Phenomenology of the Structure Function F_2 in lepton proton scattering

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1. INTRODUCTION

With the advent of high energy neutrino beams at CERN, BNL, ANL and electron beams at SLAC, DESY an intensive investigation of the proton structure was initiated. The differential cross section of νp resp. ep scattering contains 3 resp. 2 structure functions $F_i(x,Q^2)$ corresponding to the helicity states of the exchanged intermediate vector boson. The early measurements played an important rôle in establishing the quark-parton model of the nucleon, the existence of a sea component and a substantial neutral component not coupling directly to γ or W. This neutral component was attributed to gluons, the carriers of the strong force which found in the quantum chromo dynamics (QCD) a sound theoretical basis. Together with the discovery of the weak neutral current the foundation was laid for a comprehensive description of electroweak and strong phenomena. A worldwide program at the large research centers was set up to study the properties of the gauge forces and the nucleon structure. Intense ν and μ beams reaching in the few hundred GeV region were built and employed in many new experiments. The detectors developped into facilities and simultaneously the collaborations increased in size. The fixed target experiments had their culmination in the 1980-ies with high accuracy and high statistics, then a new era started with colliders.

A wealth of data on F_2 and xF_3 was obtained in neutrino experiments using bubble chambers (Gargamelle, BEBC, 15') and calorimeters (HPWF, CDHS, CHARM, CCFR) and in parallel on F_2 in muon experiments (EMC, BCDMS, NMC, E665). Apart from a discrepancy at small x the F_2 measurements in neutrino and muon experiments agreed well after accounting for the relevant charges. These data served for the double purpose of determining the parton distributions of the nucleon and of testing QCD. In the perturbative regime, i.e. for Q^2 larger then a few GeV^2, QCD predicts a characteristic Q^2- dependent deviation from the originally observed scaling behaviour. The comparison with the data gives access to a determination of the strong interaction coupling constant α_s and its running. The neutrino experiments with their measurements of xF_3 had the advantage of providing a clean test of QCD, since the Q^2-evolution is independent of the gluon. However, these tests were limited by statistics. On the other hand, the QCD interpretation of the high statistics F_2 data remained uncertain because of the unavoidable assumptions on the unknown form of the gluon distribution.

Since the beginning of the 1990-ies the ep collider HERA is operating and explored F_2 by the two experiments H1 and ZEUS in a considerably extended phase space region. At present, the high Q^2-region reaches up to 40000 GeV^2 for x in the valence region, while in the low Q^2-region x-values as low as 10^{-6} were reached.

2. DISPLAY OF THE F_2-DATA

F_2 is a function of the two kinematic variables x and Q^2. The data are usually displayed in terms of Q^2 on a logarithmic scale, as for instance shown in the contribution by D. Reyna[1]. Another choice is the variable

