F_2^{ep} in the Deep Sea and DGLAP *

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The consistency of the DGLAP equations is tested in the deep sea region using the HERA F_2 data.

1. Introduction

The theory of strong interactions is now 30 years old. Lepton-nucleon experiments have contributed decisively to the understanding of QCD and to the structure of the proton. During the last decade the experiments at the ep-collider HERA have extended the available phase space to very high values of Q^2 in the valence region and have opened at values of Q^2 below 100 GeV^2 a hitherto unexplored region, the *deep sea*, i.e. x <0.001. The observed strong rise of F_2 at low values of x was unexpected and so was the successful inclusion of the low-x data into global QCD analyses [1] without loosing apparently in fit quality.

A phenomenological study of the F_2 data in the deep sea revealed two prominent features, when plotting the data in terms of the variable $q = \log_{10}(1 + Q^2/Q_0^2)$ [2] (with Q_0^2=0.5 GeV^2) rather than the usual ln Q^2 : (i) Within the experimental precision the data [3] are well represented by $F_2(x,q) = u_0(x) + u_1(x)\ (q - \langle q \rangle)$. For x <0.001 the linear extrapolation to q=0 satisfies $F_2(x,0) = 0$ as required by the conservation of the electromagnetic current, while for x >0.001 the valence contribution gets increasingly important and makes a linear extrapolation inappropriate. (ii) The data covering the range above Q^2=0.05 GeV^2 do not indicate any change of behaviour in the transition region from non-perturbative to perturbative physics. This empirical fact[3] challenges the question of how the linear behavior of F_2 in q is brought about as a result of intrinsic properties of the kernels in the validity region of the DGLAP equations.

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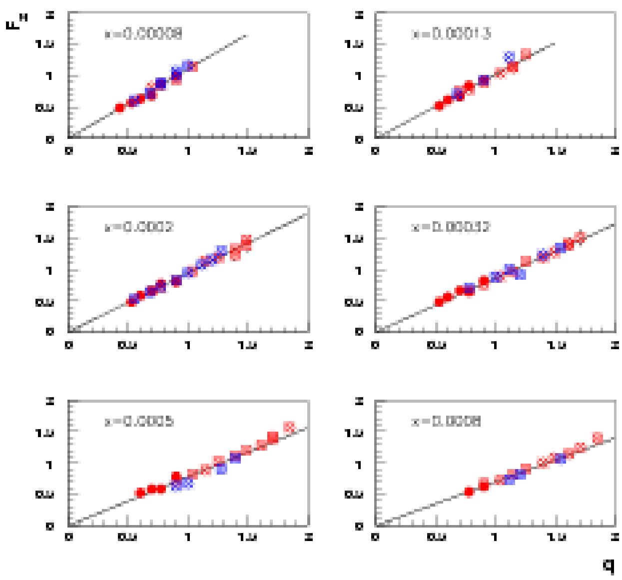

Fig. 1. F_2 data from H1 and ZEUS for 6 fixed x-bins versus q.

2. The DGLAP equations and F_2^{ep}

The formalism describing the evolution of parton distributions is well known [4]. In order to take advantage of the properties of q, the coupled DGLAP equations for the singlet (S) and the gluon (G) distributions are expressed in this variable:

$$\frac{\partial S(x,q)}{\partial q} = a(q)\Big(P_{qq}\otimes S(x,q) + P_{qg}\otimes G(x,q)\Big)\,, \tag{1a}$$

$$\frac{\partial G(x,q)}{\partial q} = a(q)\Big(P_{gq}\otimes S(x,q) + P_{gg}\otimes G(x,q)\Big)\,, \tag{1b}$$

