THE TRANSITION FROM $\sigma(\gamma^* p)$ TO $\sigma(\gamma p)$

DIETER HAIDT
DESY, Notke Strasse 85,D-22603 HAMBURG, Germany
*E-mail: dieter.haidt@desy.de**

1 Relation between $\sigma(\gamma^* p)$ and F_2

In electron-proton scattering the structure function F_2^{ep} and the cross section $\sigma(\gamma^* p)$ are related. For small x one finds :

$$\sigma^{\gamma^* p}(W^2, Q^2) \approx \frac{4\pi^2\alpha}{Q^2} \cdot F_2^{ep}(W^2, Q^2) \tag{1}$$

Empirically[1] the HERA F_2-data (F_2 short for F_2^{ep}) are well described by :

$$F_2(x, Q^2) = m \cdot \log\,(x_0/x) \cdot \log\,(1 + Q^2/Q_0^2) \tag{2}$$

(m=0.40, x_0=0.04 and Q_0^2=0.50 GeV^2) for $x < 0.001$ without restriction in Q^2. Therefore, it is convenient to use the quantity $q = \log\,(1{+}Q^2/Q_0^2)$ rather than Q^2 itself and cast eq. 1 in the form :

$$\sigma^{\gamma^* p}(W^2, Q^2) = \frac{4\pi^2\alpha}{Q_0^2} \cdot \frac{q}{Q^2/Q_0^2} \cdot \frac{F_2(W^2, q)}{q} \tag{3}$$

Since $Q^2 \rightarrow 0$ implies $q \sim Q^2/Q_0^2$, the observable F_2/q carries the information on the transition $\sigma(\gamma^* p) \rightarrow \sigma(\gamma\, p)$.

2 The data

For the purpose of this study the low-Q^2 F_2-data[2,4,5] together with the recently published data[3,6] are applied requiring $x < 0.001$.

The two ZEUS samples called *bpc*[4] and *bpt*[6] have 34 F_2-measurements in common allowing for the hypothesis test :

*Talk presented at DIS2001,Bologna, April 2001

$F_2^{bpt} - F_2^{bpc} = 0$. Fig. 1, 2 shows the differences grouped in Q^2-bins. The errors are calculated from the uncorrelated uncertainties alone. The average being $\langle F_2^{bpt} - F_2^{bpc}\rangle$=0.031 ± 0.008 and χ^2/dof=50/33 imply that for obtaining consistency between the two samples the systematic uncertainties must be taken into account. The HERA Collaborations have performed a careful study of the systematics. With F_2/q linear in log $1/x$ (see fig. 3), the effect of each systematic source can be illustrated in a planar arrow diagram, each arrow representing the 1σ- shift in the average u_0

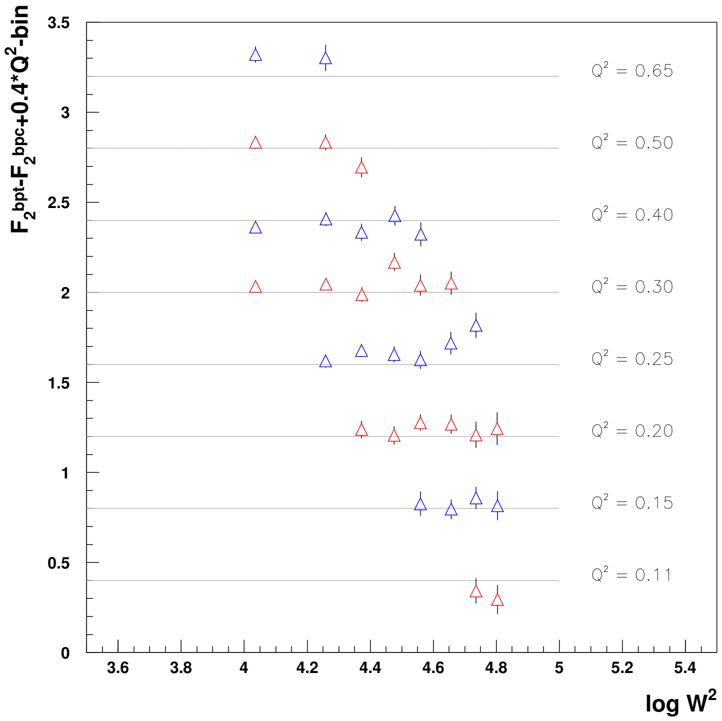

Figure 1. *Difference of* $F_2^{bpt} - F_2^{bpc}$ *versus* log W^2 *for various* Q^2*-bins; each line corresponds to 0.*

and slope u_1, as shown in fig. 2 for the *bpt*-data.

