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Sphaleron in Baryogenesis

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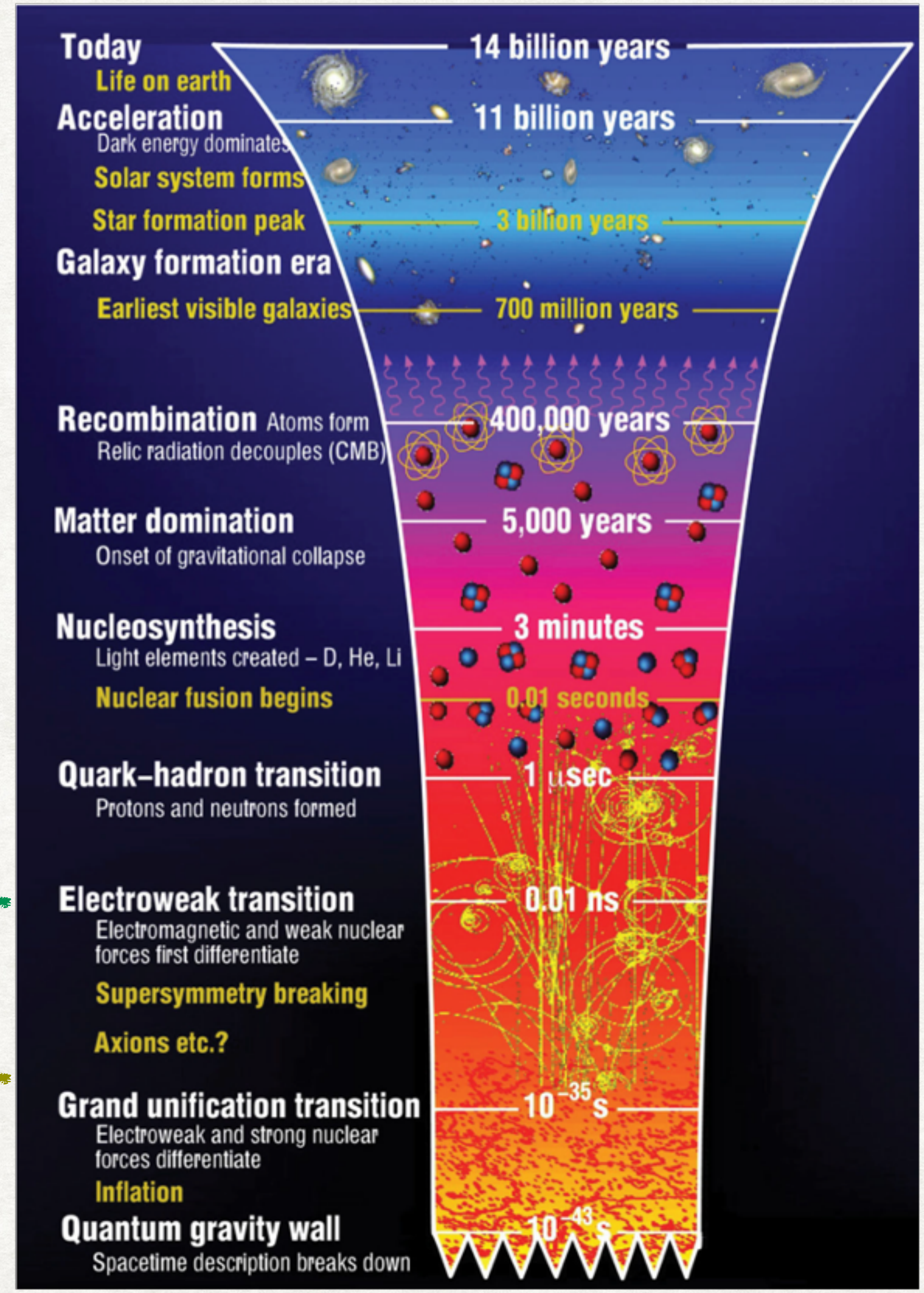
Higgs Seminar @ DESY

2025/12/4

Matter-antimatter asymmetry of our universe

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$



$$T_{EW} \simeq 100 \text{ GeV}$$

Electroweak Baryogenesis

$T \sim 10^{9-12}?$ GeV Leptogenesis

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 0$$

Credit: PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 17, 013104 (2021)

Baryogenesis Mechanisms

A. D. Sakharov, 1967, Usp. Fiz. Nauk 161,61-64

Three Sakharov conditions for baryogenesis:

	SM	EW baryogenesis	Leptogenesis
Baryon number violation	✔	**Sphaleron**	**Sphaleron**
Sufficient C & CP violation	✘	**New C & CP violation sources**	**New C & CP violation sources**
Departure from thermal equilibrium	✘	**First-order PT** **new particles**	**Decay of heavy neutrinos**

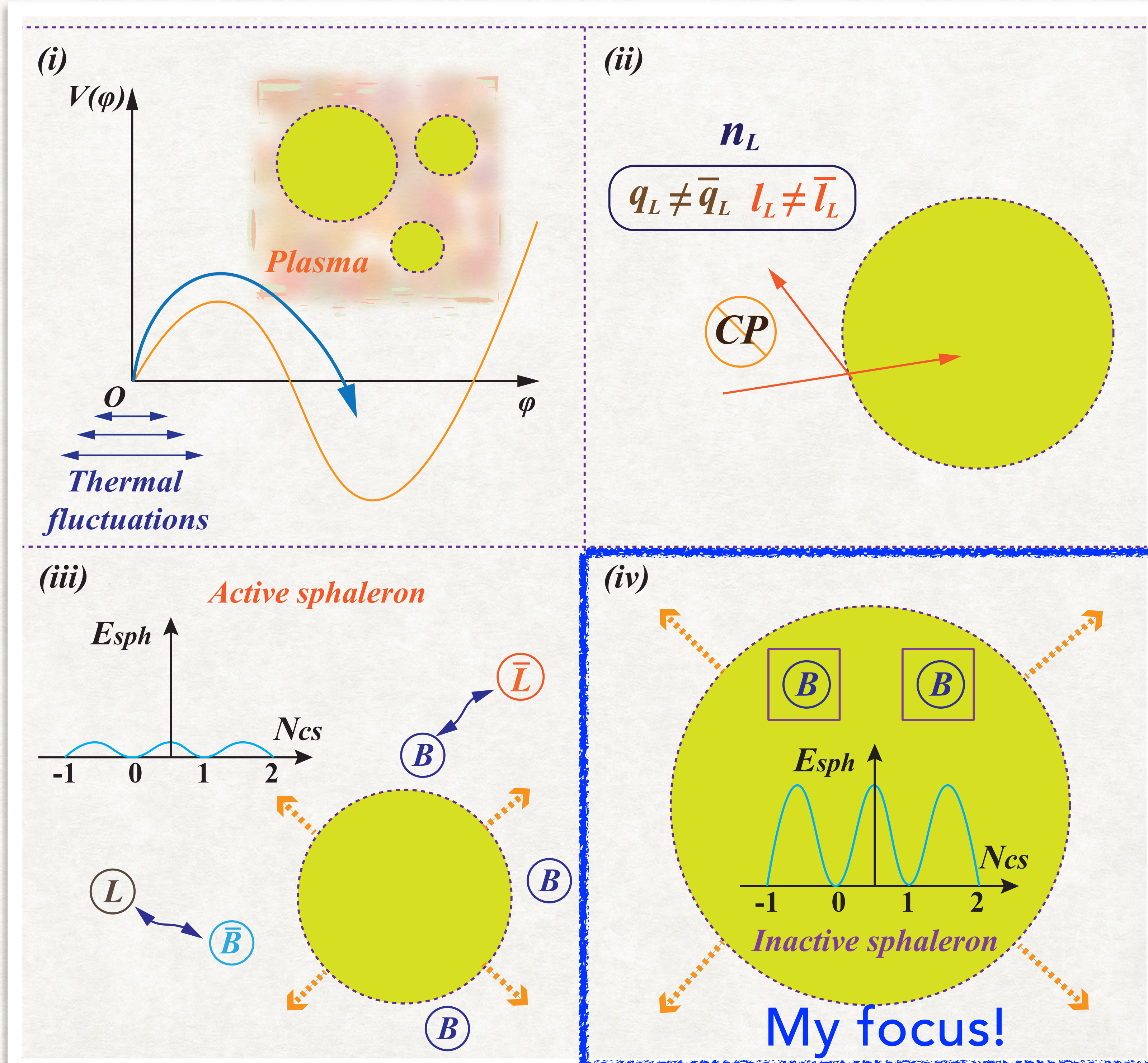
★ Collider Search!

