Vacuum Stability Constraints in EVADE and Vevacious++

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Motivation

- Extend the baseline for constraints in phenomenological studies
- Go beyond Boundedness from Below constraints
- Find likelihood of a given parameter point to tunnel to a deeper minimum of the potential

Outline

- Theoretical framework of the THDMS
- Testing for constraints
- Which one to use?

Theoretical framework of THDMS

Two Higgs doublets

$$\Phi_1 = \begin{pmatrix} \chi_1^+ \\ v_1 + \frac{\rho_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \chi_2^+ \\ v_2 + \frac{\rho_2 + i\eta_2}{\sqrt{2}} \end{pmatrix} \tag{1}$$

Additional complex singlet

$$S = v_S + \frac{\rho_S + i\eta_S}{\sqrt{2}} \tag{2}$$

$$\tan\beta = \frac{v_2}{v_1}, \qquad v = \sqrt{v_1^2 + v_2^2} = 174 \text{ GeV} \tag{3}$$

Symmetry

Fields	$\mathbb{Z}_2$	$\mathbb{Z}_3$
Φ_1	$+1$	$+1$
Φ_2	-1	$e^{i2\pi/3}$
S	$+1$	$e^{-i2\pi/3}$

Theoretical framework of THDMS

Higgs potential:

$$\begin{aligned} V =& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\ &+ \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) - m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.} \\ &+ m_S^2 S^\dagger S + \lambda_1'(S^\dagger S)(\Phi_1^\dagger\Phi_1) + \lambda_2'(S^\dagger S)(\Phi_2^\dagger\Phi_2) \\ &+ \frac{\lambda_3''}{4}(S^\dagger S)^2 + \left(\frac{\mu_{S1}}{6}S^3 + \mu_{12}S\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right) \end{aligned} \tag{4}$$

12 free parameters:

$$\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_1',\ \lambda_2',\ \lambda_3'',\ m_{12},\ \mu_{S1},\ \mu_{12},\ v_S,\ \tan\beta \tag{5}$$

m_{12} softly break the $\mathbb{Z}_2$, $\mathbb{Z}_3$ symmetry

Theoretical framework of THDMS

Tree level Higgs mass matrices:

$$
\begin{aligned}
M^2_{S11} &= 2\lambda_1 v^2 \cos^2\beta + (m^2_{12} - \mu_{12} v_S)\tan\beta \\
M^2_{S22} &= 2\lambda_2 v^2 \sin^2\beta + (m^2_{12} - \mu_{12} v_S)\cot\beta \\
M^2_{S12} &= (\lambda_3 + \lambda_4) v^2 \sin 2\beta - (m^2_{12} - \mu_{12} v_S) \\
M^2_{S13} &= (2\lambda'_1 v_S \cos\beta + \mu_{12}\sin\beta) v \\
M^2_{S23} &= (2\lambda'_2 v_S \sin\beta + \mu_{12}\cos\beta) v \\
M^2_{S33} &= \frac{\mu_{S1}}{2} v_S + \lambda''_3 v_S^2 - \mu_{12}\frac{v^2}{2v_S}\sin 2\beta
\end{aligned}
\qquad
\begin{aligned}
M^2_{P11} &= (m^2_{12} - \mu_{12} v_S)\tan\beta \\
M^2_{P22} &= (m^2_{12} - \mu_{12} v_S)\cot\beta \\
M^2_{P12} &= -(m^2_{12} - \mu_{12} v_S) \\
M^2_{P13} &= \mu_{12} v \sin\beta \\
M^2_{P23} &= -\mu_{12} v \cos\beta \\
M^2_{P33} &= -\frac{3}{2}\mu_{S1} v_S - \mu_{12}\frac{v^2}{2v_S}\sin 2\beta \\
M^2_{C} &= 2(m^2_{12} - \mu_{12} v_S)\csc 2\beta - \lambda_4 v^2
\end{aligned}
\tag{6}
$$

Diagonalization $\longrightarrow$ mixing matrices R_{ij} $\longrightarrow$ parameterized by rotation angles α_i