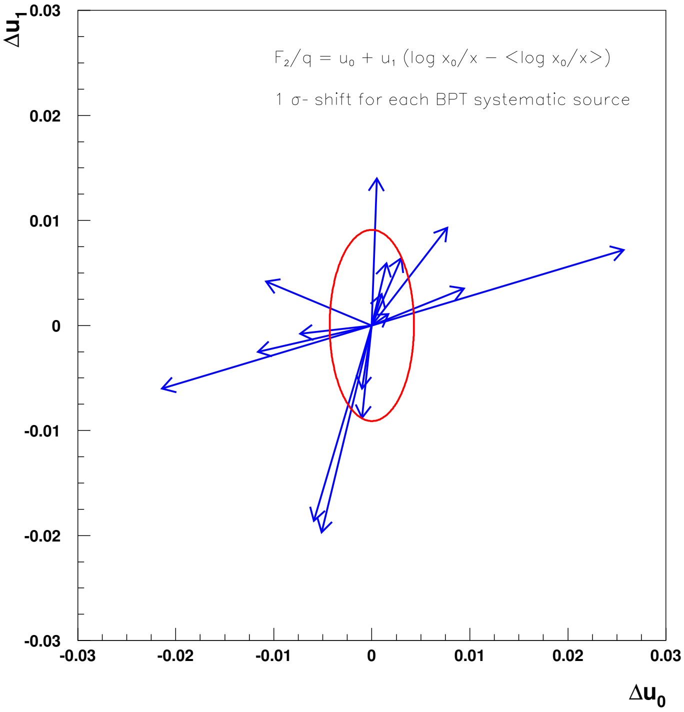

Figure 2. *Shifts induced by each systematic source for the ZEUS-bpt data; the ellipse refers to the statistical precision.*

3 Properties of F_2/q

The measured F_2/q values are displayed in fig. 3 versus log x_0/x for all Q^2 supporting again eq. 2. There is no observable Q^2-dependence.

For the study of $\sigma(\gamma^* p) \rightarrow \sigma(\gamma p)$ F_2/q is reexpressed in terms of (q, W^2) instead of (x, Q^2). In the selected phase space region $W^2 = Q^2 \cdot \frac{1}{x}$, thus $F_2/q \sim \log 1/x$ implies $F_2/q \sim \log W^2$. The q-dependence of the data is therefore analysed in the form :

$$\frac{F_2(W^2, q)}{q} = u_0(q) + u_1(q) \cdot (\log W^2 - \langle \log W^2 \rangle)$$

