text{ GeV}^{-2}$.

Thus, after integrating over the transverse momenta of the scattered protons we have

$$\frac{d\sigma}{dy} \approx \frac{1}{256\pi b^2} \frac{\alpha_s G_F \sqrt{2}}{9} \left[\int \frac{d^2 \mathbf{Q}_T}{\mathbf{Q}_T^4} f(x_1, Q_T) f(x_2, Q_T) \right]^2 \quad (8)$$

where $f(x, Q) \equiv \partial G(x, Q) / \partial \ln Q^2$ and we have neglected the exchanged transverse momentum in the integrand. Notice that in determining the total rate we have introduced uncertainty in the normalisation arising from our lack of knowledge of b . This uncertainty, as we shall soon see, is somewhat diminished as the result of a similar b -dependence in the gap survival factor.

We should about the fact that our integral diverges in the infra-red. Fortunately we have missed some crucial physics. The lowest order diagram is not enough, virtual graphs possess logarithms in the ratio Q_T/m_H which are very important as $Q_T \rightarrow 0$; these logarithms need to be summed to all orders. This is Sudakov physics: thinking in terms of real emissions we must be sure to forbid real emissions into the final state. Let's worry about real gluon emission off the two gluons which fuse to make the Higgs. The emission probability for a single gluon is (assuming for the moment a fixed coupling α_s)

$$\frac{C_A \alpha_s}{\pi} \int_{Q_T^2}^{m_H^2/4} \frac{dp_T^2}{p_T^2} \int_{p_T}^{m_H/2} \frac{dE}{E} \sim \frac{C_A \alpha_s}{4\pi} \ln^2 \left(\frac{m_H^2}{Q_T^2} \right).$$

The integration limits are kinematic except for the lower limit on the p_T integral. The fact that emissions below Q_T are forbidden arises because the gluon not involved in producing the Higgs completely screens the colour charge of the fusing gluons if the wavelength of the emitted radiation is long enough, i.e. if $p_T < Q_T$. Now we see how this helps us solve our infra-red problem: as $Q_T \rightarrow 0$ so the screening gluon fails to screen and real emission off the fusing gluons cannot be suppressed. To see this argument through to its conclusion we realise that multiple real emissions exponentiate and so we can write the non-emission probability as

$$e^{-S} = \exp \left(- \frac{C_A \alpha_s}{\pi} \int_{Q_T^2}^{m_H^2/4} \frac{dp_T^2}{p_T^2} \int_{p_T}^{m_H/2} \frac{dE}{E} \right). \quad (9)$$

As $Q_T \rightarrow 0$ the exponent diverges and the non-emission probability vanishes faster than any power of Q_T . In this way our integral over Q_T becomes (its value is finite):

$$\int \frac{dQ_T^2}{Q_T^4} f(x_1, Q_T) f(x_2, Q_T) e^{-S}. \quad (10)$$

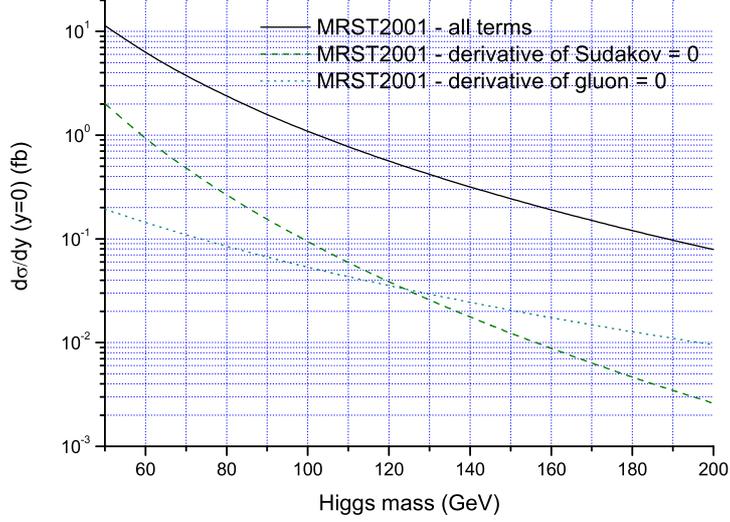


Fig. 3: The Higgs cross-section at zero rapidity, and the result obtained if one were to assume that $\partial G(x, Q)/\partial Q = 0$ or that $\partial S/\partial Q = 0$.

There are two loose ends to sort out before moving on. Firstly, note that emission off the screening gluon is less important since there are no associated logarithms in m_H/Q_T . Secondly, (9) is correct only so far as the leading double logarithms. It is of considerable practical importance to correctly include also the single logarithms. To do this we must re-instate the running of α_s and allow for the possibility that quarks can be emitted. Including this physics means we ought to use

$$e^{-S} = \exp \left(- \int_{Q_T^2}^{m_H^2/4} \frac{dp_T^2}{p_T^2} \frac{\alpha_s(p_T^2)}{2\pi} \int_0^{1-\Delta} dz [z P_{gg}(z) + \sum_q P_{qg}(z)] \right) \quad (11)$$

where $\Delta = 2p_T/m_H$, and $P_{gg}(z)$ and $P_{qg}(z)$ are the leading order DGLAP splitting functions. To correctly sum all single logarithms requires some care in that what we want is the distribution of gluons in Q_T with no emission up to m_H , and this is in fact [25]

$$\tilde{f}(x, Q_T) = \frac{\partial}{\partial \ln Q_T^2} \left(e^{-S/2} G(x, Q_T) \right).$$

The integral over Q_T is therefore

$$\int \frac{dQ_T^2}{Q_T^4} \tilde{f}(x_1, Q_T) \tilde{f}(x_2, Q_T) \quad (12)$$

which reduces to (10) in the double logarithmic approximation where the differentiation of the Sudakov factor is subleading.