Theoretical framework of THDMS

Express the free parameters in terms of tree level masses and mixing angles

$$\tilde{\mu}^2 = \cos^2\alpha_4 m_{a1}^2 + \sin^2\alpha_4 m_{a2}^2$$

$$\begin{aligned}
\lambda_1 &= \frac{1}{2v^2\cos^2\beta}\left[\sum_i m_{h_i}^2 R_{i1}^2 - \tilde{\mu}^2\sin^2\beta\right] & \mu_{12} &= \frac{m_{a1}^2 - m_{a2}^2}{v}\sin\alpha_4\cos\alpha_4 \\
\lambda_2 &= \frac{1}{2v^2\sin^2\beta}\left[\sum_i m_{h_i}^2 R_{i2}^2 - \tilde{\mu}^2\cos^2\beta\right] & \lambda'_1 &= \frac{1}{2v_S v\cos\beta}\left[\sum_i m_{h_i}^2 R_{i1}R_{i3} - \mu_{12}v\sin\beta\right] \\
\lambda_3 &= \frac{1}{v^2}\left[\frac{1}{\sin 2\beta}\sum_i m_{h_i}^2 R_{i1}R_{i2} + m_{h^\pm}^2 - \tilde{\mu}^2\right] & \lambda'_2 &= \frac{1}{2v_S v\sin\beta}\left[\sum_i m_{h_i}^2 R_{i2}R_{i3} - \mu_{12}v\cos\beta\right] \\
\lambda_4 &= \frac{\tilde{\mu}^2 - m_{h^\pm}^2}{v^2} & \lambda''_3 &= \frac{1}{v_S^2}\left[\sum_i m_{h_i}^2 R_{i3}^2 + \mu_{12}\frac{v^2}{2v_S}\sin 2\beta - \frac{\mu_{S1}}{2}v_S\right] \\
m_{12}^2 &= \mu_{12}v_S + \tilde{\mu}^2\sin\beta\cos\beta & \mu_{S1} &= -\frac{2}{3v_S}\left[\sin^2\alpha_4 m_{a1}^2 + \cos^2\alpha_4 m_{a2}^2 + \frac{v}{2v_S}\sin 2\beta\,\mu_{12}\right]
\end{aligned} \quad (7)$$

Inputs of mass basis (12 parameters):

$$m_{h_{1,2,3}},\ m_{a_{1,2}},\ m_{h^\pm},\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4,\ v_S,\ \tan\beta \quad (8)$$

Constraints

- Tree-level unitarity $\longrightarrow$ `SARAH` & `SPheno` checks
- Boundedness from Below $\longrightarrow$ N2HDM
- Perturbative Unitarity Constraints

Benchmark point

Type II Yukawa structure

- Implement the THDMS in the SARAH
- Use SPheno to generate the spectra ($\overline{\mathrm{DR}}$ scheme)
- Benchmark point

$$
\begin{aligned}
\lambda_1 &= 0.5519 & \lambda_2 &= 0.06848 & \lambda_3 &= 0.3574 \\
\lambda_4 &= -0.04036 & \lambda_1' &= 0.06433 & \lambda_2' &= -0.07785 \\
\lambda_3'' &= 0.8431 & \mu_{S1} &= -440.5 & \mu_{12} &= -4.84 \\
v_S &= 300.2 & m_{12} &= 259537 & \tan\beta &= 1.45
\end{aligned}
$$

$$
\begin{aligned}
m_{h_1} &= 96.1 \text{ GeV}, & m_{h_2} &= 125.2 \text{ GeV}, & m_{a_1} &= 446.3 \text{ GeV} \\
m_{h^\pm} &= 745.5 \text{ GeV}, & m_{a_2} &= 744 \text{ GeV}, & m_{h_3} &= 743.7 \text{ GeV}
\end{aligned}
$$

Comparing EVADE and Vevacious(++)

- Scan benchmark point around cubic parameters μ_{12} and μ_{S2}
- $\mu_{12} \in \{-400, 400\}$ and $\mu_{S2} \in \{100, -1000\}$

Vevacious(++)