- In both mechanisms, sphaleron plays an important role.
- For EW baryogengesis, how does the new particles affect the sphaleron energy (BNV rate)?

The general picture for EW baryogenesis

EW baryogenesis	
Sphaleron (BNV)	*(iii, iv)*
New C & CP violation sources	*(ii)*
First-order PT new particles	*(i)*

(iv): Does sphaleron erase all the generated baryon asymmetry?



X.X. Li, M. J. Ramsey-Musolf, T. V. I. Tenkanen, **YW**, 2506.01585

Aspects of sphaleron

★ • What is sphaleron? Topological nature; Connection to ABJ anomaly; Connection to ’t Hooft-Polyakov monopole.

• How BSM physics will modify sphaleron (leading order)? The influence of additional Higgs scalars to the sphaleron energy / monopole mass.

• How to overcome the gauge dependence issue beyond the leading order? Computation of sphaleron induced BNV rate under 3D EFT. Organize the gauge-invariant result with power countering.

What is sphaleron?

Perturbative QFT configurations | Topological field configurations

$$S = \int dx \left[\mathscr{L}_{\text{free}} + \mathscr{L}_{\text{interaction}}\right]$$

$$\frac{\delta \mathscr{L}_{\text{free}}}{\delta \phi, A_i} = 0$$

$$\frac{\delta(\mathscr{L}_{\text{free}} + \mathscr{L}_{\text{interaction}})}{\delta \phi, A_i} = 0$$

$$\phi, A_i \sim a_p e^{-ipx}$$

$$\phi, A_i \sim \text{complicated}$$

$$\int dx\ \partial_\mu j_5^\mu \sim \int dx\ F\tilde{F} = 0$$

$$\int dx\ \partial_\mu j_5^\mu \sim \int dx\ F\tilde{F} = \int dx\ \partial_\mu K^\mu \neq 0$$

S. L. Adler, Phys. Rev. 177, 2426 (1969);
J. S. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969)

SU(2) Yang-Mills + SM Higgs : Sphaleron

N. S. Manton, Phys.Rev.D 28 (1983) 2019

What is sphaleron?

F. R. Klinkhamer, N. S. Manton, Phys.Rev.D 30 (1984) 2212

SU(2) Yang-Mills + SM Higgs : Sphaleron

Being an static solution, we care about its Euclidean energy (neglect $U(1)$).

$$E = \int \left[\frac{1}{4} F_{ij}^a F_{ij}^a + \left(D_i \Phi\right)^\dagger \left(D_i \Phi\right) + V(\Phi) \right] d^3x$$

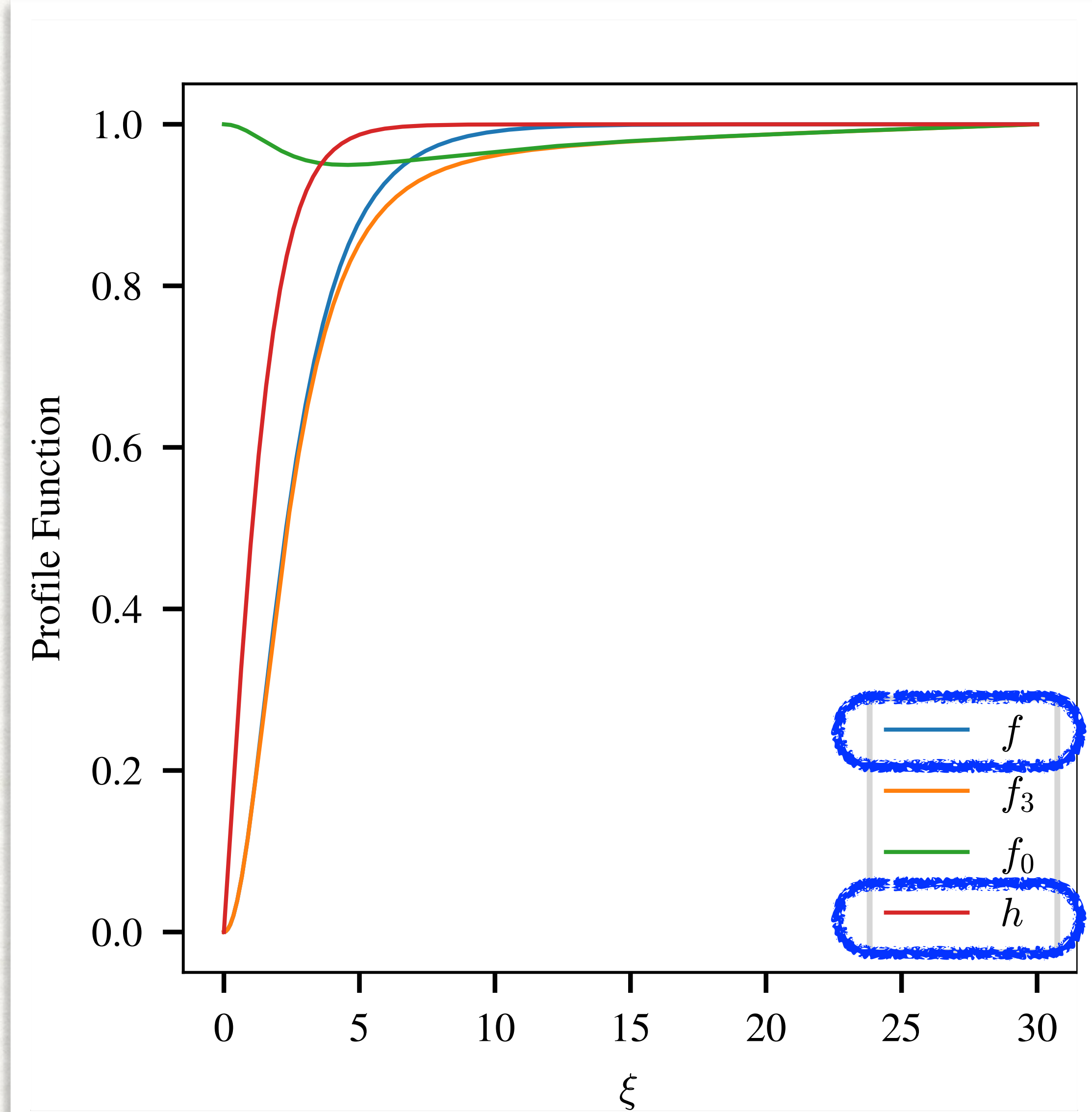
$$A_i^a T^a dx^i = -\frac{i}{g} f(\xi)(\partial_i U_{\text{sph}}) U_{\text{sph}}^{-1} dx^i$$

$$\Phi = \frac{v_\phi}{\sqrt{2}} h(\xi) U_{\text{sph}}(\mu, \theta, \phi) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U_{\text{sph}}(\mu, \theta, \phi) =$$

$$\begin{pmatrix} e^{i\mu}(\cos\mu - i\cos\theta\sin\mu) & e^{i\varphi}\sin\theta\sin\mu \\ -e^{-i\varphi}\sin\theta\sin\mu & e^{-i\mu}(\cos\mu + i\cos\theta\sin\mu) \end{pmatrix}$$

To make E finite, the integrand should vanish when $r \to \infty$. This is the situation with $f \to 1$, $h \to 1$.