The numerical effect of correctly including the single logarithms is large. For production of a 120 GeV Higgs at the LHC, there is a factor ~ 30 enhancement compared to the double logarithmic approximation, with a large part of this coming from terms involving the derivative of the Sudakov. Figure 3 shows just how important it is to keep those single logarithmic terms coming from differentiation of the Sudakov factor. For the numerical results we used the MRST2001 leading order gluon [26], as included in LHAPDF [27]. Here and elsewhere (unless otherwise stated), we use a NLO QCD K-factor of 1.5 and the one-loop running coupling with $n_f = 4$ and $\Lambda_{\text{QCD}} = 160$ MeV. As discussed in the next paragraph, we also formally need an infra-red cut-off Q_0 for the Q_T -integral; we take $Q_0 = 0.3$ GeV

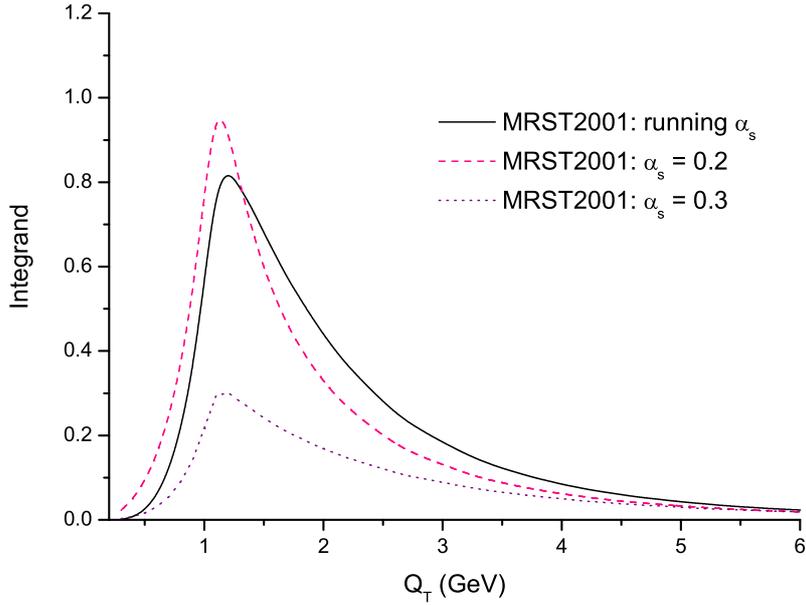


Fig. 4: The integrand of the Q_T integral for three different treatments of α_s and $m_H = 120$ GeV.

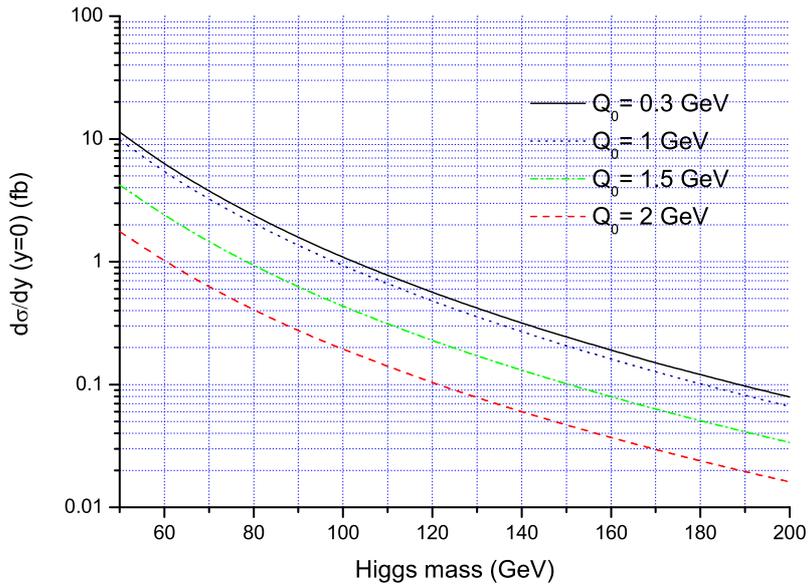


Fig. 5: The Higgs cross-section dependence upon the infra-red cutoff Q_0 .

although as we shall see results are insensitive to Q_0 provided it is small enough. Finally, all our results include an overall multiplicative “gap survival factor” of 3% (gap survival is discussed shortly).

Formally there is the problem of the pole in the QCD coupling at $p_T = \Lambda_{\text{QCD}}$. However, this problem can be side-stepped if the screening gluon has “done its job” sufficiently well and rendered an integrand which is peaked at $Q_T \gg \Lambda_{\text{QCD}}$ since an infra-red cutoff on p_T can then safely be introduced. We must be careful to check whether or not this is the case in processes of interest. Indeed, a saddle point estimate of (10) reveals that

$$\exp(\langle \ln Q_T \rangle) \sim \frac{m_H}{2} \exp\left(-\frac{c}{\alpha_s}\right) \quad (13)$$

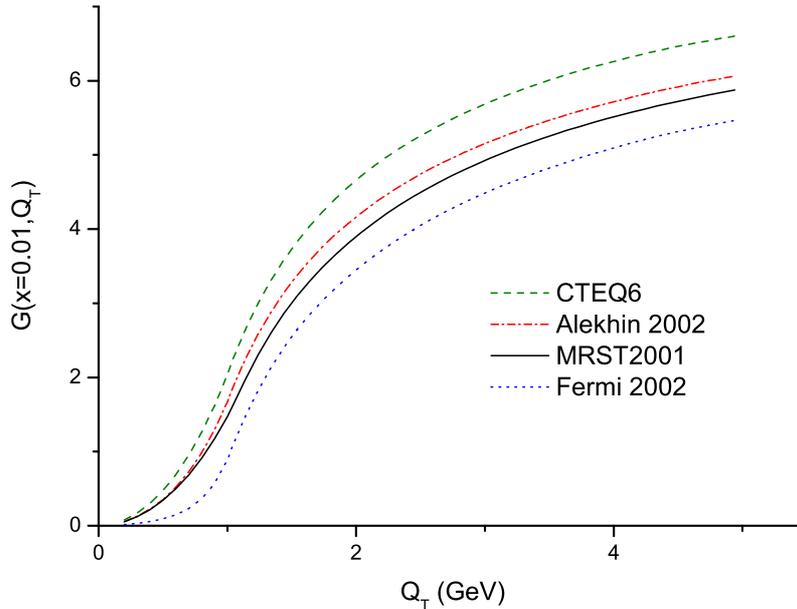


Fig. 6: The gluon density function in four different parameterisations.

where c is a constant if the gluon density goes like a power of Q_T^2 . Clearly there is a tension between the Higgs mass, which encourages a large value of the loop momentum, and the singular behaviour of the $1/Q_T^4$ factor which encourages a low value. Also, as α_s reduces so real emission is less likely and the Sudakov suppression is less effective in steering Q_T away from the infra-red. Putting in the numbers one estimates that $\exp(\langle \ln Q_T^2 \rangle) \approx 4 \text{ GeV}^2$ for the production of a 120 GeV scalar at the LHC which is just about large enough to permit an analysis using perturbative QCD. Figure 4 provides the quantitative support for these statements in the case of a Higgs of mass 120 GeV. The integrand of the Q_T integral in equation (12) is shown for both running and fixed α_s . We see that the integrand peaks just above 1 GeV and that the Sudakov factor becomes increasingly effective in suppressing the cross-section as α_s increases. Although it isn't too easy to see on this plot, the peak does move to higher values of Q_T as α_s increases in accord with (13). This plot also illustrates quite nicely that the cross-section is pretty much insensitive to the infra-red cutoff for $Q_0 < 1 \text{ GeV}$ and this is made explicit in Figure 5.

