

# QED $\otimes$ QCD Exponentiation and Shower/ME Matching at the LHC\*

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## Abstract

We present the elements of QED $\otimes$ QCD exponentiation and its interplay with shower/ME matching in precision LHC physics scenarios. Applications to single heavy gauge boson production at hadron colliders are illustrated.

In the LHC environment, precision predictions for the effects of multiple gluon and multiple photon radiative processes will be needed to realize the true potential of the attendant physics program. For example, while the current precision tag for the luminosity at FNAL is at the  $\sim 7\%$  level [1], the high precision requirements for the LHC dictate an experimental precision tag for the luminosity at the 2% level [2]. This means that the theoretical precision tag requirement for the corresponding luminosity processes, such as single W,Z production with the subsequent decay into light lepton pairs, must be at the 1% level in order not to spoil the over-all precision of the respective luminosity determinations at the LHC. This theoretical precision tag means that multiple gluon and multiple photon radiative effects in the latter processes must be controlled to the stated precision. With this objective in mind, we have developed the theory of QED  $\otimes$  QCD exponentiation to allow the simultaneous resummation of the multiple gluon and multiple photon radiative effects in LHC physics processes, to be realized ultimately by MC methods on an event-by-event basis in the presence of parton showers in a framework which allows us to systematically improve the accuracy of the calculations without double-counting of effects in principle to all orders in both  $\alpha_s$  and  $\alpha$ .

Specifically, the new QED  $\otimes$  QCD exponentiation theory is an extension of the QCD exponentiation theory presented in Refs. [3]<sup>1</sup>. We recall that in the latter references it has been established that the following result holds for a process such as  $q + \bar{q}' \rightarrow V + n(G) + X \rightarrow \bar{\ell}\ell' + n(g) + X$ :

$$d\hat{\sigma}_{\text{exp}} = \sum_n d\hat{\sigma}^n = e^{\text{SUM}_{\text{IR}}(\text{QCD})} \sum_{n=0}^{\infty} \int \prod_{j=1}^n \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy \cdot (P_1 + P_2 - Q_1 - Q_2 - \sum k_j) + D_{\text{QCD}}} * \tilde{\beta}_n(k_1, \dots, k_n) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0} \quad (1)$$

where gluon residuals  $\tilde{\beta}_n(k_1, \dots, k_n)$ , defined by Ref. [3], are free of all infrared divergences to all orders in  $\alpha_s(Q)$ . The functions  $SUM_{\text{IR}}(\text{QCD})$ ,  $D_{\text{QCD}}$ , together with the basic infrared functions  $B_{\text{QCD}}^{\text{nls}}$ ,  $\tilde{B}_{\text{QCD}}^{\text{nls}}$ ,  $\tilde{S}_{\text{QCD}}^{\text{nls}}$  are specified in Ref. [3]. Here  $V = W^\pm, Z$ , and  $\ell = e, \mu$ ,  $\ell' = \nu_e, \nu_\mu(e, \mu)$  respectively for  $V = W^+(Z)$ , and  $\ell = \nu_e, \nu_\mu$ ,  $\ell' = e, \mu$  respectively for  $V = W^-$ . We call attention to the essential compensation between the left over genuine non-Abelian IR virtual and real singularities between  $\int dPh \tilde{\beta}_n$  and  $\int dPh \tilde{\beta}_{n+1}$  respectively that really allows us to isolate  $\tilde{\beta}_j$  and distinguishes QCD from QED, where no such compensation occurs. The result in (1) has been realized by Monte Carlo methods [3]. See also Refs. [5–7] for exact  $\mathcal{O}(\alpha_s^2)$  and Refs. [8–10] for exact  $\mathcal{O}(\alpha)$  results on the W,Z production processes which we discuss here.

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<sup>1</sup>We stress that the formal proof of exponentiation in non-Abelian gauge theories *in the eikonal approximation* is given in Ref. [4]. The results in Ref. [3] are in contrast *exact* but have an exponent that only contains the leading contribution of the exponent in Ref. [4].

