

# Log-Log behaviour of $F_2$ at low $x$ in the $Q^2$ range from 0.1 to 35 $\text{GeV}^2$

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## 1. INTRODUCTION

In previous DIS workshops[1] it has been shown that in the low- $x$  region the structure function  $F_2$  is linear in terms of the empirical variable :

$$\xi = \log(x_0/x) \log(1 + Q^2/Q_0^2) \quad (1)$$

## 2. DATA

New data from the H1[2,3] and ZEUS[4] Collaborations are considered (see fig 1). In or-

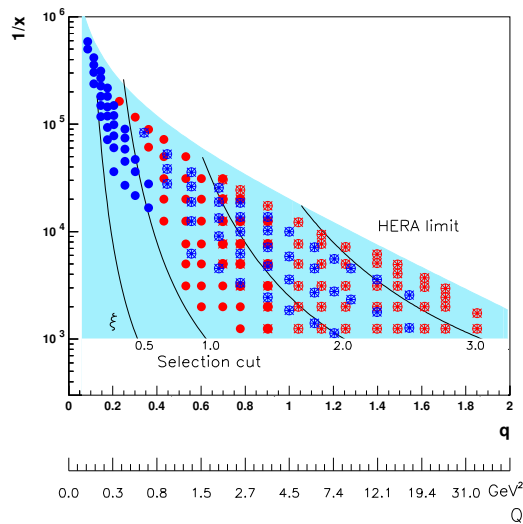


Figure 1. Phase space considered for  $F_2$ -analysis.

der to stay safely away from the valence region only data satisfying  $x < 0.001$  are selected. The data samples cover nearly the whole phase space with  $Q^2$ -values extending up to the HERA kinematic limit which increases with  $x$ . The lowest  $Q^2$

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value is 0.11  $\text{GeV}^2$ [5]. All experiments have carefully studied various sources of systematic errors. Since the  $F_2$  measurements are statistically quite accurate, the account of systematic uncertainties in comparing the 4 data sets is indispensable,

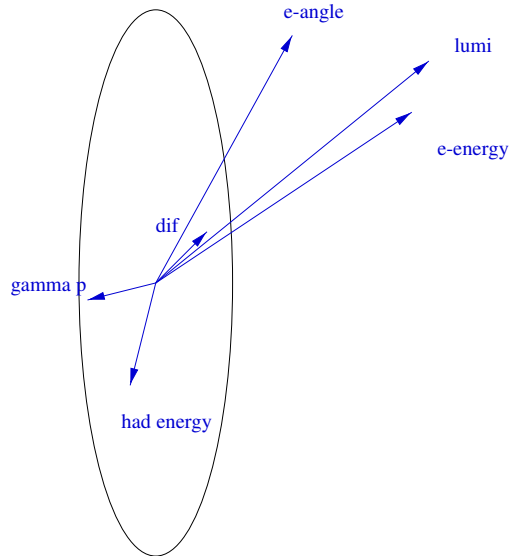


Figure 2. Shift of  $(u_0, u_1)$  ellipse due to systematic errors.

when testing the hypothesis

$$F_2(x, Q^2) = u_0 + u_1 * (\xi - \langle \xi \rangle) \quad (2)$$

The arrows in fig.2 illustrate the displacement of the  $(u_0, u_1)$ -ellipse (uncorrelated errors) due to a  $1\sigma$  shift induced by each systematic source in the H1-1995 data[2].