Discussion of the infra-red sensitivity would not be complete without returning to the issue of the unintegrated gluon density. In all our calculations we model the off-diagonality as discussed below equation (7) and we shan't discuss this source of uncertainty any further here.³ Figure 6 shows the gluon density $G(x, Q)$ as determined in four recent global fits (rather arbitrarily chosen to illustrate the typical variety) [26, 28–30]. Apart from the Fermi2002 fit, they are all leading order fits. Now, none of these parameterisations go down below $Q = 1 \text{ GeV}$, so what is shown in the figure are the gluons extrapolated down to $Q = 0$. We have extrapolated down assuming that the gluon and its derivative are continuous at $Q = 1 \text{ GeV}$ and that $G(x, Q) \sim Q^2$ at $Q \rightarrow 0$.⁴ The gluons plotted in Figure 6 are all determined at $x = 0.01$ which would be the value probed in the production of a 120 GeV Higgs at $y = 0$ at the LHC. The key point is to note that it is hard to think of any reasonable parameterisation of the gluon below 1 GeV which could give a substantial contribution to the cross-section. The Sudakov factor suppresses the low Q^2 region and also the size of the gluon and its derivative are crucial, and one cannot keep both of these large for $Q < 1 \text{ GeV}$. Figure 7 shows the integrand of the Q_T integral for different fits to the gluon. In all cases the contribution below 1 GeV is small, although there are clearly important uncertainties

³We actually assume a constant enhancement factor of 1.2 per gluon density.

⁴To be precise we extrapolate assuming $G(x, Q) \sim Q^{2+(\gamma-2)Q}$.

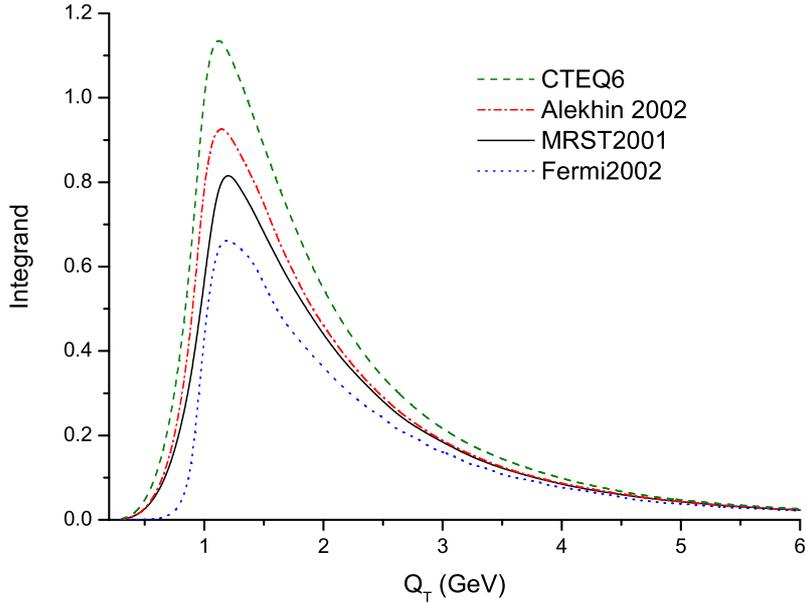


Fig. 7: The integrand of the Q_T integral for four recent global fits to the gluon.

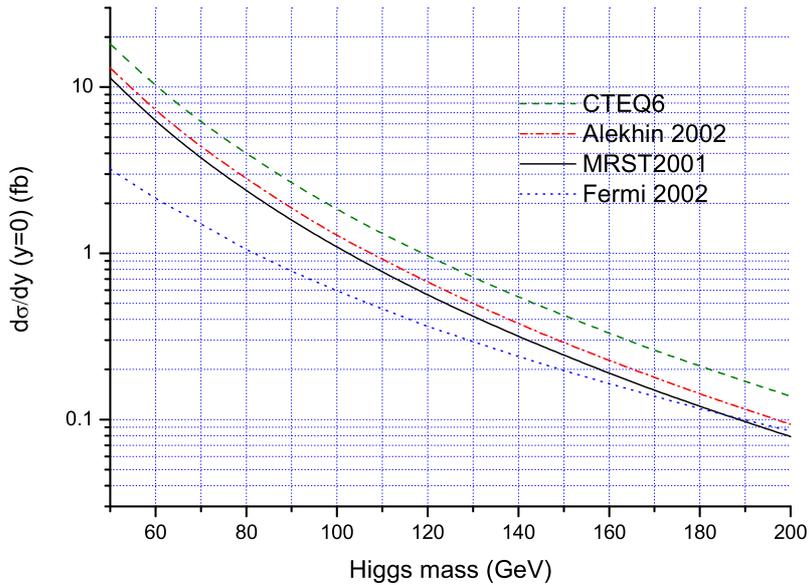


Fig. 8: The Higgs cross-section for four recent global fits to the gluon.

in the cross-section. These uncertainties are better seen in Figure 8 which illustrates that one might anticipate a factor of a few uncertainty from this source.

We note that although a variety of parameterizations are presented in Figure 8 the way that the actual Q_T dependence of the integrand is obtained is the same in each case. In [31, 32] the uncertainties arising from the way the unintegrated parton densities are obtained from the integrated ones are examined. Here we have followed the prescription presented in [33] which amounts to performing one backward step in a DGLAP parton shower. However, it is known that such showers tend to underestimate the hardness of, for example, the W/Z p_\perp spectra in hadron colliders unless a large intrinsic transverse momentum is added to the perturbative k_\perp distribution of the colliding partons [34, 35]. In [32] it was

shown that adding such an intrinsic transverse momentum would harden the Q_T distribution of the integrand in (12) for small Q_T which in turn lowers the cross-section for central exclusive Higgs production by a factor 2 (for a Gaussian intrinsic transverse momentum with $\langle k_{\perp}^2 \rangle = 2 \text{ GeV}^2$). Investigations into how one could use unintegrated gluon densities obtained by CCFM [36] and LDC [37] evolution for central exclusive Higgs production have also been performed [32]. However, as discussed in more detail elsewhere in these proceedings [23], the available parameterizations, which are all fitted to HERA data only, are not constrained enough to allow for reliable predictions for Higgs production at the LHC.