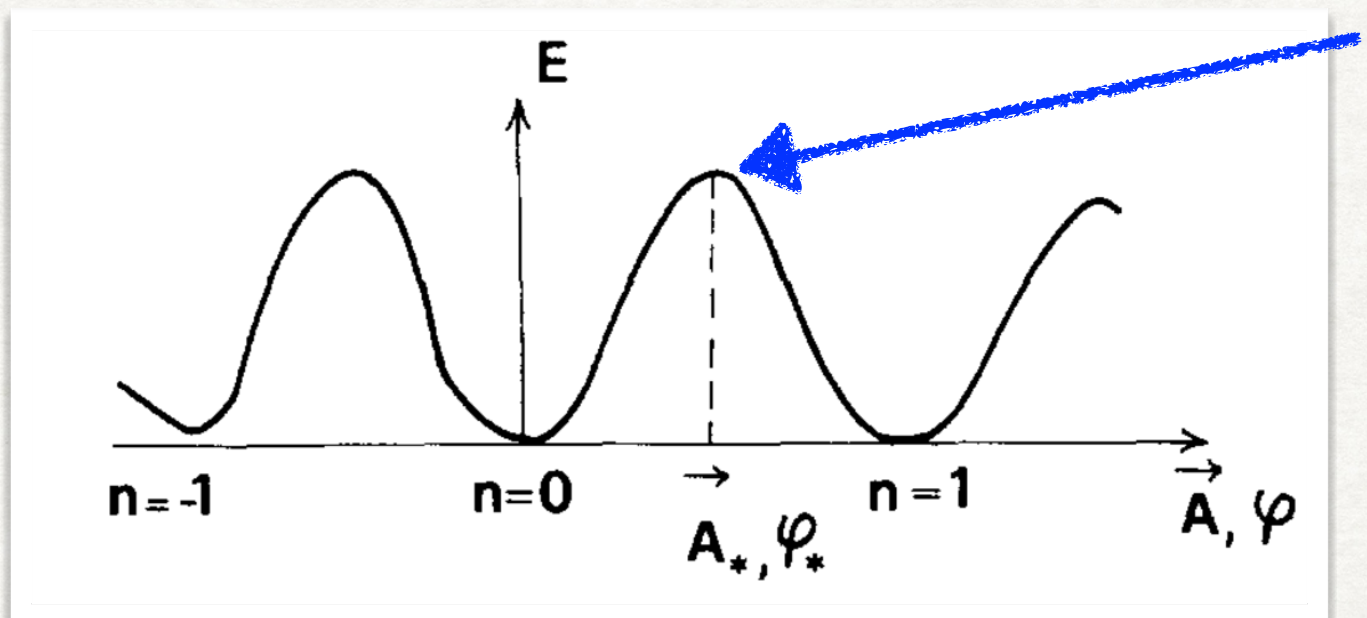
What is sphaleron?

F. R. Klinkhamer, N. S. Manton, Phys.Rev.D 30 (1984) 2212

Connection of sphaleron energy to baryon number violation rate.

Variation of Baryon and Lepton number:

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L \sim F\tilde{F} \qquad n = \int dx\; F\tilde{F} \in Z$$



Sphaleron configuration: (1) Half-integer winding number; (2) Unstable.

R. Jackiw, Rev.Mod.Phys. 49 (1977) 681-706

A. D. Linde, Nucl.Phys.B 216 (1983) 421

V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, Phys.Lett.B 155 (1985) 36

Rate of baryon number violation: transitions from left to $(\vec{A}_*, \varphi_*)$ to that right to $(\vec{A}_*, \varphi_*)$. Analogous to vacuum decay.

$$\frac{dn_B}{dt} = Ae^{-E(T)/T}$$

Aspects of sphaleron

- What is sphaleron? Topological nature; Connection to ABJ anomaly; Connection to 't Hooft-Polyakov monopole.

★

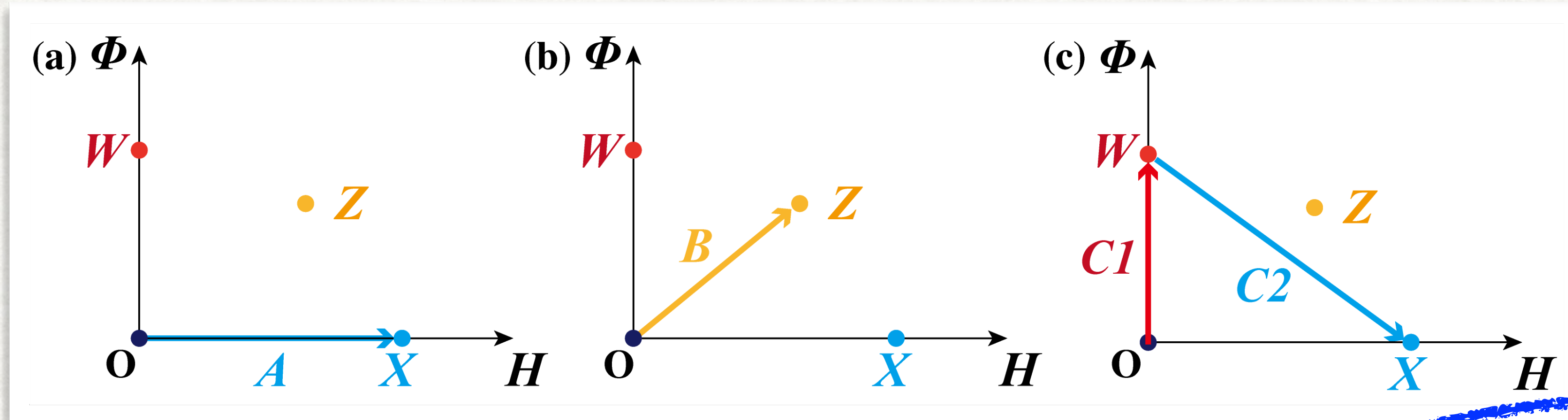
- How BSM physics will modify sphaleron (leading order)? The influence of additional Higgs scalars to the sphaleron energy / monopole mass.

- How to overcome the gauge dependence issue beyond the leading order? Computation of sphaleron induced BNV rate under 3D EFT. Organize the gauge-invariant result with power countering.

Sphaleron energy under multiple Higgs

Consider the SM Higgs (H) and a general SU(2) multiplet (Φ) with zero hypercharge. (under zero temperature)

$$E = \frac{4\pi\Omega}{g} \underbrace{\int d\xi \left[\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}f^a_{ij}f^a_{ij} + (D_iH)^\dagger(D_iH) + (D_i\Phi)^\dagger(D_i\Phi) + V(H,\Phi)\right]}_{B_{\rm sph}}$$



	X	Z	W
v_Φ	0	$\mathcal{O}(1)$ GeV	$[100-800]$ GeV
v_H	246 GeV	~ 246 GeV	0
$B_{\rm sph}$	1.9	~ 1.9	$\sim [1-8]$

Situation will be quite different for 2HDM, G-M model.

Interesting scenario for 2-step EWPT.

YW, W. Zhang, M. J. Ramsey-Musolf, Phys.Rev.D 112 (2025) 5, 053003

Aspects of sphaleron

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- How BSM physics will modify sphaleron (leading order)? The influence of additional Higgs scalars to the sphaleron energy / monopole mass.