The new  $QED \otimes QCD$  theory is obtained by simultaneously exponentiating the large IR terms in QCD and the exact IR divergent terms in QED, so that we arrive at the new result

$$\begin{aligned}
d\hat{\sigma}_{\text{exp}} &= e^{\text{SUM}_{\text{IR}}(\text{QCED})} \\
&\sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^n \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^m \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} \\
&e^{iy \cdot (p_1 + q_1 - p_2 - q_2 - \sum k_{j_1} - \sum k'_{j_2}) + D_{\text{QCED}}} \\
&\tilde{\beta}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0},
\end{aligned} \tag{2}$$

where the new YFS [11, 12] residuals, defined in Ref. [13],  $\tilde{\beta}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m)$ , with  $n$  hard gluons and  $m$  hard photons, represent the successive application of the YFS expansion first for QCD and subsequently for QED. The functions  $\text{SUM}_{\text{IR}}(\text{QCED})$ ,  $D_{\text{QCED}}$  are determined from their analogs  $\text{SUM}_{\text{IR}}(\text{QCD})$ ,  $D_{\text{QCD}}$  via the substitutions

$$\begin{aligned}
B_{\text{QCD}}^{\text{nls}} &\rightarrow B_{\text{QCD}}^{\text{nls}} + B_{\text{QED}}^{\text{nls}} \equiv B_{\text{QCED}}^{\text{nls}}, \\
\tilde{B}_{\text{QCD}}^{\text{nls}} &\rightarrow \tilde{B}_{\text{QCD}}^{\text{nls}} + \tilde{B}_{\text{QED}}^{\text{nls}} \equiv \tilde{B}_{\text{QCED}}^{\text{nls}}, \\
\tilde{S}_{\text{QCD}}^{\text{nls}} &\rightarrow \tilde{S}_{\text{QCD}}^{\text{nls}} + \tilde{S}_{\text{QED}}^{\text{nls}} \equiv \tilde{S}_{\text{QCED}}^{\text{nls}}
\end{aligned} \tag{3}$$

everywhere in expressions for the latter functions given in Refs. [3]. The residuals  $\tilde{\beta}_{n,m}$  are free of all infrared singularities and the result in (2) is a representation that is exact and that can therefore be used to make contact with parton shower MC's without double counting or the unnecessary averaging of effects such as the gluon azimuthal angular distribution relative to its parent's momentum direction.

In the respective infrared algebra (QCED) in (2), the average Bjorken  $x$  values

$$\begin{aligned}
x_{\text{avg}}(\text{QED}) &\cong \gamma(\text{QED}) / (1 + \gamma(\text{QED})) \\
x_{\text{avg}}(\text{QCD}) &\cong \gamma(\text{QCD}) / (1 + \gamma(\text{QCD}))
\end{aligned}$$

where  $\gamma(A) = \frac{2\alpha_A C_A}{\pi} (L_s - 1)$ ,  $A = \text{QED}, \text{QCD}$ , with  $C_A = Q^2, C_F$ , respectively, for  $A = \text{QED}, \text{QCD}$  and the big log  $L_s$ , imply that QCD dominant corrections happen an order of magnitude earlier than those for QED. This means that the leading  $\tilde{\beta}_{0,0}$ -level gives already a good estimate of the size of the interplay between the higher order QED and QCD effects which we will use to illustrate (2) here.

More precisely, for the processes  $pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \bar{\ell}\ell' + n'(\gamma) + m(g) + X$ , where  $V = W^\pm, Z$ , and  $\ell = e, \mu$ ,  $\ell' = \nu_e, \nu_\mu(e, \mu)$  respectively for  $V = W^+(Z)$ , and  $\ell = \nu_e, \nu_\mu$ ,  $\ell' = e, \mu$  respectively for  $V = W^-$ , we have the usual formula (we use the standard notation here [13])

$$\begin{aligned}
d\sigma_{\text{exp}}(pp \rightarrow V + X \rightarrow \bar{\ell}\ell' + X') &= \\
&\sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{\text{exp}}(x_i x_j s),
\end{aligned} \tag{4}$$

and we use the result in (2) here with semi-analytical methods and structure functions from Ref. [14]. A Monte Carlo realization will appear elsewhere [15].