This is perhaps a good place to mention pseudo-scalar production, as might occur in an extension to the Standard Model. The scalar product, $\mathbf{k}_{1T} \cdot \mathbf{k}_{2T}$, in (6) now becomes $(\mathbf{k}_{1T} \times \mathbf{k}_{2T}) \cdot \mathbf{n}$, where \mathbf{n} is a unit vector along the beam axis. After performing the angular integral the only surviving terms are proportional to the vector product of the outgoing proton transverse momenta, i.e. $\mathbf{q}_1' \times \mathbf{q}_2'$. Notice that this term vanishes, in accord with the spin-0 selection rule, as $\mathbf{q}_i' \rightarrow 0$. Notice also that the integrand now goes like $\sim 1/Q_T^6$ (in contrast to the $1/Q_T^4$ in the scalar case). As a result c in (13) is larger (in fact it is linearly proportional to the power of Q_T) and the mean value of Q_T smaller. This typically means that pseudo-scalar production is not really accessible to a perturbative analysis.

The Sudakov factor has allowed us to ensure that the exclusive nature of the final state is not spoiled by perturbative emission off the hard process. What about non-perturbative particle production? The protons can in principle interact quite apart from the perturbative process discussed hitherto and this interaction could well lead to the production of additional particles. We need to account for the probability that such emission does not occur. Provided the hard process leading to the production of the Higgs occurs on a short enough timescale, we might suppose that the physics which generates extra particle production factorizes and that its effect can be accounted for via an overall factor multiplying the cross-section we have just calculated. This is the “gap survival factor”. Gap survival is discussed in detail elsewhere in these proceedings and so we’ll not dwell on it here [38].

The gap survival, S^2 , is given by

$$d\sigma(p + H + p | \text{no soft emission}) = d\sigma(p + H + p) \times S^2$$

where $d\sigma(p + H + p)$ is the differential cross-section computed above. The task is to estimate S^2 . Clearly this is not straightforward since we cannot utilize QCD perturbation theory. Let us at this stage remark that data on a variety of processes observed at HERA, the Tevatron and the LHC can help us improve our understanding of “gap survival”.

The model presented here provides a good starting point for understanding the more sophisticated treatments [39–41]. Dynamically, one expects that the likelihood of extra particle production will be greater if the incoming protons collide at small transverse separation compared to collisions at larger separations. The simplest model which is capable of capturing this feature is one which additionally assumes that there is a single soft particle production mechanism, let us call it a “re-scattering event”, and that re-scattering events are independent of each other for a collision between two protons at transverse separation r . In such a model we can use Poisson statistics to model the distribution in the number of re-scattering events per proton-proton interaction:

$$P_n(r) = \frac{\chi(r)^n}{n!} \exp(-\chi(r)) . \quad (14)$$

This is the probability of having n re-scattering events where $\chi(r)$ is the mean number of such events for proton-proton collisions at transverse separation r . Clearly the important dynamics resides in $\chi(r)$; we expect it to fall monotonically as r increases and that it should be much smaller than unity for r much greater than the QCD radius of the proton. Let us for the moment assume we know $\chi(r)$, then we can determine S^2 via

$$S^2 = \frac{\int dr d\sigma(r) \exp(-\chi(r))}{\int dr d\sigma(r)} \quad (15)$$

where $d\sigma(r)$ is the cross-section for the hard process that produces the Higgs expressed in terms of the transverse separation of the protons. Everything except the r dependence of $d\sigma$ cancels when computing S^2 and so we need focus only on the dependence of the hard process on the transverse momenta of the scattered protons (\mathbf{q}_i'), these being Fourier conjugate to the transverse position of the protons, i.e.

$$\begin{aligned} d\sigma(r) &\propto \left[\left(\int d^2\mathbf{q}_1' e^{i\mathbf{q}_1' \cdot \mathbf{r}/2} \exp(-b\mathbf{q}_1'^2/2) \right) \times \left(\int d^2\mathbf{q}_2' e^{-i\mathbf{q}_2' \cdot \mathbf{r}/2} \exp(-b\mathbf{q}_2'^2/2) \right) \right]^2 \\ &\propto \exp\left(-\frac{r^2}{2b}\right). \end{aligned} \quad (16)$$

Notice that since the b here is the same as that which enters into the denominator of the expression for the total rate there is the aforementioned reduced sensitivity to b since as b decreases so does S^2 (since the collisions are necessarily more central) and what matters is the ratio S^2/b^2 .

It remains for us to determine the mean multiplicity $\chi(r)$. If there really is only one type of re-scattering event⁵ independent of the hard scattering, then the inelastic scattering cross-section can be written

$$\sigma_{\text{inelastic}} = \int d^2\mathbf{r} (1 - \exp(-\chi(r))), \quad (17)$$

from which it follows that the elastic and total cross-sections are

$$\sigma_{\text{elastic}} = \int d^2\mathbf{r} (1 - \exp(-\chi(r)/2))^2, \quad (18)$$

$$\sigma_{\text{total}} = 2 \int d^2\mathbf{r} (1 - \exp(-\chi(r)/2)). \quad (19)$$

There is an abundance of data which we can use to test this model and we can proceed to perform a parametric fit to $\chi(r)$. This is essentially what is done in the literature, sometimes going beyond a single-channel approach. Suffice to say that this simple approach works rather well. Moreover, it also underpins the models of the underlying event currently implemented in the PYTHIA [42] and HERWIG [43, 44] Monte Carlo event generators which have so far been quite successful in describing many of the features of the underlying event [45–47]. Typically, models of gap survival predict S^2 of a few percent at the LHC. Although data support the existing models of gap survival there is considerable room for improvement in testing them further and in so doing gaining greater control of what is perhaps the major theoretical uncertainty in the computation of exclusive Higgs production. In all our plots we took $S^2 = 3\%$ which is typical of the estimates in the literature for Higgs production at the LHC.

3 Other Models

We'll focus in this section mainly on the model presented by what we shall call the Saclay group [13]. The model is a direct implementation of the original Bialas-Landshoff (BL) calculation [20] supplemented with a gap survival factor. It must be emphasised that BL did not claim to have computed for an exclusive process, indeed they were careful to state that “additional...interactions...will generate extra particles...Thus our calculation really is an inclusive one”.

