Double-logarithmic Behaviour of $F_2(x,Q^2)$ for all Q^2 and $x < 0.005$

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INTRODUCTION

The objective of this talk is to elaborate on the logarithmic behaviour of the proton structure function F_2 in both x and Q^2 reported at the DIS96-Workshop at Rome [1] : $F_2(x,Q^2) = a + m \log Q^2/Q_0^2 \log x_0/x$. This form was established empirically using the H1 data [2] in the phase space domain defined by $x < 0.01$ and $Q^2 > 5$ GeV^2.

The H1 Collaboration has published further F_2 data in a new regime of the phase space [3] extending the measurements towards Q^2 as low as 0.35 GeV^2, while simultaneously decreasing x to values near to 10^{-6}. Is the $log - log$ form also supported by the new data at low Q^2 ? Since for values of Q^2 near or below $Q_0^2 \approx 0.5$ GeV^2 the term $\log Q^2/Q_0^2$ becomes negative and is thus inadequate to describe the Q^2-dependence of the structure function at low Q^2, a modified variable is introduced instead :

$$\log Q^2/Q_0^2 \rightarrow \log\,(1 + Q^2/Q_0^2)$$

The modified variable still depends upon just the *one* parameter, Q_0^2, and has the desired property of approaching smoothly the old variable for $Q^2 \gg Q_0^2$, as is largely fulfilled by the previously used data of ref. [2]. On the other hand, it approaches 0 for $Q^2 \rightarrow 0$.

THE Q^2-DEPENDENCE OF F_2

Following the same method as in ref. [1] it turns out that a linear fit in $q = \log\,(1 + Q^2/Q_0^2)$ performed for each x -bin :

$$x \quad \rightarrow F_2(x,Q^2) = u_0(x) + u_1(x)\;(q - \langle q \rangle) \tag{1}$$

provides a good representation of both data sets [2,3] (marked in the figures with different symbols) in the HERA phase space domain defined by :

$$x \quad < 0.005 \;\; \text{and } \textit{all} \text{ kinematically allowed } Q^2 \tag{2}$$

This means that F_2 is consistent with being linear in $\log\,(1 + Q^2/Q_0^2)$.

THE x -DEPENDENCE OF F_2

It is convenient to cast eq. (1) in the equivalent form :

$$F_2(x, Q^2) = [u_0(x) - u_1(x)\langle q\rangle] + [u_1(x)]\ q \tag{3}$$

Choosing $Q_0^2 \approx 0.5$ GeV^2 the first term in eq. (3) is consistent with 0 within errors [1] for all accepted x -values, as shown in fig. 1a : $u_0(x) = u_1(x)\langle q\rangle$. This means that the directly measured slopes agree approximately with the slopes calculated from the point ($\langle q\rangle$,F_2(x,$\langle q\rangle$) and the origin (0,0). This is a remarkable property of the data, if plotted according to eq. (3), and

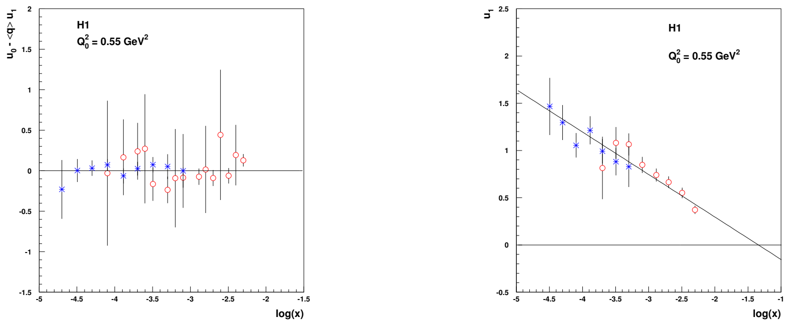

FIGURE 1. *(a)* $u_0- < q > u_1$ *versus* $\log x$; *(b)* u_1 *versus* $\log x$

implies that the x -dependence of F_2 comes from the slope $u_1(x)$ alone, thus $F_2(x, Q^2) = u_1(x)\ q$. In the next step the x -dependence of the slope u_1 is investigated. Fig. 1b exhibits for both sets a logarithmic decrease with x . It is therefore possible to cast the slope u_1 in the form : $u_1(x) = m \log x_0/x$, where the value of x_0 can be read off from the figure and is around 0.04.

SUMMARY

Combining the above empirical facts (see fig. 1), one is lead to the conclusion that

$$F_2(x, Q^2) = m\ \xi$$
$$\xi = \log\ (1 + Q^2/Q_0^2)\ \log\ x_0/x \tag{4}$$

This hypothesis is checked by plotting u_0 as a function of $\langle q\rangle \log x_0/x$ (see eq. (3)). With the choice Q_0^2= 0.55 GeV^2 and x_0= 0.04 the quantity u_0 is plotted for each pair (x,$\langle q\rangle$) in figure 2a. The χ^2/dof is 4.1/13 and 7.3/7 for the two data sets respectively. Figure 2b shows all F_2-measurements satisfying

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1) It is understood that all equalities are valid only within experimental uncertainties.

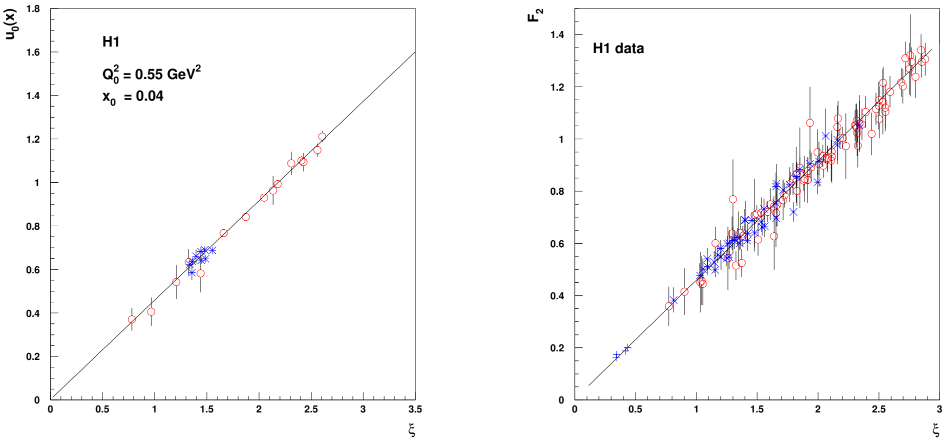

FIGURE 2. *(a)* $u_0(x) = F_2(x,\langle q\rangle)$ *versus* $\langle q\rangle \log x_0/x$ *; (b)* F_2 *versus* ξ

eq. (2) as a function of ξ. The linearity is well borne out. The line is a two-parameter fit in ξ. The linear extrapolation to $\xi = 0$ yields -0.004 $\pm$ 0.016. The χ^2/dof is 34.6/87 and 29.8/42 for the two data sets respectively. The 4 lowest points in fig. 2b corresponding to Q^2=0.11 and 0.15 GeV^2 have been taken from the analysis of the low-Q^2 data by the ZEUS Collaboration reported to this Workshop [5]. They are found to agree within errors with the linear extrapolation of the H1 data.

Some theoretical considerations are presented in ref. [4]. Also with the inclusion of the low-Q^2 data and the good quality of the fit an x -dependence of F_2 which is stronger than logarithmic is not requested by the data.

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