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Sphaleron in finite temperature

In the thermal equilibrium, final baryon asymmetry:

$$B = C(B_{\rm ini} - L_{\rm ini})$$

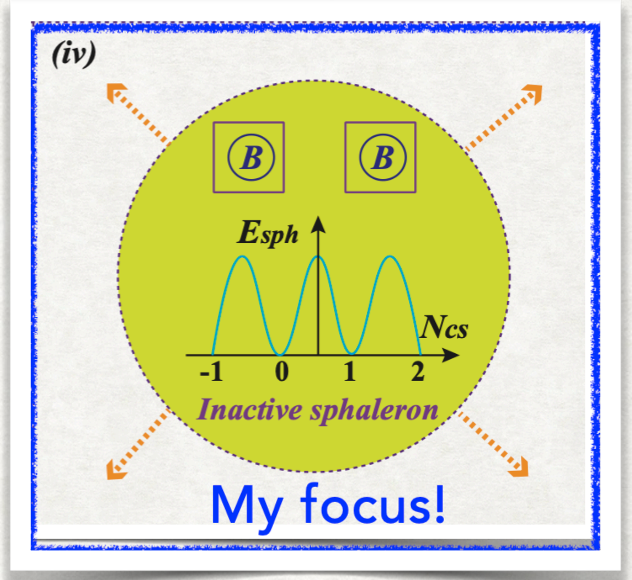
Electroweak Baryogenesis (EWBG): $B_{\rm ini} - L_{\rm ini} = 0$

EWBG: out of equilibrium from first order phase transition!

Be careful: The broken phase does NOT means out of equilibrium for baryon number violation (BNV).

BNV out of equilibrium criterion:

BNV rate (T) < Hubble constant (T)



EWBG, in the broken phase, we might have BNV rate > Hubble constant, which will erase all the baryon asymmetry.

The tradition condition

BNV out of equilibrium criterion:

BNV rate (T) < Hubble constant (T)

The traditional condition: $\frac{v_c}{T_c} \gtrsim 1$ M. Quiros, hep-ph/9901312

It comes from the traditional sphaleron rate computation:

$$\Gamma_{\text{BNV,sph}} \sim T^4 \, \kappa \, e^{-B \, v_c/T_c} \lesssim H, \quad \text{with } B \simeq 1.9$$

Drawbacks:

- v_c is the minimal of V_{eff}, which is gauge dependent under the general R_ξ gauge. Different gauges might lead into different lower bound.

  H. H. Patel, M. J. Ramsey-Musolf, JHEP 07 (2011) 029
- The B is chosen as the SM value; while BSM can modify this value, especially in 2HDM, Georgi-Machacek, etc.
- The effect of zero modes and positive modes (κ) are not been consistently considered for BSM.

Gauge invariant rigorous sphaleron rate

BNV out of equilibrium criterion: BNV rate (T) < Hubble constant (T)

Our approach: sphaleron rate under 3D EFT for SM EFT (BSM are heavy to be integrated out).

Traditional case

$$\Gamma_{\text{BNV,sph}} \sim T^4 \, \kappa \, e^{-B \, v_c/T_c}$$

$$\frac{v_c}{T_c} > 1$$

- v_c gauge dependent and fixed B
- No consistent zero and positive modes
- Condition immediately in the broken phase

Our case

$$\Gamma_{\text{BNV,sph}}(x,y) \sim T^4 \, (\text{zero modes}(x,y)) \, e^{-f(x,y)}$$

$$x = \frac{\text{Higgs selfcoupling}_3}{\text{gauge coupling}_3^2} \quad y = \frac{\text{Higgs mass}_3^2}{\text{gauge coupling}_3^4}$$

- x, y are gauge invariant quantities under power countering (to g^4);
- Two-loop matching under 3D EFT Zero modes are consistently considered.
- Quantitively control of baryon washout in the broken phase.

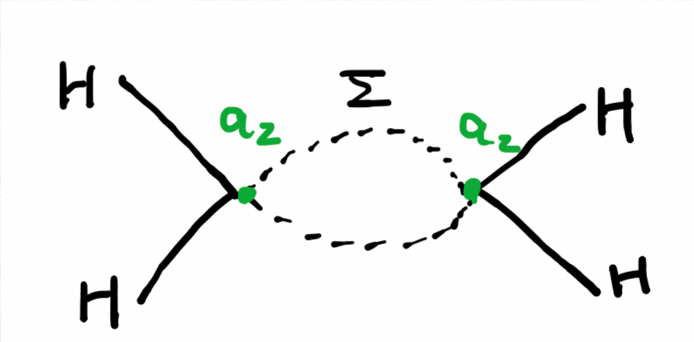
New baryon number preservation condition

Our sphaleron rate: $\Gamma_{\text{BNV,sph}}(x,y) \sim T^4\ (\text{zero modes}(x,y))\ e^{-f(x,y)}$

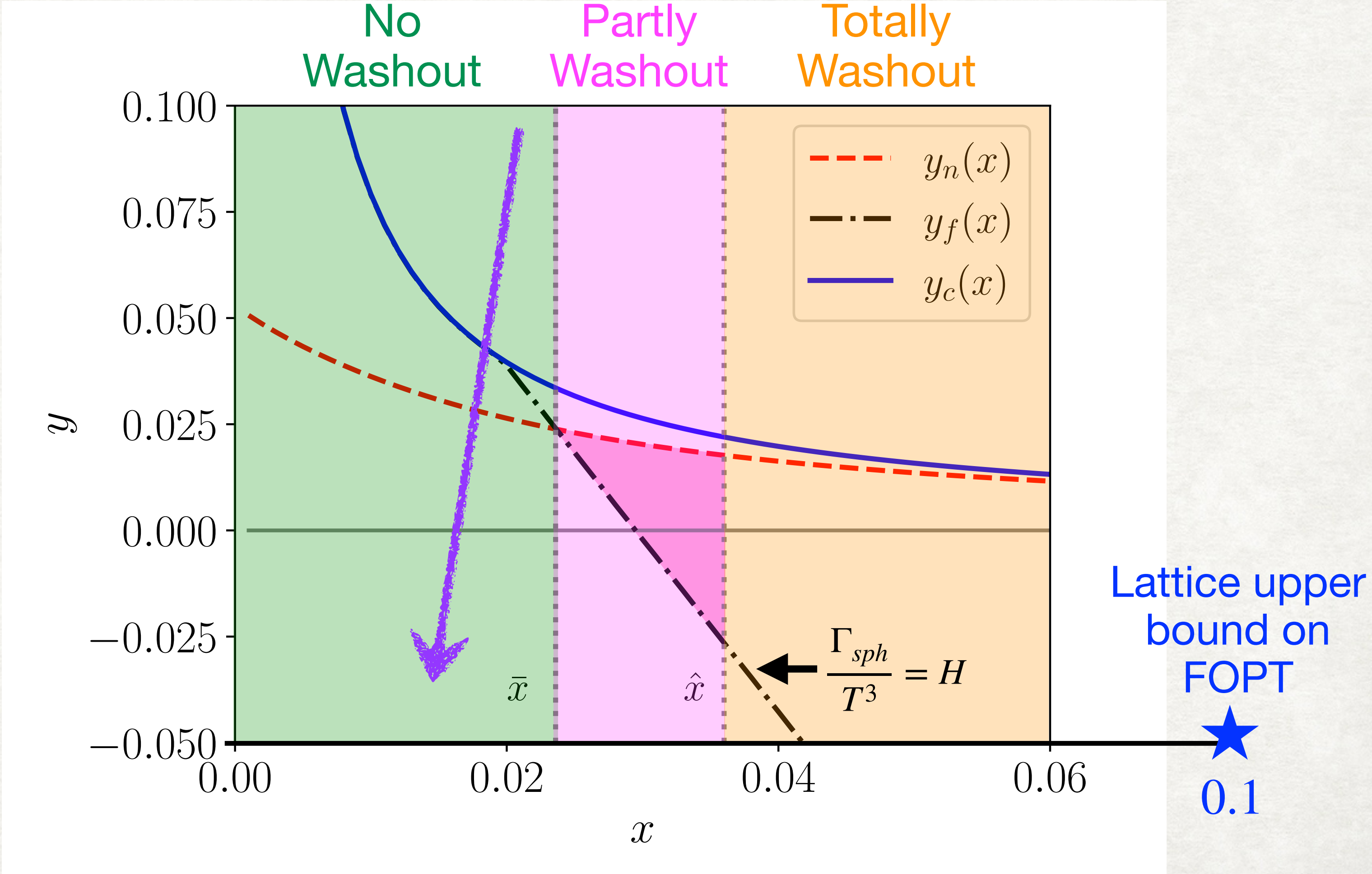
$$x = \frac{\text{Higgs selfcoupling}_3}{\text{gauge coupling}_3^2} \quad y = \frac{\text{Higgs mass}_3^2}{\text{gauge coupling}_3^4}$$

Assume BSM fields are heavy, which can be integrated out.