We do not attempt in the *present* discussion to replace HERWIG [16] and/or PYTHIA [17] – we intend *here* to combine our exact YFS calculus with HERWIG and/or PYTHIA *by using the latter to generate a parton shower starting from the initial  $(x_1, x_2)$  point at factorization scale  $\mu$  after this point is provided by the  $\{F_i\}$* . This combination of theoretical constructs can be systematically improved with exact results order-by-order in  $\alpha_s$ , where currently the state of the art in such a calculation is the work in Refs. [18] which accomplishes the combination of an exact  $\mathcal{O}(\alpha_s)$  correction with HERWIG. We note that, even in this latter result, the gluon azimuthal angle is averaged in the combination. We note that the recent alternative parton distribution function evolution MC algorithm in Refs. [19] can also be used

in our theoretical construction here. Due to its lack of the appropriate color coherence [20], we do not consider ISAJET [21] here.

To illustrate how the combination with Pythia/Herwig can proceed, we note that, for example, if we use a quark mass  $m_q$  as our collinear limit regulator, DGLAP [22] evolution of the structure functions allows us to factorize all the terms that involve powers of the big log  $L_c = \ln \mu^2/m_q^2 - 1$  in such a way that the evolved structure function contains the effects of summing the leading big logs  $L = \ln \mu^2/\mu_0^2$  where we have in mind that the evolution involves initial data at the scale  $\mu_0$ . The result is therefore independent of  $m_q$  for  $m_q \downarrow 0$ . In the context of the DGLAP theory, the factorization scale  $\mu$  represents the largest  $p_\perp$  of the gluon emission included in the structure function. In practice, when we use these structure functions with an exact result for the residuals in (2), it means that we must in the residuals omit the contributions from gluon radiation at scales below  $\mu$ . This can be shown to amount in most cases to replacing  $L_s = \ln \hat{s}/m_q^2 - 1 \rightarrow L_{\text{nls}} = \ln \hat{s}/\mu^2$  but in any case it is immediate how to limit the  $p_T$  in the gluon emission<sup>2</sup> so that we do not double count effects. In other words, we apply the standard QCD factorization of mass singularities to the cross section in (2) in the standard way. We may do it with either the mass regulator for the collinear singularities or with dimensional regularization of such singularities – the final result should be independent of this regulator. This would in practice mean the following: We first make an event with the formula in (4) which would produce an initial beam state at  $(x_1, x_2)$  for the two hard interacting partons at the factorization scale  $\mu$  from the structure functions  $\{F_j\}$  and a corresponding final state  $X$  from the exponentiated cross section in  $d\hat{\sigma}_{\text{exp}}(x_i x_j s)$ ; the standard Les Houches procedure [23] of showering this event  $(x_1, x_2, X)$  would then be used, employing backward evolution of the initial partons. If we restrict the  $p_T$  as we have indicated above, there would be no double counting of effects. Let us call this  $p_T$  matching of the shower from the backward evolution and the matrix elements in the QCED exponentiated cross section.

However, one could ask if it is possible to be more accurate in the use of the exact result in (2)? Indeed, it is. Just as the residuals  $\tilde{\beta}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m)$  are computed order by order in perturbation theory from the corresponding exact perturbative results by expanding the exponents in (2) and comparing the appropriate corresponding coefficients of the respective powers of  $\alpha^n \alpha_s^m$ , so too can the shower formula which is used to generate the backward evolution be expanded so that the product of the shower formula's perturbative expansion, the perturbative expansion of the exponents in (2), and the perturbative expansions of the residuals can be written as an over-all expansion in powers of  $\alpha^n \alpha_s^m$  and required to match the respective calculated exact result for given order. In this way, new shower subtracted residuals,  $\{\hat{\tilde{\beta}}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m)\}$ , are calculated that can be used for the entire gluon  $p_T$  phase space with an accuracy of the cross section that should in principle be improved compared with the first procedure for shower matching presented above. Both approaches are under investigation.