where $a(q) = \frac{\alpha_s(Q^2)}{2\pi}\,\frac{Q^2+Q_0^2}{Q^2}\ln 10$ plays the rôle of the QCD-coupling. The structure function F_2 in ep scattering evolves differently for the singlet part, $\epsilon\, S(x)$, and nonsinglet part, $N(x)$. In the Quark-Parton Model $\epsilon = \frac{1}{f}\sum_i^f e_i^2$ and $S(x) = \sum_i^f x\big(q_i(x) + \bar{q}_i(x)\big)$, where e_i^2 are the QED coupling constants for the f active flavors. In QCD at next-to-leading order the parton distributions get Q^2-dependent. Choosing the $\overline{MS}$ renormalization scheme the expression for F_2^{ep} reads [4]: $F_2^{ep} = C_F\otimes N + \epsilon\,(C_F\otimes S + C_G\otimes G)$. In the kinematic region of interest, the deep sea, $\epsilon\, S(x,q) = F_2^{ep}(x,q)(1 + \mathcal{O}(\text{few }\%))$, as long as $Q^2 > 1$ GeV^2. In the calculations below the kernels are used at next-to-leading order with 3 flavors and the singlet function $\epsilon\, S(x,q)$ is identified for $x < 0.001$ with F_2^{ep} itself, while for $x > 0.001$ S and $\partial S/\partial q$ are extended smoothly to the valence region in agreement with data. Eq. 1a is equivalent to:

$$a(q)P_{qg} \otimes G(x,q) = \frac{\partial S(x,q)}{\partial q} - a(q)P_{qq} \otimes S(x,q) \tag{1c}$$

Now the r.h.s. $\hat{S}(x,q) \equiv (\partial/\partial q - a(q)P_{qq}\otimes)\, S(x,q)$ consists of known quantities: $S, \partial S/\partial q$ by experiment and P_{qq}, α_s by theory, thus constraining the properties of the unknown gluon on the l.h.s. This information ought to be consistent with the second DGLAP equation (eq. 1b). A quantitative test in the deep sea is performed under the two hypotheses

1: *The singlet $S(x,q)$ is exactly linear in q in the deep sea.*
2: *The DGLAP equations are valid in the considered phase space region.*

using later on as test quantity :

$$a(q)P_{qg} \otimes \frac{\partial G(x,q)}{\partial q} \tag{2}$$

3. The first DGLAP equation

a.) $\hat{S}$: The term $\partial S/\partial q$ is given, in the deep sea, by the measured slopes of F_2^{ep}, i.e. $u_1(x)$ (see fig. 1). The other term $a(q)P_{qq} \otimes S(x,q)$ involves the kernel P_{qq} and so a convolution with S over the full range from x until 1. Its effect is numerically small as shown in figure 2a. The convolution with the lowest order kernel is also shown. The effect of the $1/x$-term in the NLO-part of P_{qq} gets prominent at low x. In conclusion, the r.h.s. of eq. 1c is well determined and is nearly Q^2-independent for $1 < Q^2 <$100 GeV^2. The precise shape of S in the valence region is not relevant.

b.) The gluon function satisfying eq. 1c must have a strong dependence upon q, since both $q \cdot a(q)$ and $\hat{S}$ are weakly q-dependent. Fig. 2b shows $\hat{S}(x,q)$ for q=1. Its shape is dominated by the logarithmic behaviour of $\partial S/\partial q = u_1(x) \sim \log(1/x)$ [3] with a strong suppression at large x and a small negative curvature at low x caused by $P_{qq} \otimes S(x,q)$. Eq. 1c can be approximately solved for the gluon function by noting the property of the kernel P_{qg}, which applied to a valence-like distribution produces a constant, while applied to a constant produces a logarithmic rise in the deep sea. The resulting gluon function for q=1 is displayed in fig. 2b. The decrease at low x accounts for the small negative curvature in $\hat{S}$. For verification both $\hat{S}(x,1)$ and $a(q)P_{qg} \otimes G(x,q)$ for q=1 using the reconstructed gluon $G(x,1)$ is also shown in fig. 2b by the two curves, one displaced for better visibility.