BSM fields (different BSM parameters) will modify the values of x, y.



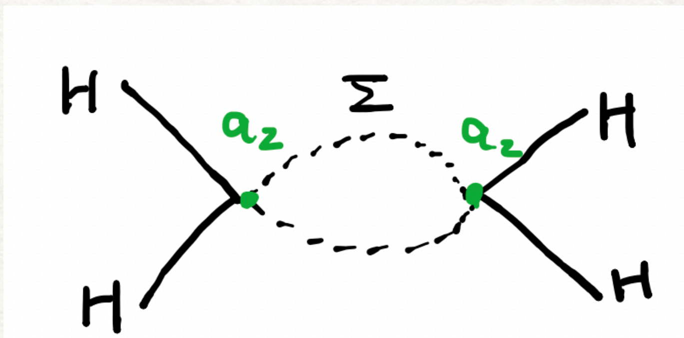
BSM field affects $\lambda(x)$

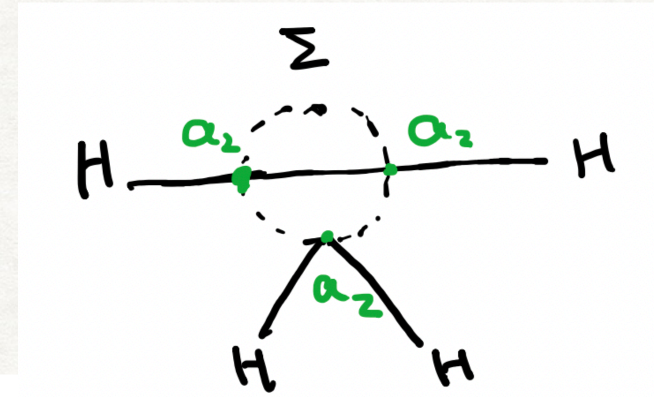


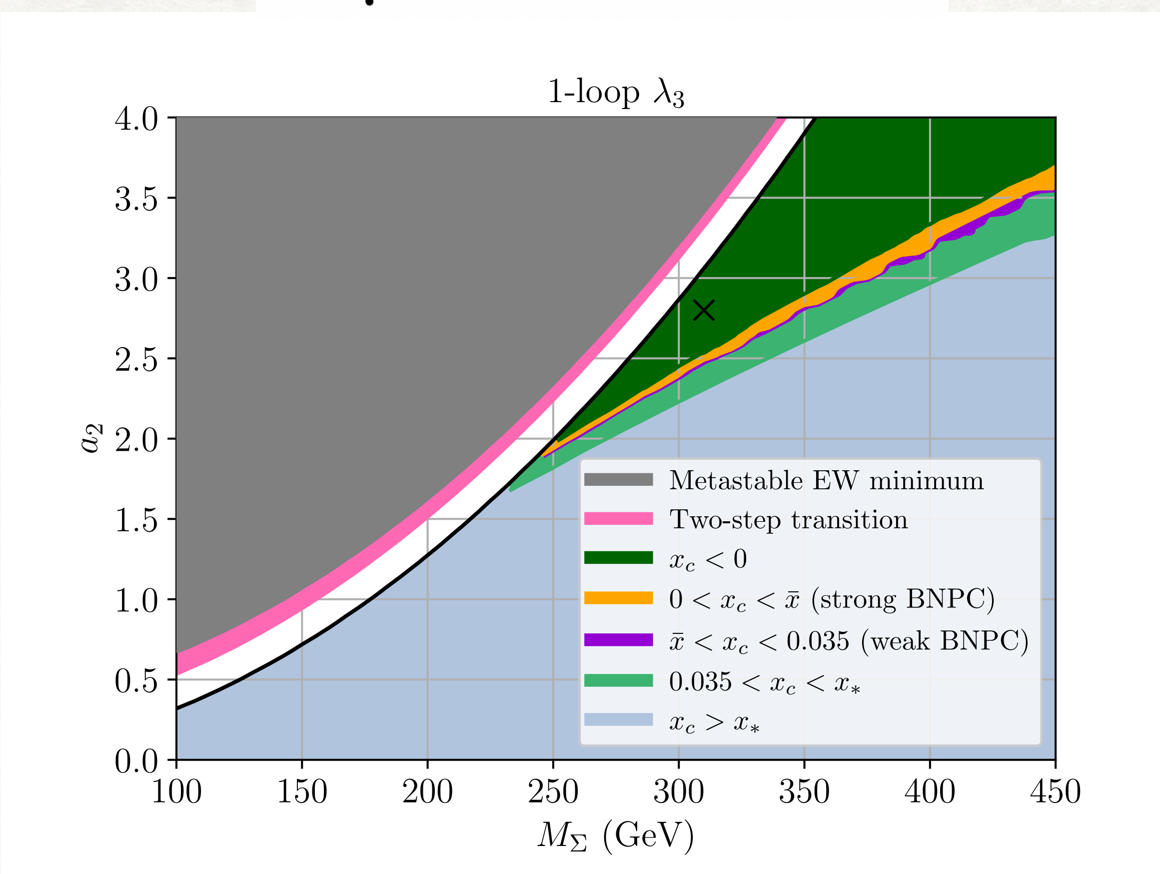
*X.X. Li, M. J. Ramsey-Musolf, T. V. I. Tenkanen, **YW**, 2506.01585*

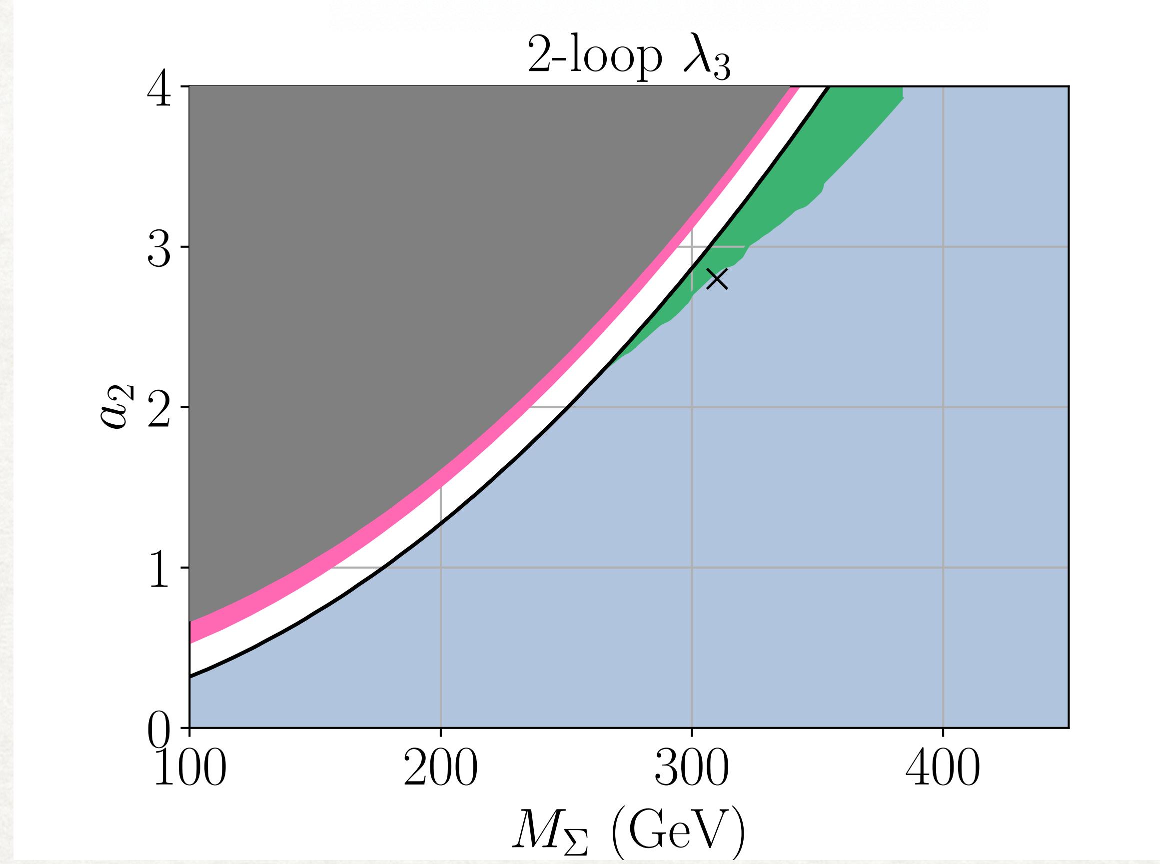
BSM applications: real triplet extension