Equation (6) is the last equation that is common to both models. BL account for the coupling to the proton in a very simple manner: they multiply the quark level amplitude by a factor of 9 (which corresponds to assuming that there are three quarks in each proton that are able to scatter off each other). Exactly like the Durham group they also include a form factor suppression factor $\exp(-bq_{iT}^2)$ for each proton at the cross-section level with $b = 4 \text{ GeV}^{-2}$. Since BL are not interested in suppressing radiation, they do have a problem with the infra-red since there is no Sudakov factor. They dealt with this

⁵Clearly this is not actually the case, but such a “single channel eikonal” model has the benefit of being simple.

by following the earlier efforts of Landshoff and Nachtmann (LN) in replacing the perturbative gluon propagators with non-perturbative ones [48,49]:

$$\frac{g^2}{k^2} \rightarrow A \exp(-k^2/\mu^2).$$

Rather arbitrarily, $g^2 = 4\pi$ was assumed, except for the coupling of the gluons to the top quark loop, where $\alpha_s = 0.1$ was used.

Following LN, μ and A are determined by assuming that the $p\bar{p}$ elastic scattering cross-section at high energy can be approximated by the exchange of two of these non-perturbative gluons between the 3×3 constituent quarks: the imaginary part of this amplitude determines the total cross-section for which there are data which can be fitted to. In order to carry out this procedure successfully, one needs to recognize that a two-gluon exchange model is never going to yield the gentle rise with increasing centre-of-mass energy characteristic of the total cross-section. BL therefore also include an additional ‘‘reggeization’’ factor of $s^{\alpha(t)-1}$ in the elastic scattering amplitude where

$$\alpha(t) = 1 + \epsilon + \alpha' t$$

is the pomeron trajectory which ensures that a good fit to total cross-section data is possible for $\epsilon = 0.08$ and $\alpha' = 0.25 \text{ GeV}^{-2}$. In this way the two-gluon system is modelling pomeron exchange. They found that $\mu \approx 1 \text{ GeV}$ and $A \approx 30 \text{ GeV}^{-2}$ gave a good fit to the data. Similarly, the amplitude for central Higgs production picks up two reggeization factors.

The inclusive production of a Higgs boson in association with two final state protons is clearly much more infra-red sensitive than the exclusive case where the Sudakov factor saves the day. Having said that, the Saclay model does not include the Sudakov suppression factor. Instead it relies upon the behaviour of the non-perturbative gluon propagators to render the Q_T integral finite. As a result, the typical Q_T is much smaller than in the Durham case. Indeed it may be sufficiently small to make the approximation $Q_T^2 \gg q_{iT}^2$ invalid which means that the spin-0 selection rule is no longer applicable.

Pulling everything together, the Saclay model of the cross-section for $pp \rightarrow p + H + p$ gives

$$\frac{d\sigma}{d^2\mathbf{q}_{1T}' d^2\mathbf{q}_{2T}' dy} \approx S^2 \left(\frac{N_c^2 - 1}{N_c^2} \right)^2 \frac{\alpha_s^2}{(2\pi)^5} \left(\frac{g^2}{4\pi} \right)^4 \frac{G_F}{\sqrt{2}} e^{-bq_{1T}^2} e^{-bq_{2T}^2} x_1^{2-2\alpha(q_{1T}^2)} x_2^{2-2\alpha(q_{2T}^2)} \left[9 \int \frac{d^2\mathbf{Q}_T}{2\pi} \mathbf{Q}_T^2 \left(\frac{A}{g^2} \right)^3 \exp(-3\mathbf{Q}_T^2/\mu^2) \frac{2}{3} \right]^2 \quad (20)$$

The reggeization factors depend upon the momentum fractions x_1 and x_2 which satisfy $x_1 x_2 s = m_H^2$ and $y = \frac{1}{2} \ln(x_1/x_2)$. The only difference⁶ between this and the original BL result is the factor of S^2 . Integrating over the final state transverse momenta and simplifying a little gives

$$\frac{d\sigma}{dy} \approx S^2 \frac{\pi}{b + 2\alpha' \ln(1/x_1)} \frac{\pi}{b + 2\alpha' \ln(1/x_2)} \left(\frac{N_c^2 - 1}{N_c^2} \right)^2 \frac{G_F}{\sqrt{2}} \frac{\alpha_s^2}{(2\pi)^5} \frac{1}{(4\pi)^4} \left(\frac{s}{m_H^2} \right)^{2\epsilon} \frac{1}{g^4} \left[\frac{A^3 \mu^4}{3} \right]^2. \quad (21)$$

Figure 9 shows how the Saclay model typically predicts a rather larger cross-section with a weaker dependence upon m_H than the Durham model. The weaker dependence upon m_H arises because the Saclay model does not contain the Sudakov suppression, which is more pronounced at larger m_H , and also because of the choice $\epsilon = 0.08$. A larger value would induce a correspondingly more rapid fall. The Durham use of the gluon density function does indeed translate into an effective value of ϵ substantially larger than 0.08. This effect is also to be seen in the dependence of the model predictions upon the centre-of-mass energy as shown in Figure 10. We have once again assumed a constant $S^2 = 3\%$ in this figure despite the fact that one does expect a dependence of the gap survival factor upon the energy.

⁶Apart from the factor 2 error previously mentioned.

