Mid term presentation for my thesis on the Higher-order contributions to the trilinear Higgs self-coupling in the SM and beyond

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A presentation for 2HDM group meeting

Overview

1. Introduction

Problems with the Standard Model

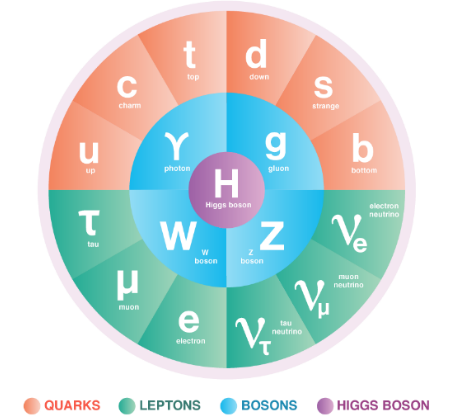

Figure 1: The Standard Model of particle physics.

Why Study the trilinear Higgs Coupling?

- **Vacuum Stability**: crucial for understanding the Higgs potential and vacuum stability in the Standard Model.
- **(S)FOEWPT**: Potentially relevant for explaining the baryon asymmetry of the universe (Nucleation, Thermal inequilibrium, Sphaleron).
- **Baryon Asymmetry**: The Standard Model does not provide a sufficient mechanism for the observed matter-antimatter asymmetry.

The Lagrangian of the Higgs Potential

- Consider the Lagrangian of the Higgs field $V(\phi)$ in the Standard Model, which can be expressed as:

$$\mathcal{L} = \partial^{\mu}\phi\partial_{\mu}\phi + \mu^2\phi^{\dagger}\phi - \frac{\lambda}{4}(\phi^{\dagger}\phi)^2 \tag{1}$$

  where ϕ is the Higgs field, μ^2 is a parameter related to the Higgs mass, and λ is the self-coupling constant.

- The trilinear Higgs coupling arises from the interaction term $\frac{\lambda}{4}(\phi^{\dagger}\phi)^2$

Spontaneous Symmetry Breaking

- In the SM, the Higgs field is initially written as

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

in terms of physical fields and Goldstones:

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H + iG^0) \end{pmatrix},$$

where v is the VEV, H is the physical Higgs field, and G^+, G^0 are the Goldstone bosons.

- Substituting and disregarding the Goldstone terms, we obtain:

$$\begin{aligned} \mathcal{L} \supset &+ H^4 \left(-\frac{1}{16}\lambda\right) \\ &+ H^3 \left(\frac{v^3\lambda}{4} - \frac{v\mu^2}{2}\right) \\ &+ H^2 \left(\frac{v^2\lambda}{8} + \frac{\mu^2}{2}\right) \\ &+ H \left(-\frac{v^3\lambda}{4} + v\mu^2\right) \end{aligned} \tag{2}$$

- trilinear coupling(H^3), mass term (H^2) the tadpole term (H).

Deriving Input Variables From The Lagrangian

Let's identify.

- The coefficient of H in (subset) $\mathcal{L}$ corresponds to t:

$$\frac{v^3\lambda}{4} - v\mu^2 = t \tag{3}$$

- The coefficient of H^2 in (subset) $\mathcal{L}$ corresponds to $-\frac{1}{2}M_H^2$:

$$\frac{1}{8}\left(-3v^2\lambda + 4\mu^2\right) = -\frac{M_H^2}{2} \tag{4}$$

- **Solve the system of equations:**
  Solving for λ and μ^2, we find:

$$\{\lambda \to \frac{2(t + M_H^2 v)}{v^3}, \mu^2 \to \frac{-3t - M_H^2 v}{2v}\} \tag{5}$$

Obtaining the countervertex

- **Substitute solutions into the Lagrangian:**
  Substituting these into the Lagrangian, we obtain:

$$\begin{aligned}\mathcal{L}_{\text{final}} \supset & + Ht \\ & - H^2 \frac{M_H^2}{2} \\ & - \frac{H^3}{2v^2}\left(\frac{t + M_H^2 v}{2}\right) \\ & - \frac{H^4}{8v^3}\left(\frac{t + M_H^2 v}{2}\right)\end{aligned} \tag{6}$$

- **(tree - level) Vertex Term:**
  Equating $v = \frac{2M_W s_w}{e}$, the vertex term is identified by the coefficient of H^3 in $\mathcal{L}$:

$$\text{Vertex} = -\frac{\mathrm{e}^2 H^3 \left(\frac{2M_H^2 M_W s_w}{\mathrm{e}} + t\right)}{8M_W^2 s_w^2} \tag{7}$$