a_2: portal coupling between triplet (Σ) and Higgs (H)









At 1-loop level, there are (No, Partly, Totally) Washout regions.

At 2-loop level, region only with Totally Washout! (All parameters fails for EWBG).

*X.X. Li, M. J. Ramsey-Musolf, T. V. I. Tenkanen, **YW**, 2506.01585*

- Future directions

Future direction 1

The measurement of Higgs trilinear self-coupling (λ) at future HL-LHC, FCC-ee, CEPC or photon-photon colliders could provide us important hints on Higgs potential.

M. Berger, J. Braathen, G. Moortgat-Pick, G. Weiglein, 2510.05012

J. Braathen, Sven Heinemeyer, Carlos Pulido Boatella, Alain Verduras Schaeidt, 2510.12569

Since sphaleron rate is very sensitive to λ, such analyses / measurements are crucial for BNV rate and EW baryogenesis (the FOPT is strong or not). Mapping λ into $x - y$ plot for SM-like EFT or model-dependently for 2HDM, G-M etc;

Search of light singlet particle at future colliders:

*Z. Wang, X. Zhu, E. E. Khoda, S. Hsu, N. Konstantinidis, K. Li, S. Li, M. J. Ramsey-Musolf, **YW**, Y. E. Zhang. LHEP 2023 (2023) 436)*

Future direction 2

The Gravitation Waves emitted by topological field configurations (sphaleron, domain wall, monopole, cosmic string, etc) can provide us important informations about the early universe.

S. Blasi, R. Jinno, T. Konstandin, H. Rubira, I. Stomberg, JCAP 10 (2023) 051

Such a careful analysis for sphaleron is still lacking. (New GWs backgounds)

Besides, a delayed FOPT can produce the primordial black hole from density perturbations. Curvature, redshift, and the Hawking temperature of PBHs can qualitatively change the sphaleron, BNV, and baryogenesis.

PBH from the delayed FOPT:

***YW**, S. Profumo, Phys.Rev.D 111 (2025) 10, 103524*

Summary

- Sphaleron plays an important role in baryon number violation many baryogengesis mechanisms.
- A gauge-invariant and accurate baryon number preservation condition is crucial for EW baryogengesis.
- New Higgs particle can have a large impact on the baryon number violation rate and EW baryogenesis.
- Looking ahead, the interplay between the Higgs self-coupling, GWs, and sphaleron would be interesting.

Thanks!

- Back Up slides

What is sphaleron?

Stability of sphaleron: unstable

$$E = \int \left[\frac{1}{4} F_{ij}^a F_{ij}^a + \left(D_i \Phi \right)^\dagger \left(D_i \Phi \right) + V(\Phi) \right] d^3 x$$

To make E finite, the integrand should vanish when $r \to \infty$. And this is the situation with $f \to 1$, $h \to 1$ in the previous page.

At $r \to \infty$, the boundary of the static space is a sphere, S^2.

On the other hand, the SU(2) gauge vacuum manifold is a three sphere, S^3.

Technique part: the stability of a field configuration is provided by the homotopy group of mapping between space boundary and vacuum manifold.

$$S^2 \to S^3 : \quad \pi_2(S^3) = 0$$

Invariance property of the 1-form

- Electroweak scalar multiplet

$$E_{sph} = \frac{4\pi v}{g}\mathscr{F}(A_i^a, H, \Phi) \quad A_i^a T^a \sim f(\xi)(\partial_i U^\infty)U^{\infty -1} \quad i(U^{\infty -1})dU^\infty = \sum_{a=1}^{3} F_a T_a$$

Ahriche et.al. (2014) use the invariance property of F_a without proof

- Construction of general dimensional sphaleron unitary matrix

Express U^∞ as the multiplication of two Wigner-D matrices

$$U^\infty_{mn}(\mu,\theta,\phi) = \sum_{m'} D^J_{mm'}(\omega_-, -\theta, \mu) D^J_{m'n}(\mu, \theta, \omega_+),$$

$$\omega_\pm = -\mu \pm (\phi - \frac{\pi}{2}) \quad .$$

We demonstrate that F_a is invariant when $J = \left[\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right]$.

Sphaleron energy under multiple Higgs

Consider the SM Higgs (H) and a general SU(2) multiplet (Φ).

$$E = \frac{4\pi\Omega}{g}\int d\xi \left[\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}f^a_{ij}f^a_{ij} + (D_iH)^\dagger(D_iH) + (D_i\Phi)^\dagger(D_i\Phi) + V(H,\Phi)\right]$$

$$f'' + \frac{2}{\xi^2}(1-f)\Big[f(f-2) + f_3\left(1+f_3\right)\Big] + (1-f)(\frac{v^2h^2}{4\Omega^2} + \alpha\phi^2) = 0,$$

$$f_3'' - \frac{2}{\xi^2}\Big[3f_3 + f(f-2)\big(1+2f_3\big)\Big] + (\frac{v^2}{4\Omega^2}h^2 + \beta\phi^2)(f_0 - f_3) = 0,$$

$$f_0'' + \frac{2}{\xi^2}\left(1-f_0\right) - \frac{g'^2}{g^2}(\frac{v^2}{4\Omega^2}h^2 + \beta\phi^2)(f_0 - f_3) = 0,$$

$$h'' + \frac{2}{\xi}h' - \frac{2}{3\xi^2}h[2(1-f)^2 + (f_0-f_3)^2] - \frac{1}{g^2v^2\Omega^2}\frac{\partial V[h,\phi]}{\partial h} = 0,$$

$$\phi'' + \frac{2}{\xi}\phi' - \frac{8\Omega^2\phi}{3v_\phi^2\xi^2}[2\alpha(1-f)^2 + \beta(f_0-f_3)^2] - \frac{1}{g^2v_\phi^2\Omega^2}\frac{\partial V[h,\phi]}{\partial \phi} = 0,$$

Topological classification of field solution

Before EWSB: $G = SU(2)_L \times U(1)_Y$

After EWSB:

- If Multiplet Φ with $Y \neq 0$: $\quad H = U(1)_{em}$

$$\frac{G}{H} = \frac{SU(2)_L \times U(1)_Y}{U(1)_{em}} \simeq S^3$$

Sphaleron topology

i.e. SM Higgs

- If Multiplet Φ with $Y = 0$: $H = U(1)_{em} \times U(1)_Y$

$$\frac{G}{H} = \frac{SU(2)_L \times U(1)_Y}{U(1)_{em} \times U(1)_Y} \simeq S^2$$