$$q = \log(1 + Q^2/Q_0^2) \tag{1}$$

The quantity Q_0^2 characterizes the transition from

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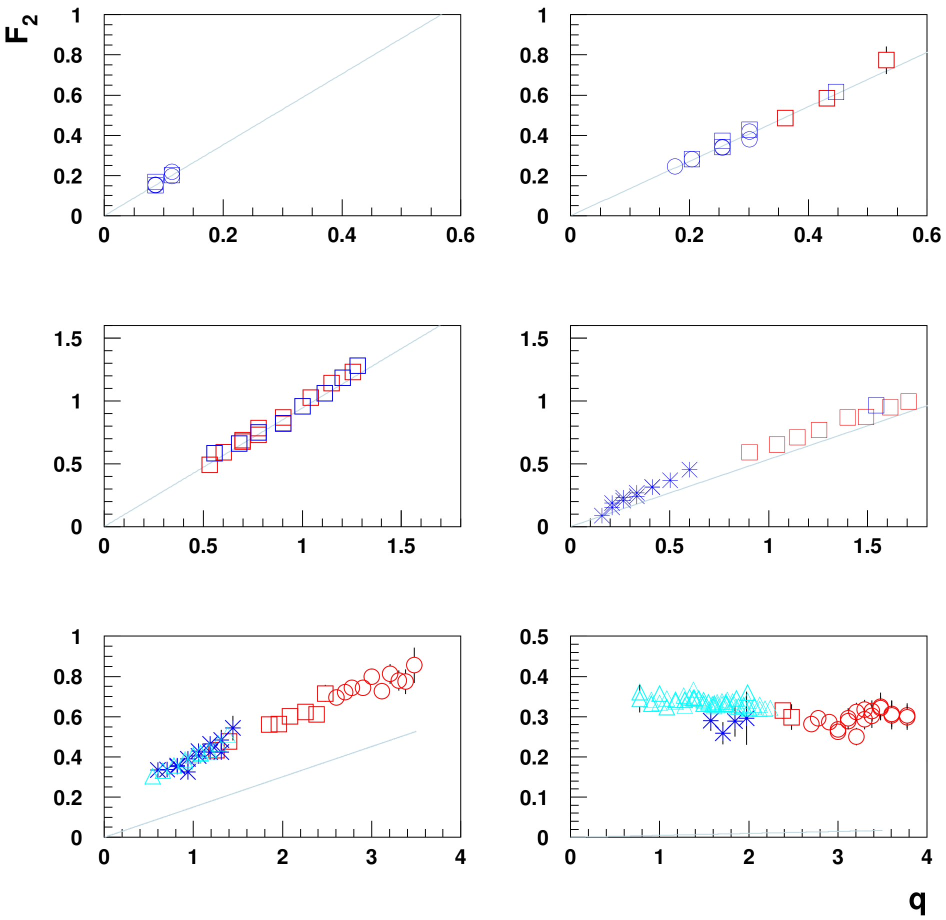

Figure 1. *F_2 as a function of q for 6 x-bins at 0.000002, 0.00002, 0.0002, 0.002, 0.02, 0.2. The HERA phase space limits the points in the smallest x-bin to $q < 0.15$. With increasing x the reach in q extends, as seen by the change in scale. The slopes steadily decrease and get negative at the largest x-value. The data at small x were obtained by the HERA collaborations H1 and ZEUS. At $x > 0.001$ also the Fixed Target data from NMC and E665 are contributing at low q. Good agreement among the independent data sets is observed. The line corresponds to $u_1(x) \cdot q$.*

nonperturbative to perturbative physics. In analyzing the first low-x data of H1 it was noticed[2] that F_2 is linear in $\log Q^2$ for fixed x and extrapolates approximately to a common value $Q_0^2 \approx 0.5$ GeV^2. This suggested $\log Q^2/Q_0^2$ as a good quantity. With the advent of more and more low-Q^2 data, also below Q_0^2, $\log Q^2/Q_0^2$ became inadequate and was minimally extended[4] to $\log(1+Q^2/Q_0^2)$, thus providing a positive definite quantity with the properties :

- for large Q^2 : $q \rightarrow \log Q^2$
- for small Q^2 : $q \rightarrow Q^2$

The F_2-measurements are displayed in figure 1 as a function of q. For reasons of restricted space they are shown just for a few x-bins across 6 orders of magnitude. In addition to previous presentations [7] the recently published very low x BPT-data by ZEUS[5] is also included. All data lie far outside the resonance region.

3. ANALYSIS

An interpretation of the data in the low-x regime is given in ref. [3].

In this section a phenomenological analysis is given combining studies reported to previous meetings of the Deep Inelastic Scattering Conferences([2,4,7,8]).