Returning to the general discussion, we compute, with and without QED,  $r_{\text{exp}} = \sigma_{\text{exp}}/\sigma_{\text{Born}}$ . For this ratio we do not use the narrow resonance approximation; for, we wish to set a paradigm for precision heavy vector boson studies. The formula which we use for  $\sigma_{\text{Born}}$  is obtained from that in (4) by substituting  $d\hat{\sigma}_{\text{Born}}$  for  $d\hat{\sigma}_{\text{exp}}$  therein, where  $d\hat{\sigma}_{\text{Born}}$  is the respective parton-level Born cross section. Specifically, we have from (1) the  $\tilde{\beta}_{0,0}$ -level result

$$\hat{\sigma}_{\text{exp}}(x_1 x_2 s) = \int_0^{v_{\text{max}}} dv \gamma_{\text{QCED}} v^{\gamma_{\text{QCED}}-1} F_{\text{YFS}}(\gamma_{\text{QCED}}) e^{\delta_{\text{YFS}}} \hat{\sigma}_{\text{Born}}((1-v)x_1 x_2 s) \quad (5)$$

where we intend the well-known results for the respective parton-level Born cross sections and the value of  $v_{\text{max}}$  implied by the experimental cuts under study. What is new here is the value for the QED $\otimes$ QCD exponent

$$\gamma_{\text{QCED}} = \left\{ 2Q_f^2 \frac{\alpha}{\pi} + 2C_F \frac{\alpha_s}{\pi} \right\} L_{\text{nls}} \quad (6)$$

where  $L_{\text{nls}} = \ln x_1 x_2 s/\mu^2$  when  $\mu$  is the factorization scale.

<sup>2</sup>Here, we refer to both on-shell and off-shell emitted gluons.

The functions  $F_{\text{YFS}}(\gamma_{\text{QCED}})$  and  $\delta_{\text{YFS}}(\gamma_{\text{QCED}})$  are well-known [12] as well:

$$\begin{aligned} F_{\text{YFS}}(\gamma_{\text{QCED}}) &= \frac{e^{-\gamma_{\text{QCED}}\gamma_E}}{\Gamma(1 + \gamma_{\text{QCED}})}, \\ \delta_{\text{YFS}}(\gamma_{\text{QCED}}) &= \frac{1}{4}\gamma_{\text{QCED}} + \left(Q_f^2 \frac{\alpha}{\pi} + C_F \frac{\alpha_s}{\pi}\right) \left(2\zeta(2) - \frac{1}{2}\right), \end{aligned} \quad (7)$$

where  $\zeta(2)$  is Riemann's zeta function of argument 2, i.e.,  $\pi^2/6$ , and  $\gamma_E$  is Euler's constant, i.e., 0.5772... Using these formulas in (4) allows us to get the results

$$r_{\text{exp}} = \begin{cases} 1.1901 & , \text{QCED} \equiv \text{QCD} + \text{QED}, \text{ LHC} \\ 1.1872 & , \text{QCD}, \text{ LHC} \\ 1.1911 & , \text{QCED} \equiv \text{QCD} + \text{QED}, \text{ Tevatron} \\ 1.1879 & , \text{QCD}, \text{ Tevatron}. \end{cases} \quad (8)$$

We see that QED is at the level of .3% at both LHC and FNAL. This is stable under scale variations [13]. We agree with the results in Refs. [5, 6, 8–10] on both of the respective sizes of the QED and QCD effects. The QED effect is similar in size to structure function results found in Refs. [24–28], for further reference.

We have shown that YFS theory (EEX and CEEX) extends to non-Abelian gauge theory and allows simultaneous exponentiation of QED and QCD, QED $\otimes$ QCD exponentiation. For QED $\otimes$ QCD we find that full MC event generator realization is possible in a way that combines our calculus with Herwig and Pythia in principle. Semi-analytical results for QED (and QCD) threshold effects agree with literature on Z production. As QED is at the .3% level, it is needed for 1% LHC theory predictions. We have demonstrated a firm basis for the complete  $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$  results needed for the FNAL/LHC/RHIC/ILC physics and all of the latter are in progress.