Monopole topology

i.e. real triplet

$Y = 0$ **multiplet can contribute to DM relic density**

Complex septuplet ($Y = 0$) extension to the SM

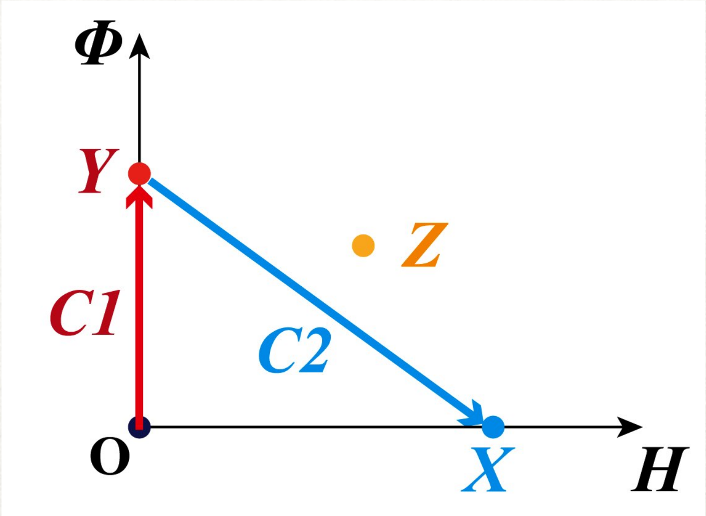
- The Higgs field and septuplet field

1812.07829

$$V = V_0(H) + V_{portal}(H, \Phi) + V_{self}(\Phi)$$

$$H = \begin{pmatrix} \omega^+ \\ \frac{1}{\sqrt{2}}(v + h + i\pi) \end{pmatrix}$$

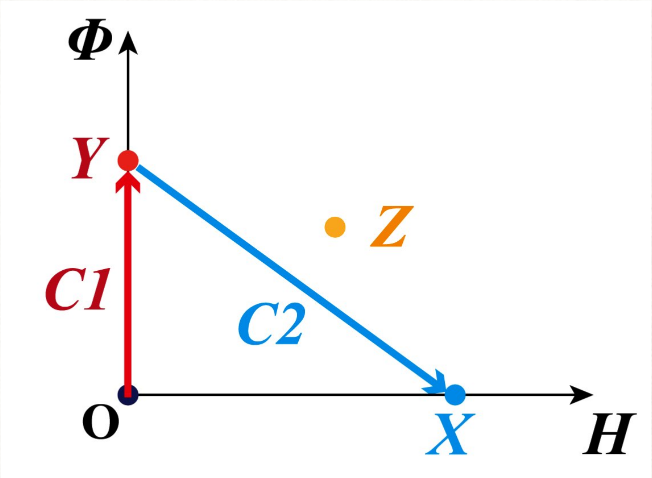
$$\Phi = \begin{pmatrix} \phi_{3,3} \\ \phi_{3,2} \\ \phi_{3,1} \\ \frac{1}{\sqrt{2}}(v_\phi + \phi + i\pi_\phi) \\ \phi_{3,-1} \\ \phi_{3,-2} \\ \phi_{3,-3} \end{pmatrix}$$



Baryon asymmetry could be made in $C1$, so it's important to know the Baryon number dilution rate (**monopole mass**) in broken phase at Y.