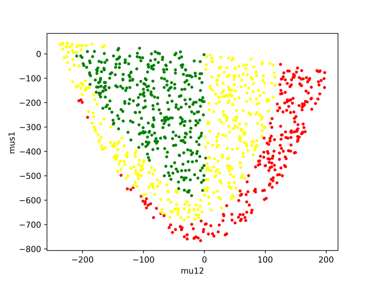
- Homotopy Continuation with HOM4PS2
- First guess of the bounce action via straight path approximation
- Optimization by CosmoTransitions (old code, rewritten in ++ version)
- Tree-level or Coleman-Weinberg one-loop effective potential

EVADE

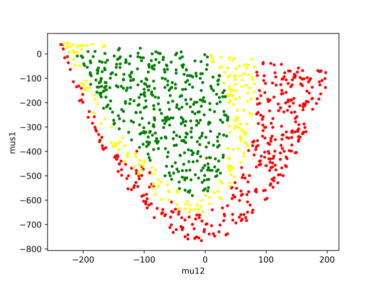
- Homotopy Continuation with HOM4PS2
- Straight path approximation for the Bounce Action
- Tree-level

Scan results

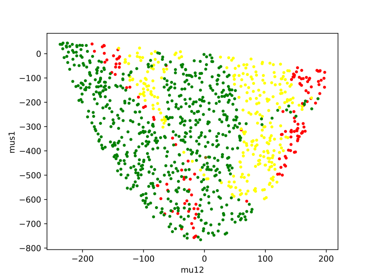
● unstable ● long-lived ● stable



EVADE



Vevacious tree-level

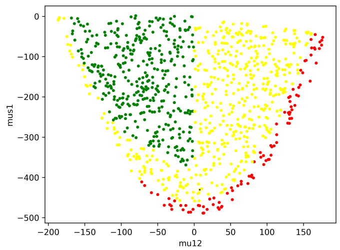


Vevacious one-loop

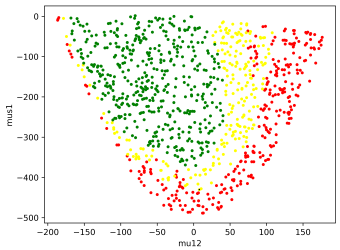
- Clear cut for the stable region at $\mu_{12} = 0$ in EVADE
- Tree-level results are in general in good agreement
- Vevacious one-loop...?

Other benchmark points I

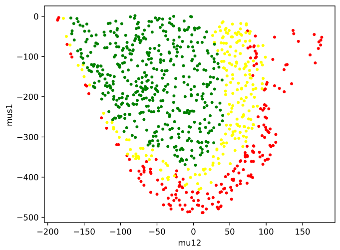
● unstable ● long-lived ● stable



EVADE



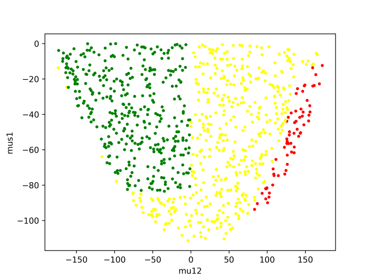
Vevacious tree-level



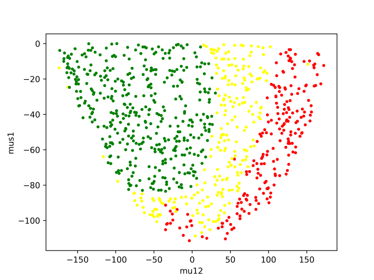
Vevacious one-loop

Other benchmark points II

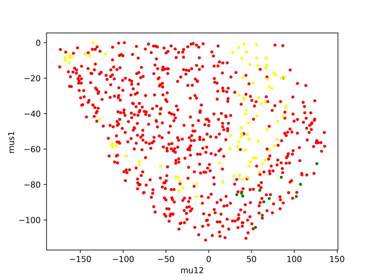
● unstable ● long-lived ● stable



EVADE



Vevacious tree-level



Vevacious one-loop

Summary

Which on to use?

- Short answer: EVADE, more recent and no/fewer numerical instabilities
- Long answer: EVADE, but if in doubt, compare results at tree level with Vevacious
- No consistent results at one-loop level with Vevacious(++) in our case

Outlook

- Bounce action plots for the same scenario
- Recheck one-loop calculations
- Compare minima locations around $\mu_{12} = 0$