The most prominent feature of the data is the linear behaviour of F_2 in q over the whole phase space considered. The data are well represented by :

$$F_2(x,q) = u_0(x) + u_1(x) \cdot (q - \langle q \rangle) \qquad (2)$$

with two uncorrelated functions $u_0(x)$ and $u_1(x)$ carrying the information on F_2. The meaning of u_0 and u_1 is then :

$$u_0(x) = F_2(x, \langle q \rangle)$$
$$u_1(x) = \partial F_2(x,q)/\partial q$$

3.1. The Slopes

The derivative $\partial F_2(x,q)/\partial q$ is equal to the slope $u_1(x)$ and follows directly from the 2-parameter fit to each x-bin. The result is plotted in figure 2. At small x the slopes vary logarithmically and can be represented by

$$\partial F_2(x,q)/\partial q = 0.41 \cdot \log x_0/x \qquad (3)$$

The parameter x_0 is 0.04 and characterizes the transition from sea to valence dominance. For increasing x the slopes change over to the typical behaviour of the well studied valence. Infact, for x above 0.1 it has been shown [8] that the slopes of the valence alone, $\partial v(x,q)^{mrst}/\partial q$, calculated from the MRST-parametrisation[6] agree with the measured slopes.

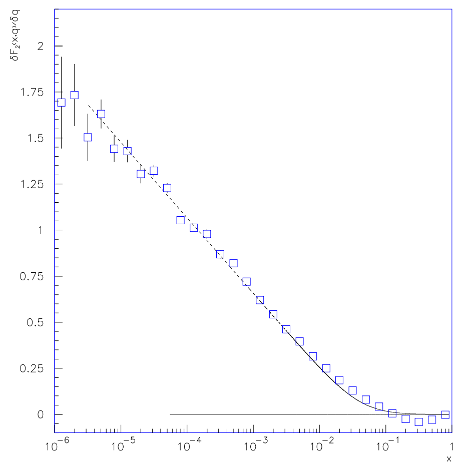

Figure 2. *The measured slopes $\partial F_2(x,q)/\partial q$ versus x. The shape is logarithmic down to about 0.04. For $x >$ 0.1 the slopes become small and negative.*

As a side remark, it is instructive[7] to compare $\partial F_2(x,q)/\partial q$ with the usual $\partial F_2(x,q)/\partial \log Q^2$. They are related by :

$$\partial F_2(x,q)/\partial \log Q^2 = \frac{Q^2}{Q^2+Q_0^2} \cdot \partial F_2(x,q)/\partial q \qquad (4)$$

The Q^2-dependent term reflects the variable change from $q = \log(1 + Q^2/Q_0^2)$ to $\log Q^2$. Its effect is negligible as long as $Q^2 \gg Q_0^2$, but for small Q^2 the explicit dependence on both x and Q^2 causes a considerable complication. Had the derivatives to be determined directly from the data a nonlinear fit in $\log Q^2$ for given x would have to be performed. It is readily seen that the appearence of the jacobian Q^2-dependent factor implies a strong drop of $\partial F_2(x,q)/\partial \log Q^2$ at small x, because for decreasing x the available phase space restricts Q^2 to smaller and smaller values, e.g. $x = 10^{-5}$ implies $Q^2 < 1$ GeV^2 and the drop gradually becomes proportional to Q^2. All these complications are avoided by the use of q as variable.

3.2. The two components of F_2

Sofar only $u_1(x)$, i.e. the information contained in the slopes of F_2, has been considered. The lines $u_1(x) \cdot q$ drawn in figure 1 reproduce well the structure function F_2 as long as x is below 0.001, which is the region dominated by the sea. Equivalently, in terms of eq. 2, the data are consistent with $u_0(x) - \langle q \rangle \cdot u_1(x) = 0$. In other words, the structure function, when extrapolated in q towards 0, approaches 0. This is a fundamental property and follows from the conservation of the electromagnetic current.

For larger values of x the linearity of F_2 with q still holds true, but another feature emerges, namely $u_0(x) - \langle q \rangle \cdot u_1(x)$ now becomes significantly non-zero reflecting the presence of the valence contribution. Figure 1 shows the offset with respect to the line $u_1(x) \cdot q$. The offset is seen to be approximately Q^2-independent for x up to about 0.2.

For still larger values of x the structure function F_2 is valence dominated and in good agreement with the perturbative QCD evolution[6].