2. Derivation Of The Vertex And The Counterterm Expression

Counterterm transformation

- **Substitute Shifts:**
  To account for higher-order corrections, we introduce the following shifts into the vertex term:

$$\text{Vertex}_C = \text{Vertex}\Bigg|_{\substack{M_H^2 \to M_H^2 + \delta^1_{M_H^2} + \delta^2_{M_H^2} \\ M_W \to M_W + \delta^1_{M_W} + \delta^2_{M_W} \\ t \to t + \delta^1_t + \delta^2_t \\ e \to e\left(1 + \delta Z^1_e + \delta Z^2_e\right) \\ s_w \to s_w\left(1 + \delta Z^1_{s_w} + \delta Z^2_{s_w}\right) \\ H \to H\sqrt{1 + \delta Z^1_H + \delta Z^2_H}}} \tag{8}$$

Counterterms and Self Energies

- We need to compute many counterterm expressions (for 2 -loop: 12 in total)
  Let us exemplify a first order calculation[1]
  We write down the Pole equation:

$$\mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}_{ii}^{\text{eff}}(\mathcal{M}_i^2) = 0, \tag{9}$$

  With $\mathcal{M}$ beeing the Complex pole given by:

$$\mathcal{M}_i^2 = M_i^2 - iM_i\Gamma_i, \tag{10}$$

  Expanding to first order around the real pole:

$$M_i^2 - m_i^2 + \text{Re}\hat{\Sigma}_{ii}^1(M_i^2) \stackrel{!}{=} 0. \tag{11}$$

  Implementing the OS scheme:

$$M_i^2 = m_i^2, \tag{12}$$

  Expressing the ren. SE through the bare SE and CTs:

$$\text{Re}\hat{\Sigma}_{ii}^1(M_i^2) = \text{Re}\Sigma_{ii}^1(M_i^2) - \delta^1 M_i^2 \tag{13}$$

  To finally recover the mass counterterm (for instance with the Z-Boson):

$$\delta^1 M_Z^2 = \text{Re}\Sigma_{ZZ}^{T,1}(M_Z^2). \tag{14}$$

[1] Of course, 2-loop counterterms are more involved. [2] and [3] discuss this in more detail.

Expanding the Vertex Term to first order

- We expand the shifted vertex term to the first loop order, substitute $t = 0$ and the equation for the counterterm of the W mass as given by:

$$\delta^1_{M_W} \to \frac{\delta^1 M_W^2}{2M_W} \tag{15}$$

- **Vertex:**
  The expression, as can be cross-checked in [1]:

$$C_0 + C_1 = -\frac{3e}{2s_w}\frac{M_H^2}{M_W}\left[1 + \delta Z_e - \frac{\delta s_w}{s_w} + \frac{\delta M_H^2}{M_H^2} + \frac{e}{2s_w}\frac{\delta t}{M_W M_H^2} - \frac{1}{2}\frac{\delta M_W^2}{M_W^2} + \frac{3}{2}\delta Z_H\right] \tag{16}$$

Results for first order

- Calculating every counterterm and inserting in (16) gives numerical contributuion to lambda
- We express the deviance from the tree level with κ_λ:

$$\kappa_\lambda = \frac{C_0 + N_{\mathcal{O}(l)}}{C_0} \tag{17}$$

  where C_0 is the tree-level vertex value and $N_{\mathcal{O}(l)}$ is the numerical contribution to the vertex up to order l
- For the top - and bottom contributions in the SM ($l = 1$) we have:

$$\kappa_{\lambda_{\mathrm{SMtb}}} \approx 0.899124 \tag{18}$$

- For the full SM ($l = 1$)we have:

$$\kappa_{\lambda_{\mathrm{SM}}} \approx 0.941966 \tag{19}$$

  For both we used the input values given in the Particle Data Group.