### 3. $Q^2$ -DEPENDENCE

$F_2(x, q)$  behaves linearly in  $q = \log(1 + Q^2/Q_0^2)$  with  $Q_0^2$  chosen to be  $0.5 \text{ GeV}^2$ , as seen in the fig.3 for the 6 low  $x$ -bins and in fig.4 for the following 6  $x$ -bins up to  $x=0.001$ . A 2-parameter fit of the form  $u_0(x) + u_1(x) \cdot (q - \langle q \rangle)$  is performed for each  $x$ -bin. The extrapolation of  $F_2(x, q)$  towards  $q=0$ , and thus  $Q^2=0$ , is well consistent with 0 for all  $x$ -bins. This is in agreement with the theoretical expectation based on the conservation of the electromagnetic current. The fit quality for

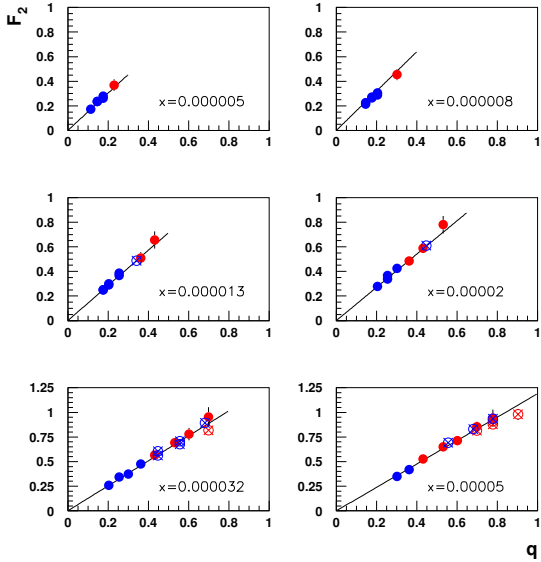


Figure 3.  $F_2$  versus  $q$  for fixed lower  $x$ -bins in increasing order.

the 12  $x$ -bins is good apart from two bins, as seen in fig. 4. The range of the measurements in  $q$  is  $x$ -dependent (see fig. 1). In particular the lowest  $x$ -bins are strongly phase space limited. It is worth noting that the linearity in  $q$  is valid down to the smallest measured  $x$ -bins. The variable  $q$  has the property of interpolating smoothly between  $Q^2$  and  $\log Q^2$ . The transition region is governed by the value of  $Q_0^2$ .

The fitted slopes  $u_1(x)$  of the lines in figs. 3, 4 are plotted in fig. 5 and are consistent with

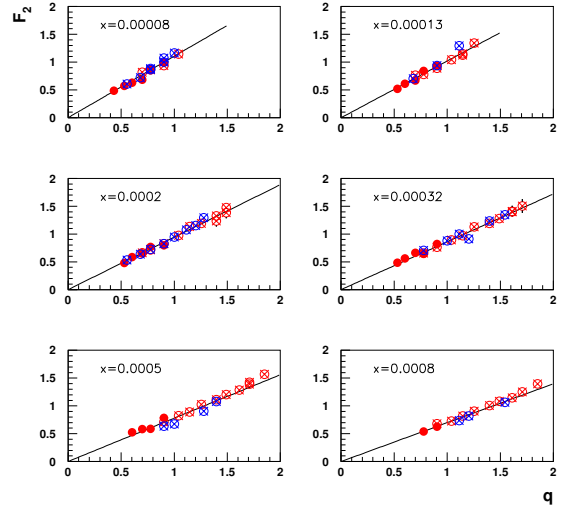


Figure 4.  $F_2$  versus  $q$  for fixed higher  $x$  in increasing order.

a linear logarithmic behaviour, i.e.  $u_1(x) = m \log(x_0/x)$ , where  $x_0=0.04$  and  $m=0.41$ . The lines drawn in the two figures. 3,4 correspond to  $0.41 \log(0.04/x)$ .