It is then possible to distinguish two components to F_2 closely related to u_0 and u_1 :

$$S(x,q) = [\partial F_2/\partial q - \partial v^{mrst}/\partial q] \cdot q$$
$$V(x,q) = F_2(x,q) - S(x,q)$$

The first component is obtained from the measured slopes by subtracting the calculated slopes, $\partial v^{mrst}/\partial q$, of the valence distribution. The size of the subtraction is on the order of 0.02 below $x \approx 0.1$, while for $x > 0.1$ the data are explained almost entirely by the valence slopes. Thus the component $S(x,q)$ approaches $u_1(x) \cdot q$ for small x, i.e. reproduces F_2, while it tends smoothly to 0 for $x \to 1$.

The second component is obtained by subtracting from the measured structure function $F_2(x,q)$ the q-dependent component $S(x,q)$ involving only information on slopes. It is shown in figure 3. By construction, $V(x,q)$ is very little

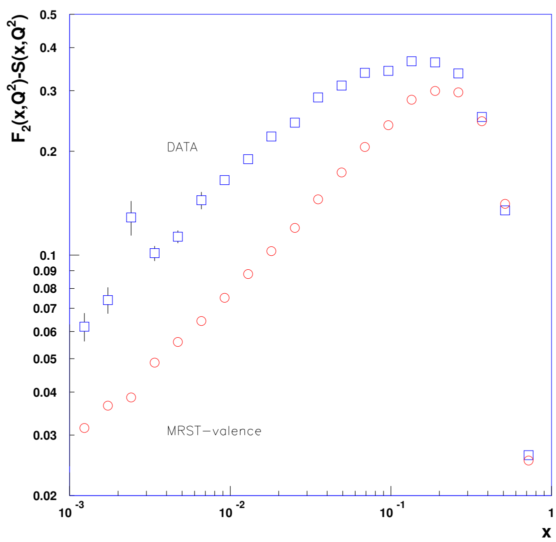

Figure 3. *The large-x component of F_2 at $\langle Q^2 \rangle$. The inner points correspond to the valence distribution v^{mrst} predicted by MRST for the same points (with $Q^2 >$ 1.3 GeV^2).*

dependent on q and exploits the complementary information provided by $u_0(x)$. The low-x falloff behaves approximately as $\sqrt{x}$. The large-x side agrees well with the v^{mrst}-distribution. At $x < 0.2$ the component V significantly exceeds the MRST-valence distribution. The excess distribution looks similar to the valence distribution, but its peak position is shifted to 0.08. For consistency, the evaluation of S and V should be iterated.

4. CONCLUSIONS

The observed linear behaviour of F_2 in the variable q has been used to delineate a logarithmic low-x component proportional to q from a large-x valence-like component.

Acknowledgement

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Questions and Answers

S. Bethke, MPI München : In your introduction you presented the groundbreaking discoveries and measurements from DIS in the past 3 decades. Would you dare to foresee the development for the coming decade ?

Compared to the exciting discovery phase we are now in a phase with many precise data covering a large area of phase space. A solid theoretical basis exists for QCD in the perturbative regime.

An immediate step is the upgrade program of HERA which will enable high statistics data taking and thus a detailed exploration of both the high-Q^2 and low-Q^2 regions. The structure functions F_2 and xF_3 can then be evaluated simultaneously in neutral and charged current interactions. The large lever arm in Q^2 should allow for an unambigous determination of the gluon distribution in the proton. On the other hand, the HERA data in the transition region from large to small Q^2 are challenging QCD as a theory also for nonperturbative physics.

A. Contogouris, Athens : You find that $F_2(x, q)$ is linear in your parameter $q = \log(1 + Q^2/Q_0^2)$. Has the linearity, and the use of it, any theoretical foundation or is it an empirical parameter which, though, might eventually prove to be useful to theorists ?

The parameter has been developped from the data themselves (see section 2). It has the virtue of bridging two scales in Q^2, namely a linear Q^2 scale for small Q^2 and a logarithmic scale otherwise. QCD applied to the transition region from nonperturbative to perturbative physics may shed light on the empirically found linear behaviour of F_2 with q.