Expanding the Vertex Term to the second order

- **Our first result**

$$
\begin{aligned}
C_0 + C_1 + C_2 = -\frac{2eM_H^2}{M_W s_w}\Bigg(\Bigg[\frac{3}{4} + \frac{3}{8M_H^2 M_W s_w}\Bigg\{ & M_H^2\left(2M_W s_w \delta^1 Z_e + 3M_W s_w \delta^1 Z_H\right. \\
& \left. - M_W s_w \delta^1_{M_W^2} - 2M_W \delta^1 s_w\right) \\
& + e\delta^1 t + 2M_W s_w \delta^1 M_H^2 \Bigg\}\Bigg] \frac{3}{32M_H^2 M_W^2 s_w^2}\Bigg\{12M_H^2 M_W^2 s_w^2 \delta^1 Z_e \delta^1 Z_H \\
& + \frac{4M_W s_w \delta^1 M_H^2}{8M_H^2 M_W^2 s_w^2}\left(2M_W s_w \delta^1 Z_e + 3M_W s_w \delta^1 Z_H - M_W s_w \delta^1_{M_W^2} - 2M_W \delta^1 s_w\right) \\
& + 2e\delta^1 t\left(M_W\left(4s_w \delta^1 Z_e + 3s_w \delta^1 Z_H - 4\delta^1 s_w\right) - 2M_W s_w \delta^1_{M_W^2}\right) \\
& - 4M_H^2 M_W^2 s_w^2 \delta^1 Z_e \delta^1_{M_W^2} + 8M_H^2 M_W^2 s_w^2 \delta^2 Z_e \\
& - 8M_H^2 M_W^2 s_w \delta^1 Z_e \delta^1 s_w + 4eM_W s_w \delta^2 t \\
& - 6M_H^2 M_W^2 s_w^2 \delta^1 Z_H \delta^1_{M_W^2} + 3M_H^2 M_W^2 s_w^2 (\delta^1 Z_H)^2 \\
& + 12M_H^2 M_W^2 s_w^2 \delta^2 Z_H - 12M_H^2 M_W^2 s_w \delta^1 Z_H \delta^1 s_w \\
& + 2M_H^2 M_W^2 s_w^2 (\delta^1_{M_W^2})^2 - 4M_H^2 M_W^2 s_w^2 \delta^2 M_W^2 \\
& + 4M_H^2 M_W^2 s_w \delta^1_{M_W^2} \delta^1 s_w + 8M_W^2 s_w^2 \delta^2 M_H^2 \\
& + 8M_H^2 M_W^2 (\delta^1 s_w)^2 - 8M_H^2 M_W^2 s_w \delta^2 s_w \Bigg\}\Bigg)
\end{aligned}
\tag{20}
$$

Subloop (CT) Diagrams

- We could potentially start with $\mathcal{O}(N_c^2)$ contributions
- Counterterms: Generate self energy amplitudes
- Focus on contributions from the top and bottom quarks.
- Restriction to $\mathcal{O}(N_c^2)$ contributions: only two-loop corrections involving CT insertions contribute.
- Present two key types of diagrams:
  - The first type involves a counterterm insertion at the vertex of external to internal particles
  - The second type includes a counterterm insertion at the propagator.

Subloop (CT) Diagrams

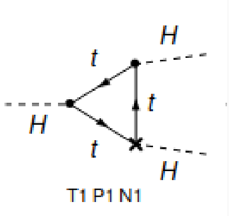

Figure 2: Feynarts diagram of the trilinear Higgs coupling with a subloop insertion at the vertex of f to H

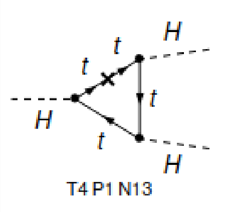

Figure 3: Feynarts diagram of the trilinear Higgs coupling with a subloop insertion at the propagator

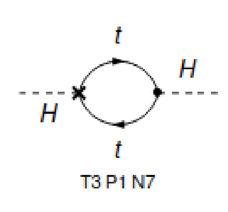

Figure 4: Feynarts diagram of a self-energy with a subloop insertion

3. Outlook

What is about to come

- **Analytic expressions**: Amplitudes of all relevant diagrams (even beyond $\mathcal{O}(N_c^2)$).
- **Counterterms**: Functions of self-energies, their derivatives, first-order counterterms.
- **Finiteness**: Eliminating $1/\epsilon^2$, $1/\epsilon$ terms, computing finite contributions.
- **Constraints**: Numerical contributions, accounting for constraints from unitarity, stability, electroweak precision observables...

4. Literature

Literature

- A. Denner, https://arxiv.org/abs/0709.1075v1 [1]
- D. Meuser, Complete electroweak $\mathcal{O}(N_c^2)$ two-loop contributions to the Higgs boson masses in the MSSM and aspects of two-loop renormalisation [2]
- A. Stremplat, 2-Schleifen-Beitraege zu leptonischen Praezisionsobservablen [3]