Complex septuplet ($Y = 0$) extension to the SM

- Monopole mass in the broken phase



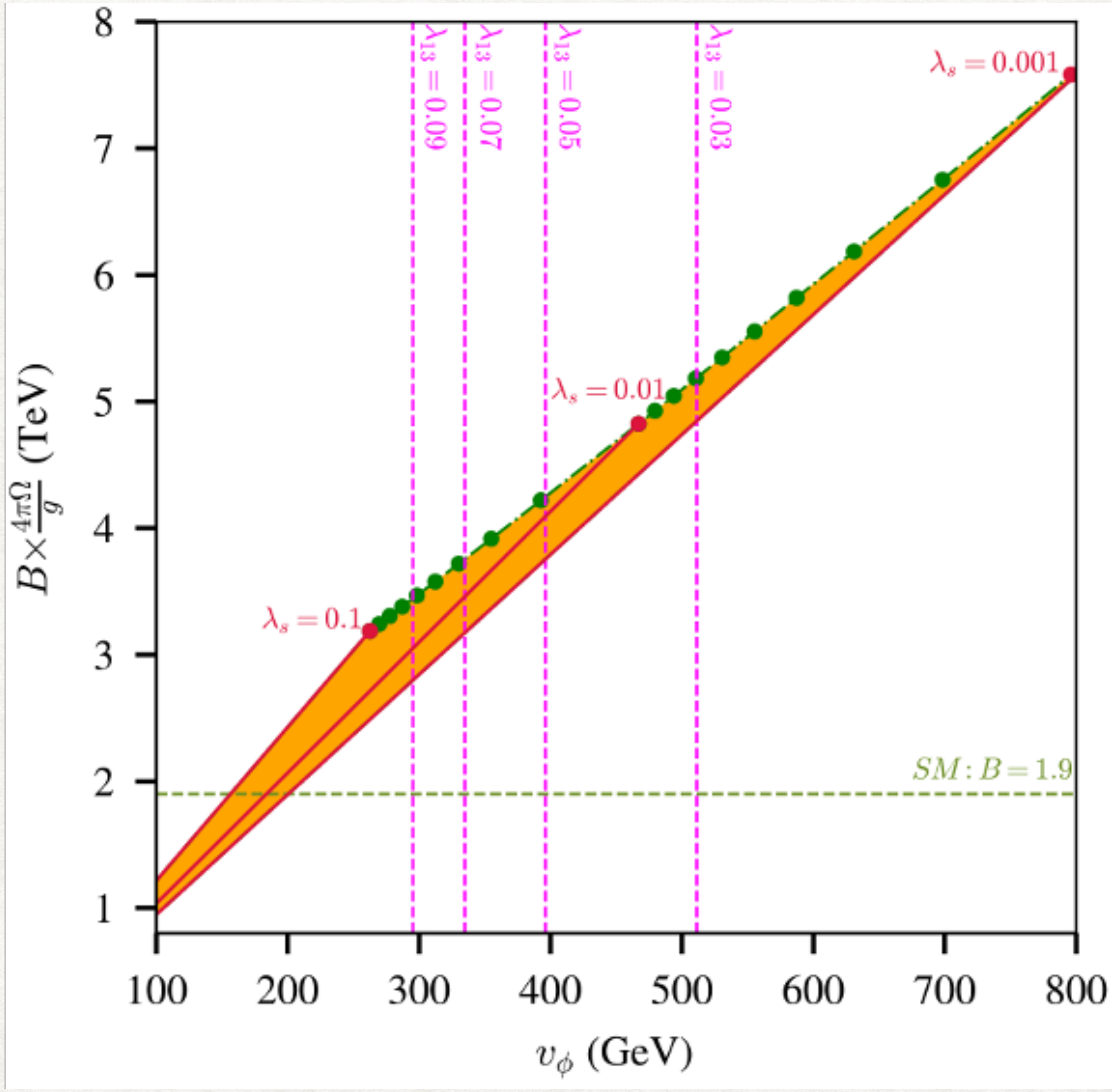
λ_{13}: effective portal coupling

λ_s : effective self coupling

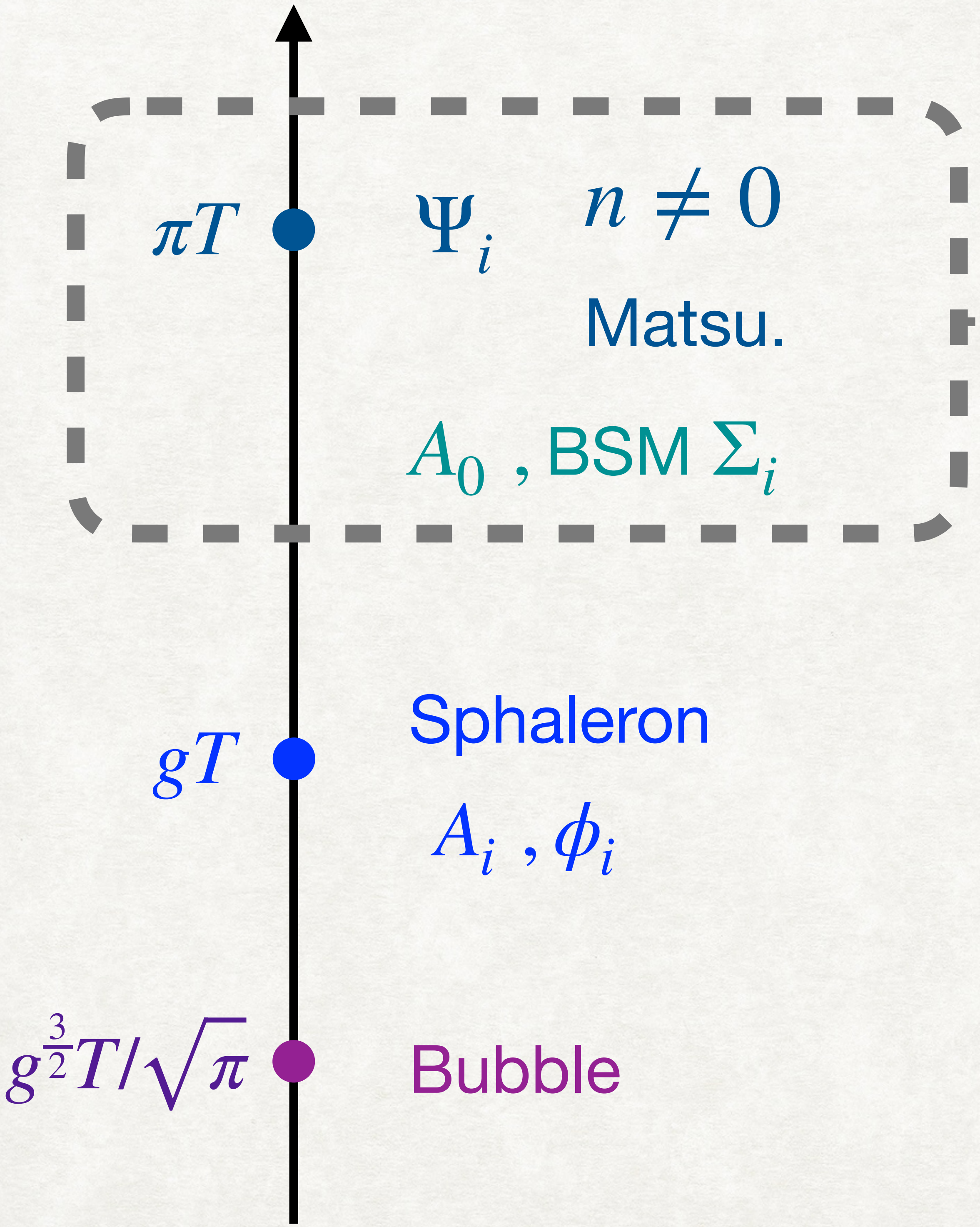
$$\Gamma_{BNV} \sim e^{-m/T}$$

$$m = B \times \frac{4\pi v}{g}$$

m :Sphaleron energy or monopole mass



Sphaleron 3D EFT



Integrated out

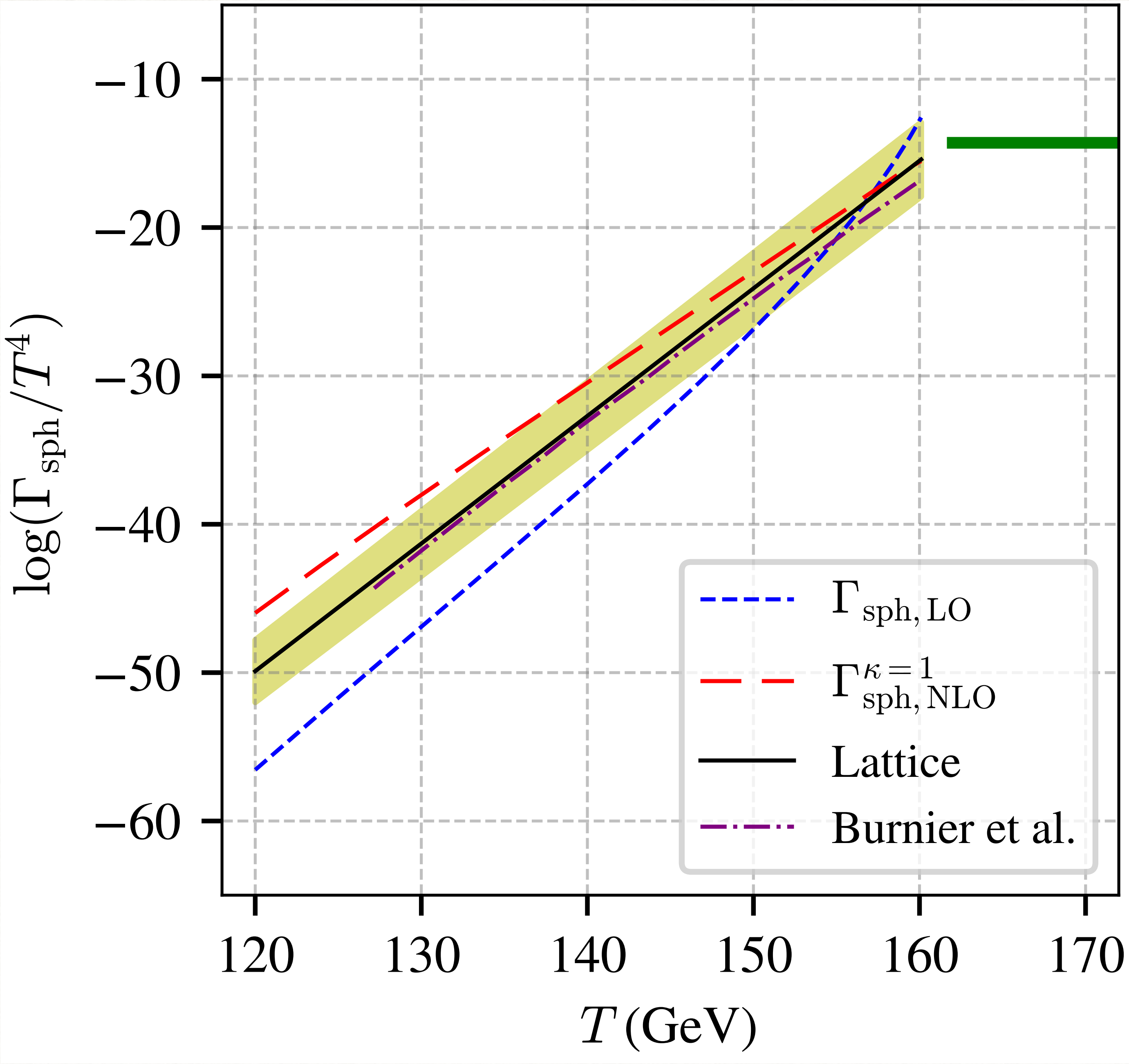
2-loop matching

$$S_3 = \int d^3x \left[\frac{1}{4} F^2 + (D_i \phi)^2 + V_3 \right] ,$$

$$V_3 = y\phi^\dagger \phi - q(\phi^\dagger \phi)^{3/2} + x(\phi^\dagger \phi)^2 .$$

Sphaleron 3D EFT

$\Gamma_{sph,LO}(x,y) = T^4 e^{-S_3}$, $\Gamma_{sph,NLO}(x,y) = T[det]_0 e^{-S_3}$.



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(hep-ph/0511246)