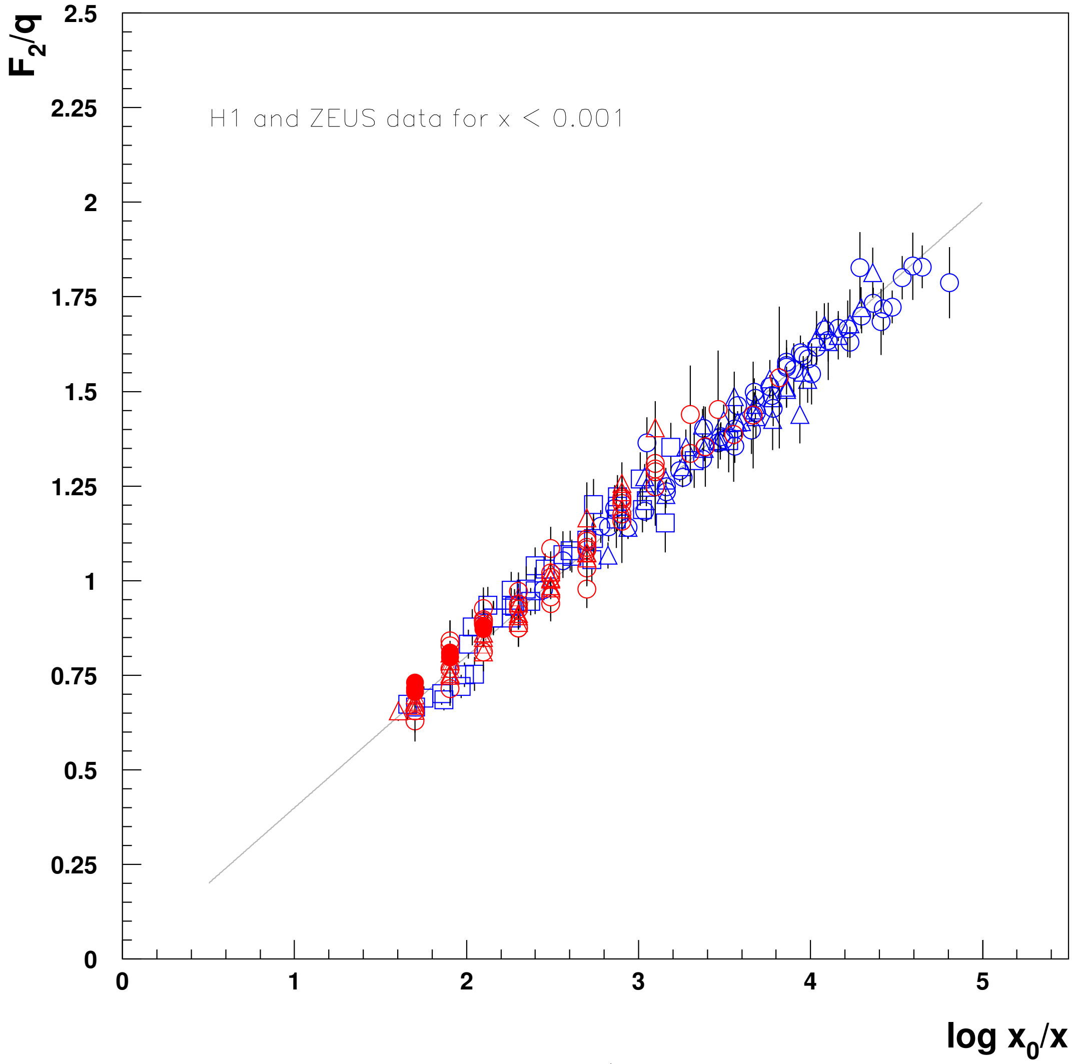

Figure 3. *The x-dependence of F_2/q.*

The slopes of F_2/q for fixed values of $\log W^2$ are plotted in fig. 4(left) and are q-independent. In fig. 4(right) the F_2/q points

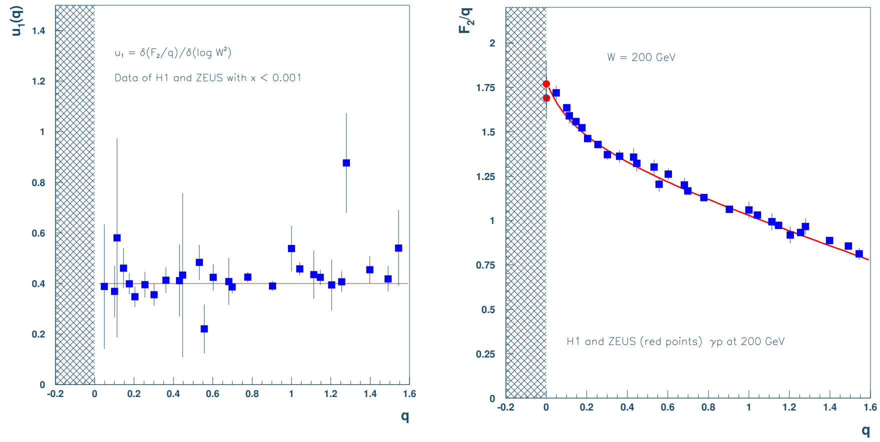

Figure 4. *Left : Slopes for constant* W^2. *Right :* F_2/q *versus* q *for* $W = 200$ *GeV compared with extended parametrisation; also displayed are the two direct measurements of* $\sigma(\gamma p)$ *by H1 and ZEUS.*