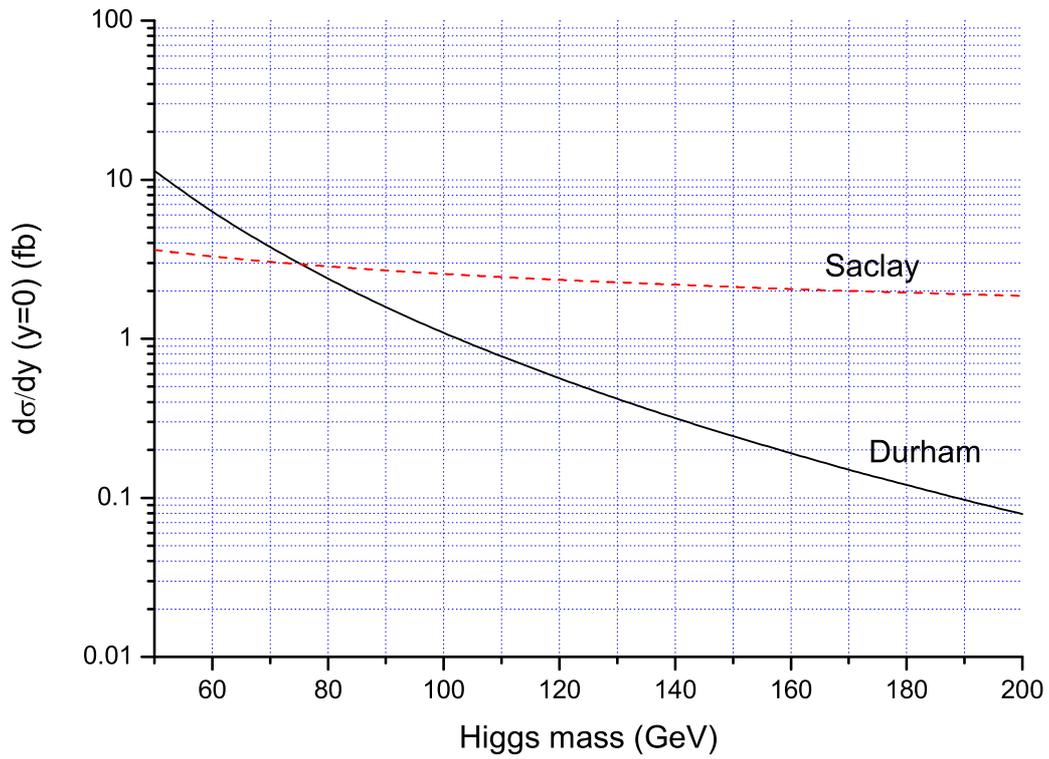


Fig. 9: Comparing dependence upon m_H of the Saclay and Durham predictions. $S^2 = 3\%$ in both cases.

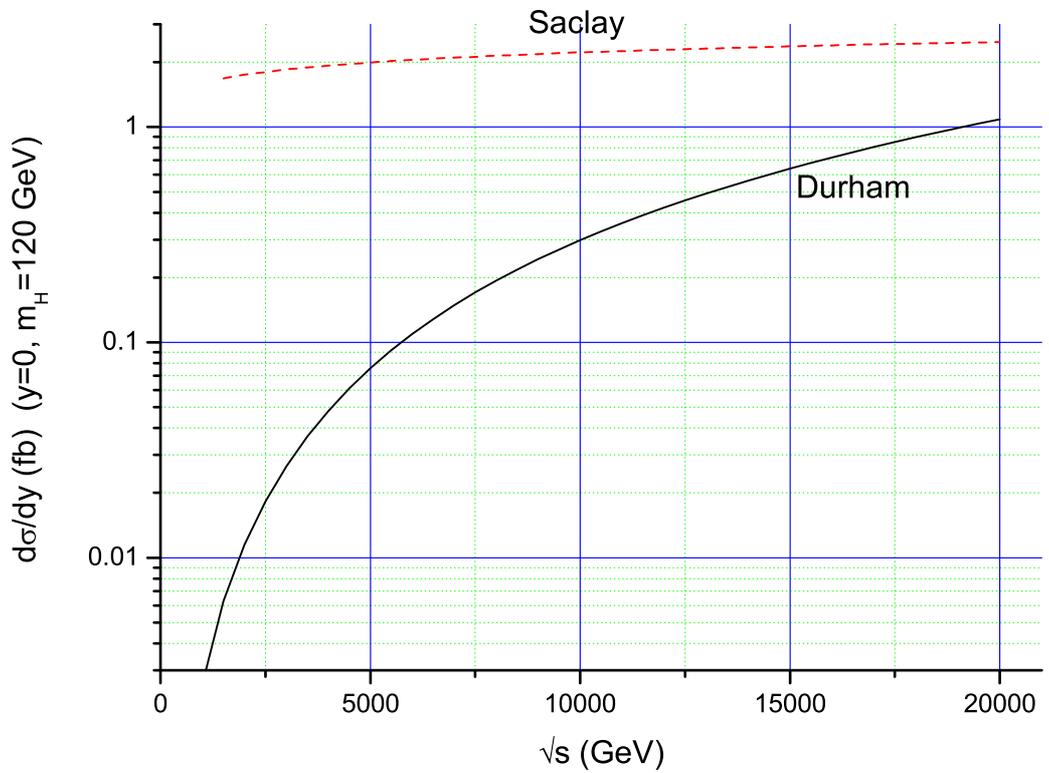


Fig. 10: Comparing dependence upon \sqrt{s} of the Saclay and Durham predictions for $m_H = 120$ GeV.

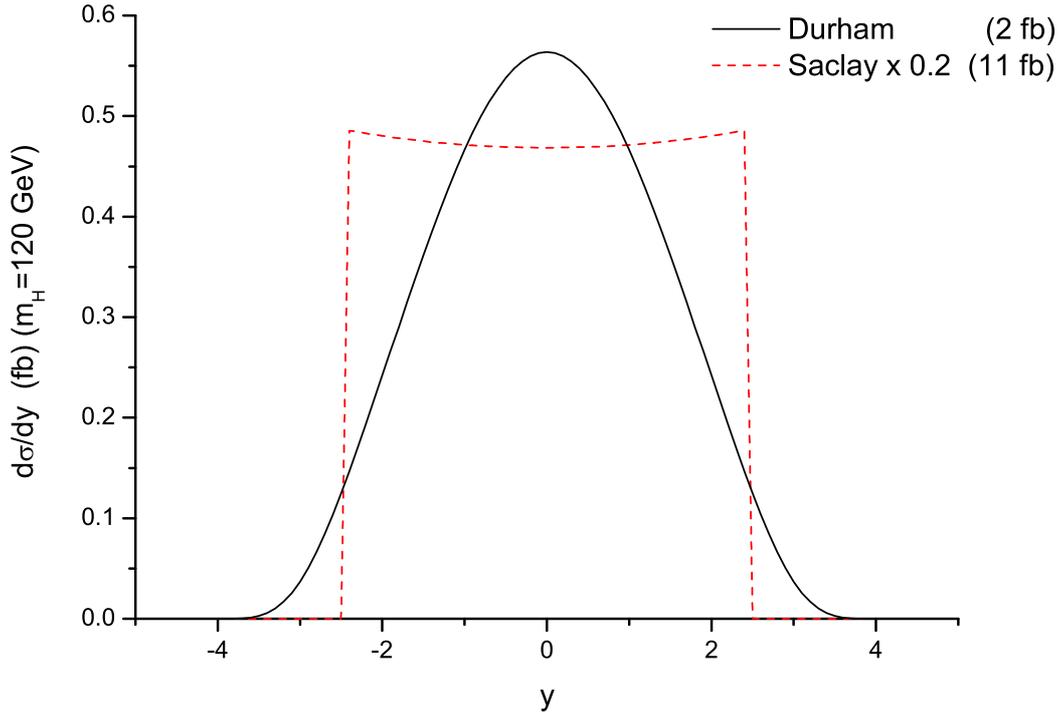


Fig. 11: Comparing the y dependence of the Saclay and Durham predictions for $m_H = 120$ GeV. Note that the Saclay prediction has been reduced by a factor 5 to make the plot easier to read. The numbers in parenthesis are the total cross-sections, i.e. integrated over rapidity.