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## References

- [1] S. Klimenko, in *Proc. HCP2002*, ed. M. Erdmann,( Karlsruhe, 2002 ) p.413.
- [2] M. Dittmar, F. Pauss and D. Zurcher,*Phys. Rev. D***56** (1997) 7284; M. Rijssenbeek, in *Proc. HCP2002*, ed. M. Erdmann,( Karlsruhe, 2002 )p. 424; M. Dittmar, *ibid.*,p.431.
- [3] B.F.L. Ward and S. Jadach, *Acta Phys.Polon.* **B33** (2002) 1543; in *Proc. ICHEP2002*, ed. S. Bentvelsen *et al.*,( North Holland, Amsterdam, 2003 ) p. 275 and references therein.
- [4] J.G.M. Gatherall, *Phys. Lett.* **B133** (1983) 90.
- [5] R. Hamberg, W. L. van Neerven and T Matsuura, *Nucl. Phys.***B359** (1991) 343.
- [6] W.L. van Neerven and E.B. Zijlstra, *Nucl. Phys.* **B382** (1992) 11; *ibid.* **B680** (2004) 513; and, references therein.
- [7] C. Anastasiou *et al.*,*Phys. Rev. D***69** (2004) 094008.
- [8] U. Baur, S. Keller and W.K. Sakumoto, *Phys. Rev. D* **57** (1998) 199; U. Baur, S. Keller and D. Wackerroth, *ibid.***59** (1998) 013002; U. Baur *et al.*, *ibid.***65** (2002) 033007, and references therein.
- [9] S. Dittmaier and M. Kramer, *Phys. Rev. D***65** (2002) 073007, and references therein
- [10] Z. A. Zykunov,*Eur. Phys. J.***C3** (2001) 9, and references therein.
- [11] D. R. Yennie, S. C. Frautschi, and H. Suura,*Ann. Phys.* **13** (1961) 379; see also K. T. Mahanthappa, *Phys. Rev.* **126** (1962) 329, for a related analysis.

- [12] See also S. Jadach *et al.*, *Comput. Phys. Commun.* **102** (1997) 229; S. Jadach, M. Skrzypek and B.F.L. Ward, *Phys. Rev. D* **55** (1997) 1206; S. Jadach, B.F.L. Ward and Z. Was, *Phys. Rev. D* **63** (2001) 113009; S. Jadach, B.F.L. Ward and Z. Was, *Comp. Phys. Commun.* **130** (2000) 260; S. Jadach *et al.*, *ibid.* **140** (2001) 432, 475.
- [13] C. Glosser *et al.*, *Mod. Phys. Lett. A* **19** (2004) 2119.
- [14] A.D. Martin *et al.*, *Phys. Rev. D* **51** (1995) 4756.
- [15] S. Jadach *et al.*, to appear.
- [16] G. Corcella *et al.*, hep-ph/0210213, and references therein.
- [17] T. Sjostrand *et al.*, hep-ph/0308153.
- [18] S. Frixione and B. Webber, *J. High Energy Phys.* **0206**(2002) 029; S. Frixione, P. Nason and B. Webber, *ibid.* **0308** (2003) 007, and references therein.
- [19] S. Jadach and M. Skrzypek, *Acta Phys. Pol.* **B35** (2004) 735; hep-ph/0504263, 0504205.
- [20] Michelangelo Mangano, private communication, 2004.
- [21] F. Paige *et al.*, hep-ph/0312045, and references therein.
- [22] G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298; Yu. L. Dokshitzer, *Sov. Phys. JETP* **46** (1977) 641; L. N. Lipatov, *Yad. Fiz.* **20** (1974) 181; V. Gribov and L. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 675, 938.
- [23] E. Boos *et al.*, hep-ph/0109068.
- [24] S. Haywood, P.R. Hobson, W. Hollik and Z. Kunszt, in *Proc. 1999 CERN Workshop on Standard Model Physics ( and more ) at the LHC, CERN-2000-004*, eds. G. Altarelli and M.L. Mangano, ( CERN, Geneva, 2000 ) p. 122.
- [25] H. Spiesberger, *Phys. Rev. D* **52** ( 1995 ) 4936.
- [26] W.J. Stirling, "Electroweak Effects in Parton Distribution Functions", talk presented at ESF Exploratory Workshop, *Electroweak Radiative Corrections to Hadronic Observables at TeV Energies* , Durham, Sept., 2003.
- [27] M. Roth and S. Weinzierl, *Phys. Lett.* **B590** (2004) 190.
- [28] W. J. Stirling *et al.*, in *Proc. ICHEP04*, eds. H. Chen *et al.* (World Sci. Publ., Singapore, 2005) p. 527.