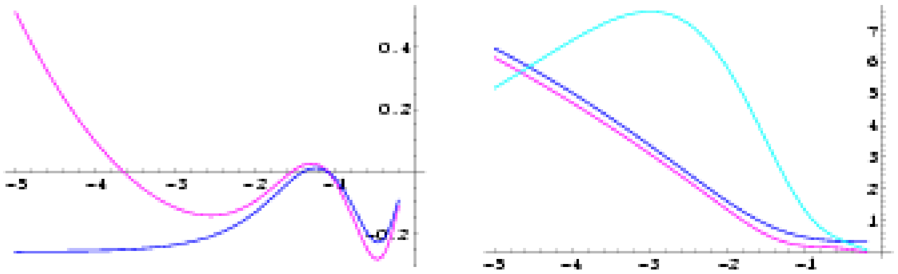

Fig. 2. Left: $a(q)P_{qq} \otimes S(x,q)$ for q=1 vs log(x) at NLO (upper) and LO (lower); Right: Display of the reconstructed gluon function $G(x,1)$; the lower curve represents $\hat{S}(x,1)$ and the curve shifted upward for visibility by 0.3 verifies that $G(x,1)$ approximately satisfies eq. 1c.

4. Consistency test for q=1

The test quantity $T(x,q) = a(q)P_{qg} \otimes \partial G(x,q)/\partial q$ (eq. 2) is evaluated for q=1 in two ways. It appears as one of the terms, denoted by T_I, when forming the derivative of the first DGLAP equation w.r.t. q :

$$T_I = \frac{\partial \hat{S}(x,q)}{\partial q} - \frac{\partial \ln a(q)}{\partial q}\ \hat{S}(x,q) - \frac{\partial \ln \alpha_s(q)}{\partial q}\ a(q)\,(P_{qg} - P_{qg}^{LO}) \otimes G(x,q)$$

On the other hand, substituting in T for $\partial G(x,q)/\partial q$ directly the second DGLAP equation (1b) yields :

$$T_{II} = a(q)\,P_{qg} \otimes a(q) \Big(P_{gq} \otimes S(x,q) + P_{gg} \otimes G(x,q) \Big)$$

The very low x behaviour is different for T_I and T_{II}, since the second one consists of a product of two kernels, while the first one involves only one kernel.

With the gluon distribution function satisfying the first DGLAP equation for q=1 one obtains the following numbers for T_I and T_{II} at 3 x-values:

x	T_I	T_{II}
10^{-3}	3.4	3.3
10^{-4}	5.6	9.3
10^{-5}	7.8	18.5

5. Results

A transparent analysis has been carried out confronting the observed form of the structure function F_2^{ep} at low x with the form implied by the DGLAP kernels. No evolution is performed, but rather the interplay of the derivative w.r.t. q and the convolution is investigated locally. The two main results are :

- In the deep sea region the linear q-dependence of F_2^{ep} is inconsistent with the DGLAP equations.
- $a(q)P_{qg} \otimes G(x, q)$ varies very little with Q^2 for $1 < Q^2 < 100$ GeV^2.

The first hypothesis regarding the linearity is not strictly satisfied. Indeed, the mere measurement uncertainties of the data do not exclude a small departure from linearity in q, which, however, is too small to invalidate the large deviation of the ratio T_I/T_{II} from unity. Furthermore, this ratio is insensitive to the assumptions made in the analysis.

The observed inconsistency is hidden in global fits [1], since the majority of the F_2 data is in the valence dominated phase space region and only the small fraction of the HERA samples in the deep sea probe the critical $1/x$ terms in the DGLAP kernels. As Q^2 becomes smaller than 100 GeV^2 the low-x behaviour affects the fits increasingly and unavoidably induces large gluon driven curvatures $\partial^2 F_2^{ep}/\partial q^2$ in conflict with the predominant linearity of F_2^{ep} in q borne out by the data.

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