Figure 11 compares the rapidity dependence of the Higgs production cross-section in the two models. The Saclay prediction is almost y -independent. Indeed the only y -dependence is a consequence of $\alpha' \neq 0$. In both models the calculations are really only meant to be used for centrally produced Higgs bosons, i.e. $|y|$ not too large since otherwise one ought to revisit the approximations implicit in taking the high-energy limit. Nevertheless, the Durham prediction does anticipate a fall as $|y|$ increases, and this is coming because one is probing larger values of x in the gluon density. In contrast, the Saclay prediction does not anticipate this fall and so a cutoff in rapidity needs to be introduced in quoting any cross-section integrated over rapidity. In Figure 11 a cut on $x_{1,2} < 0.1$ is made (which is equivalent to a cut on $|y| < 2.5$) for the Saclay model. After integrating over rapidity, the Durham model predicts a total cross-section of 2 fb for the production of a 120 GeV Higgs boson at the LHC whilst the Saclay model anticipates a cross-section a factor ~ 5 larger.

The essentially non-perturbative Saclay prediction clearly has some very substantial uncertainties associated with it. The choice of an exponentially falling gluon propagator means that there is no place for a perturbative component. However, as the Durham calculation shows, there does not seem to be any good reason for neglecting contributions from perturbatively large values of Q_T . It also seems entirely reasonable to object on the grounds that one should not neglect the Sudakov suppression factor and that including it would substantially reduce the cross-section.

In [18], the Sudakov factor of equation (11) is included, with the rest of the amplitude computed following Bialas-Landshoff. The perturbative Sudakov factor is also included in the approach of [19], albeit only at the level of the double logarithms. This latter approach uses perturbative gluons throughout the calculation but Regge factors are included to determine the coupling of the gluons into the protons, i.e. rather than the unintegrated partons of the Durham model. In both cases the perturbative Sudakov factor, not surprisingly, is important.

4 Concluding remarks

We hope to have provided a detailed introduction to the Durham model for central exclusive Higgs production. The underlying theory has been explained and the various sources of uncertainty highlighted with particular emphasis on the sensitivity of the predictions to gluon dynamics in the infra-red region. We also made some attempt to mention other approaches which can be found in the literature.

The focus has been on the production of a Standard Model Higgs boson but it should be clear that the formalism can readily be applied to the central production of any system X which has a coupling to gluons and invariant mass much smaller than the beam energy. There are many very interesting possibilities for system X which have been explored in the literature and we have not made any attempt to explore them here [2, 3, 8, 11, 15–17]. Nor have we paid any attention to the crucial challenge of separating signal events from background [5, 9]. The inclusion of theoretical models into Monte Carlo event generators and a discussion of the experimental issues relating to central exclusive particle production have not been considered here but can be found in other contributions to these proceedings [50, 51].

It seems that perturbative QCD can be used to compute cross-sections for processes of the type $pp \rightarrow p + X + p$. The calculations are uncertain but indicate that rates ought to be high enough to be interesting at the LHC. In the case that the system X is a pair of jets there ought to be the possibility to explore this physics at the Tevatron [52]. Information gained from such an analysis would help pin down theoretical uncertainties, as would information on the rarer but cleaner channel where X is a pair of photons [53]. Of greatest interest is when X contains “new physics” whence this central exclusive production mechanism offers new possibilities for its exploration.

5 Acknowledgments

Special thanks to Hannes Jung for all his efforts in making the workshop go so well. Thanks also to Brian Cox, Markus Diehl, Valery Khoze, Peter Landshoff, Leif Lönnblad, James Monk, Leszek Motyka, Andy Pilkington and Misha Ryskin for very helpful discussions.

References

- [1] M.G. Albrow and A. Rostovtsev, *Searching for the Higgs at hadron colliders using the missing mass method*. E-Print Archive: hep-ph/0009336.
- [2] M.G. Albrow et al, *FP420: An R&D proposal to investigate the feasibility of installing proton tagging detectors in the 420-m region at LHC*. CERN-LHCC-2005-025.
- [3] B.E. Cox, AIP Conference Proceedings **753**, 103 (2005). E-Print Archive: hep-ph/0409144.
- [4] J. Kallioopuska, T. Mäki, N. Marola, R. Orava, K. Österberg and M. Ottela, *Resolution studies of the leading proton measurement in exclusive central diffraction at LHC*.
- [5] V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C19**, 477 (2001). Erratum-ibid. **C20**, 599 (2001). E-Print Archive: hep-ph/0011393.
- [6] V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C24**, 581 (2002). E-Print Archive: hep-ph/0203122.
- [7] V.A. Khoze, A.D. Martin and M.G. Ryskin, Phys. Lett. **B401**, 330 (1997). E-Print Archive: hep-ph/9701419;
V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C14**, 525 (2000). E-Print Archive: hep-ph/0002072.
- [8] V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C23**, 311 (2002). E-Print Archive: hep-ph/0111078.
- [9] A. De Roeck, V.A. Khoze, A.D. Martin, R. Orava and M.G. Ryskin, Eur. Phys. J. **C25**, 391 (2002). E-Print Archive: hep-ph/0207042.