### 4. RESULTS

The linearity of  $F_2(x, q)$  in  $q$  implies that the derivative of  $F_2$  with respect to  $q$  is a function of  $x$  alone. Then, the derivative of  $F_2$  with respect to  $\log Q^2$  is bound to exhibit a characteristically different behaviour at low values of  $Q^2$ , namely :

$$\begin{aligned} \partial F_2 / \partial \log Q^2 &= \partial F_2 / \partial q \cdot \delta q / \delta \log Q^2 \quad (3) \\ &= m \log e \log(x_0/x) \frac{Q^2}{Q^2 + Q_0^2} \end{aligned}$$

The appearance of the Jacobian factor  $\delta q / \delta \log Q^2$  causes a sharp decrease of  $\partial F_2 / \partial \log Q^2$  as  $x$  decreases, which corresponds to small values of  $Q^2$  (see fig.1), in contrast to the smooth behaviour of  $\partial F_2 / \partial q$  in  $x$ , as seen in fig.5.

The empirical properties of the 2 quantities, average ( $u_0$ ) and slope ( $u_1$ ), characterizing the linear fits, show that  $F_2$  actually depends only on

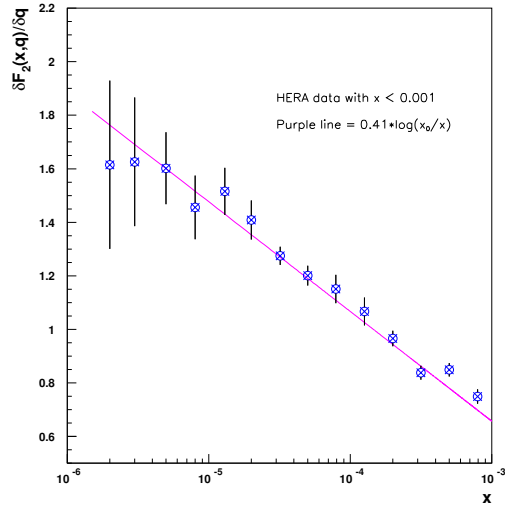


Figure 5. Fitted slopes from 2-parameter fit in  $q$  to  $F_2(x, q)$  for each  $x$ -bin.

the product of  $\log(x_0/x)$  and  $q = \log(1 + Q^2/Q_0^2)$ , i.e. on the scaling variable  $\xi$  (see eq. 1). All  $F_2$  measurements are plotted in fig. 6 as a function of  $\xi$  as calculated from eq. 1. A 2-parameter fit in  $\xi$  to the  $F_2$  data yields  $\chi^2/\text{dof} = 215/171$  including 2 points contributing 34.1. The extrapolation towards 0 is consistent with 0.

In the kinematic range investigated the structure function  $F_2(x, Q^2) = F_2(\xi, q)$  has - within experimental uncertainties - no measurable dependence on  $q$ , if expressed in terms of  $\xi$ . This yields the simple form :

$$F_2(\xi) = 0.41 \xi \quad (4)$$

This representation holds equally well in the perturbative as in the nonperturbative regime. It has been checked that it is consistent with the prediction of the MRST parametrisation[6], which is based on the QCD evolution and applicable for  $Q^2 > 1.25 \text{ GeV}^2$  and  $x > 10^{-5}$ .

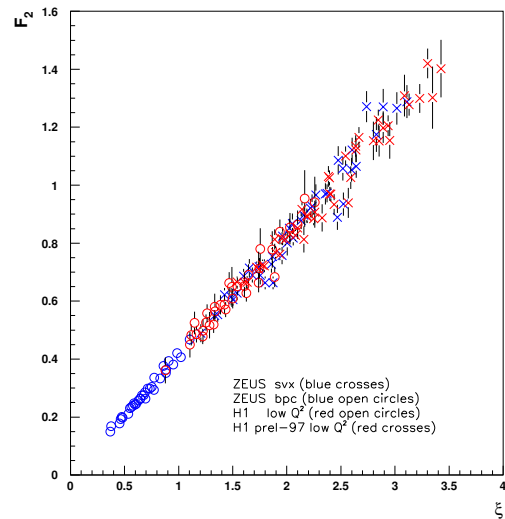


Figure 6.  $F_2$  data from 4 HERA experiments plotted in terms of the scaling variable  $\xi = \log(x_0/x) \log(1 + Q^2/Q_0^2)$ .

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