are displayed for the value $W = 200$ GeV, since there the corresponding phase space region is populated and direct $\gamma\, p$ measurements exist. The lowest measured Q^2 being 0.05 GeV^2 is quite close to 0 and the trend of the data suggests to bridge the gap linearly. Physically, the transition has to be smooth.

The prediction using eq. 2 is consistent with the F_2-data. Its differential form is : $\mathrm{d}\, F_2/q = m \ \mathrm{d}\log W^2$ - $m(1 + Q_0^2/Q^2)\ \mathrm{d}\, q$. The slopes $u_1(q)$ are consistent with m (the line in fig. 4 left). The slopes of F_2/q for W=200 GeV are Q^2-independent for large q, while for smaller q an enhancement due to the Q_0^2/Q^2-term occurs. For yet smaller values, outside the measured region, the formula (eq. 2) is bound to fail, since F_2/q is a function of x alone in the measured region, whereas $\sigma(\gamma\, p)$ is a function of W^2 alone. This asks for a quantity turning from x to W^2, as q approaches 0, which is achieved by the replacement $\frac{x_0}{x} \longrightarrow \frac{x_0}{x}\frac{Q^2}{Q^2+Q_w^2}$; for consistency $0 < Q_w^2 \ll Q_0^2$.

H1[7] and ZEUS[8] have measured $\sigma(\gamma\, p)$ at $W = 200$ and 207 GeV with q=$\mathcal{O}(10^{-4}) \approx 0$. The two points expressed as F_2/q (see eq. 3) are also displayed in fig. 4(right). The line in the figure shows the prediction of the extended formula with $Q_w^2 \approx 0.05$ GeV^2.

4 Conclusions

All F_2-data considered are consistent with each other. The systematic uncertainties of the H1 and ZEUS data samples are well estimated and need to be taken into account in detail.

F_2/q, the key quantity in the study of $\sigma(\gamma^* p) \to \sigma(\gamma\, p)$, is well described by eq. 2 and can be extended to the $\gamma\, p$–limit with a small modification in agreement with the direct $\sigma(\gamma\, p)$ measurements of H1 and ZEUS. The W^2-dependence of $\sigma(\gamma\, p)$ turns out to be *logarithmic*.

Acknowledgments

It is a pleasure to thank Prof. A.Zichichi, Prof. R.Nania and their collaborators for creating a fruitful atmosphere. I like to thank the organisers of the Structure Function Working Group Profs. J.Stirling, K.Long and R.Nisius. I enjoyed discussions with Prof. E.Lohrmann and Prof. P.Zerwas.

References

1. D. Haidt, *Proceedings of DIS 97* , Chicago, April 1997, p.386.
2. H1 Collaboration, C. Adloff *et al*, *Nucl. Phys.* B **497**, 3 (1997).
3. H1 Collaboration, C. Adloff *et al*, accepted by *Eur. Phys. J.* **C** (2001); DESY Preprint 00-181 and hep-ex/0012052.
4. ZEUS-Collaboration, J. Breitweg *et al*, *Phys. Lett.* B **407**, 432 (1997) (bpc).
5. ZEUS-Collaboration, J. Breitweg *et al*, *Eur. Phys. J.* C **7**, 609 (1999) (svx).
6. ZEUS-Collaboration, J. Breitweg *et al*, DESY Preprint 00-71 (May 2000) (bpt); hep-ex/0005018 and *Phys. Lett.* B **487**, 53 (2000).

7. H1 Collaboration, S. Aid *et al*, *Z. Phys.* C **69**, 27 (1995).
8. ZEUS-Collaboration, preliminary result published in $http://www-zeus.desy.de/conferences00/2000\_DIS\_ginsburg.ps.gz$.