- [10] V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C26**, 229 (2002). E-Print Archive: hep-ph/0207313;
A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C31**, 387 (2003). E-Print Archive: hep-ph/0307064.
- [11] A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C33**, 261 (2004). E-Print Archive: hep-ph/0311023;
V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C34**, 327 (2004). E-Print Archive: hep-ph/0401078;
K. Belotsky, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C36**, 503 (2004). E-Print Archive: hep-ph/0406037.
- [12] B.E. Cox et al, *Detecting the standard model Higgs boson in the WW decay channel using forward proton tagging at the LHC*. E-Print Archive: hep-ph/0505240.
- [13] M. Boonekamp, R. Peschanski and C. Royon, Nucl. Phys. **B669**, 277 (2003). Erratum-ibid. **B676**, 493 (2004). E-Print Archive: hep-ph/0301244.
- [14] M. Boonekamp, R. Peschanski and C. Royon, Phys. Lett. **B598**, 243 (2004). E-Print Archive: hep-ph/0406061;
M. Boonekamp, J. Cammin, R. Peschanski and C. Royon. E-Print Archive: hep-ph/0504199.
- [15] M. Boonekamp, J. Cammin, S. Lavignac, R. Peschanski and C. Royon. E-Print Archive: hep-ph/0506275.
- [16] J.R. Ellis, J.S. Lee and A. Pilaftsis, Phys. Rev. **D71**, 075007 (2005). E-Print Archive: hep-ph/0502251.
- [17] B.E. Cox, J.R. Forshaw, J.S. Lee, J. Monk and A. Pilaftsis, Phys. Rev. **D68**, 075004 (2003). E-Print Archive: hep-ph/0303206.
- [18] A. Bzdak, *Exclusive Higgs and dijet production by double pomeron exchange: The CDF upper limits*. E-print Archive: hep-ph/0504086.
- [19] V.A. Petrov and R.A. Ryutin, Eur. Phys. J. **C36**, 509 (2004). E-Print Archive: hep-ph/0311024.
- [20] A. Bialas and P.V. Landshoff, Phys. Lett. **B256**, 540 (1991).
- [21] B.E. Cox and J.R. Forshaw, Comput. Phys. Commun. **144**, 104 (2002). E-Print Archive: hep-ph/0010303.
- [22] M. Boonekamp and T. Kucs, Comput. Phys. Commun. **167**, 217 (2005). E-Print Archive: hep-ph/0312273.
- [23] J.C. Collins et al, *Unintegrated parton density functions*. These proceedings.
- [24] A.G. Shuvaev, K.J. Golec-Biernat, A.D. Martin and M.G. Ryskin, Phys. Rev. **D60**, 014015 (1999). E-Print Archive: hep-ph/9902410.
- [25] M.A. Kimber, A.D. Martin, M.G. Ryskin, Phys. Rev. **D63**, 114027 (1999). E-Print Archive: hep-ph/0101348;
M.A. Kimber, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C12**, 655 (2000). E-Print Archive: hep-ph/9911379;
A.D. Martin and M.G. Ryskin, Phys. Rev. **D64**, 094017 (2001). E-Print Archive: hep-ph/0107149.
- [26] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Eur. Phys. J. **C23**, 73 (2002). E-print Archive: hep-ph/0110215.
- [27] <http://durpdg.dur.ac.uk/1hapdf4/>.
- [28] J. Pumplin, D.R. Stump, J. Huston, H.L. Lai, P. Nadolsky and W.K. Tung, JHEP **0207**, 012 (2002). E-print Archive: hep-ph/0201195.
- [29] S. Alekhin, Phys. Rev. **D68**, 014002 (2003). E-print Archive: hep-ph/0104052.
- [30] W.T. Giele, S.A. Keller and D.A. Kosower. E-print Archive: hep-ph/0104052.
- [31] L. Lönnblad and M. Sjödahl, JHEP **02**, 042 (2004). E-Print Archive: hep-ph/0311252.

- [32] L. Lönnblad and M. Sjö Dahl, JHEP **05**, 038 (2005). E-Print Archive: hep-ph/0412111.
- [33] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Eur. Phys. J. **C4**, 463 (1998). E-Print Archive: hep-ph/9803445.
- [34] E. Thome, *Perturbative and nonperturbative effects in transverse momentum generation*. E-Print Archive: hep-ph/0401121.
- [35] E.L. Nurse, *A measurement of the inclusive $z/\gamma^* \rightarrow \mu^+\mu^-$ cross section and study of W and Z events in $p\bar{p}$ collisions at D0*. FERMILAB-THESIS-2005-05.
- [36] M. Ciafaloni, Nucl. Phys. **B296**, 49 (1988);
S. Catani, F. Fiorani and G. Marchesini, Phys. Lett. **B234**, 339 (1990);
S. Catani, F. Fiorani and G. Marchesini, Nucl. Phys. **B336**, 18 (1990);
G. Marchesini, Nucl. Phys. **B445**, 49 (1995). E-Print Archive: hep-ph/9412327.
- [37] B. Andersson, G. Gustafson and J. Samuelsson, Nucl. Phys. **B467**, 443 (1996);
B. Andersson, G. Gustafson and H. Kharraziha, Phys. Rev. **D57**, 5543 (1998). E-Print Archive: hep-ph/9711403.
- [38] E. Gotsman et al, *Survival probabilities of large rapidity gaps*. These proceedings.
- [39] E. Gotsman, E. Levin and U. Maor, Phys. Lett. **B438**, 229 (1998). E-Print Archive: hep-ph/9804404;
E. Gotsman, E. Levin and U. Maor, Phys. Lett. **B452**, 387 (1999). E-Print Archive: hep-ph/9901416;
E. Gotsman, E. Levin and U. Maor, Phys. Rev. **D60**, 094011 (1999). E-Print Archive: hep-ph/9902294;
U. Maor, *LRG production of di-jets as a probe of s-channel unitarity*. E-Print Archive: hep-ph/0406303.
- [40] V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C18**, 167 (2000). E-Print Archive: hep-ph/0007359;
A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. **C21**, 521 (2001). E-Print Archive: hep-ph/0105145.
- [41] M.M. Block and F. Halzen, Phys. Rev. **D63**, 114004 (2001). E-Print Archive: hep-ph/0101022.
- [42] T. Sjöstrand and M. van Zijl, Phys. Rev. **D36**, 2019 (1987).
- [43] J.M. Butterworth and J.R. Forshaw, J. Phys. **G19**, 1657 (1993).
- [44] J.M. Butterworth, J.R. Forshaw and M.H. Seymour, Z. Phys. **C72**, 637 (1996). E-Print Archive: hep-ph/9601371.
- [45] I. Borozan and M.H. Seymour, JHEP **0209**, 015 (2002). E-Print Archive: hep-ph/0207283.
- [46] T. Sjöstrand and P.Z. Skands, JHEP **0403**, 053 (2004). E-Print Archive: hep-ph/0402078.
- [47] K. Odagiri, JHEP **0408**, 019 (2004). E-Print Archive: hep-ph/0407008.
- [48] P.V. Landshoff and O. Nachtmann, Z. Phys. **C35**, 405 (1987).
- [49] A. Donnachie and P.V. Landshoff, Nucl. Phys. **B311**, 509 (1989).
- [50] M. Boonekamp et al, *Monte Carlo generators for central exclusive diffraction*. These proceedings.
- [51] B.E. Cox et al, *Experimental aspects of central exclusive diffraction*. These proceedings.
- [52] B.E. Cox and A. Pilkington, *Central exclusive dijet production at the Tevatron*. E-Print Archive: hep-ph/0508249.
- [53] V.A. Khoze, A.D. Martin, M.G. Ryskin and W.J. Stirling, Eur. Phys. J. **C38**, 475 (2005). E-Print Archive: hep-ph/0409037.