

# SUSY Parameter Determination for Dark Matter Candidates

Nele Peters | 27.03.2026

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⇒ accuracy of parameter reconstruction from measurements at future  $e^+e^-$  collider, including DM relic density

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Here: NMSSM electroweak sector, including dark matter candidate

⇒ accuracy of **parameter reconstruction** from measurements at future  $e^+e^-$  collider, including **DM relic density**

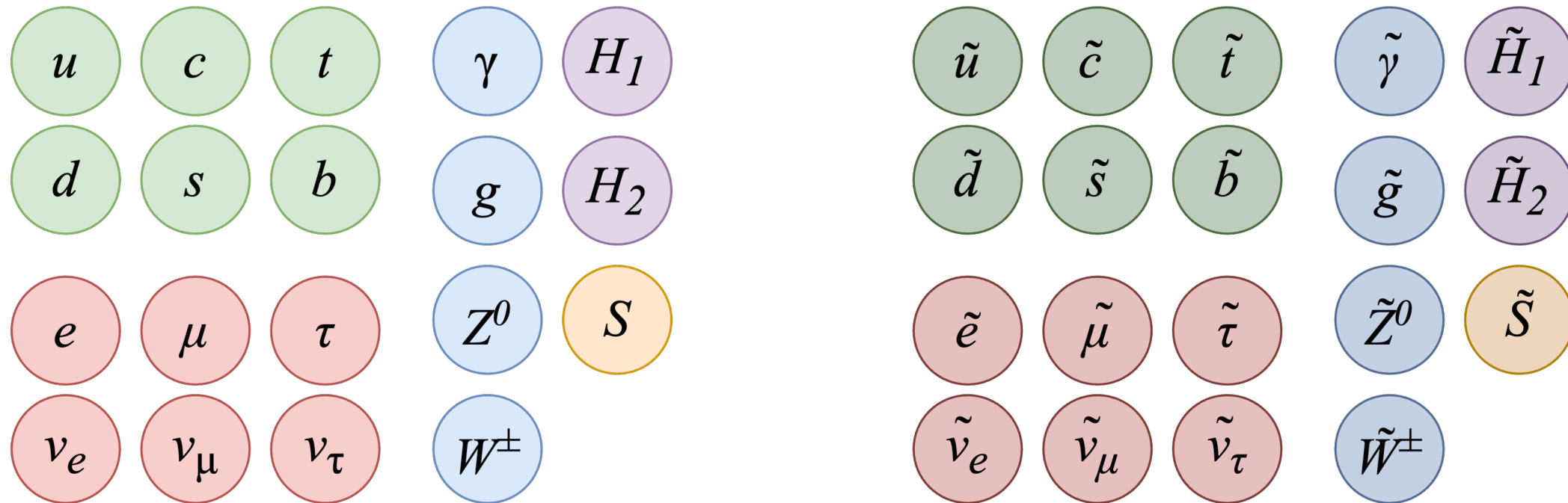
Scenario: excess at LHC (CMS + ATLAS)

Development of the method:

- First construction (2004)  
[K. Desch et al., arXiv: hep-ph/0312069]
  - Automation (2023)  
[Florian Lika, Master's thesis]
  - Analysis of **LHC excesses in the MSSM** (2026)  
[Jasmin Becks, Master's thesis]
- Analysis of the excesses in the **NMSSM**

# The NMSSM

SM particles + 2 Higgs-Doublets + singlet (left) and their supersymmetric partners (right)

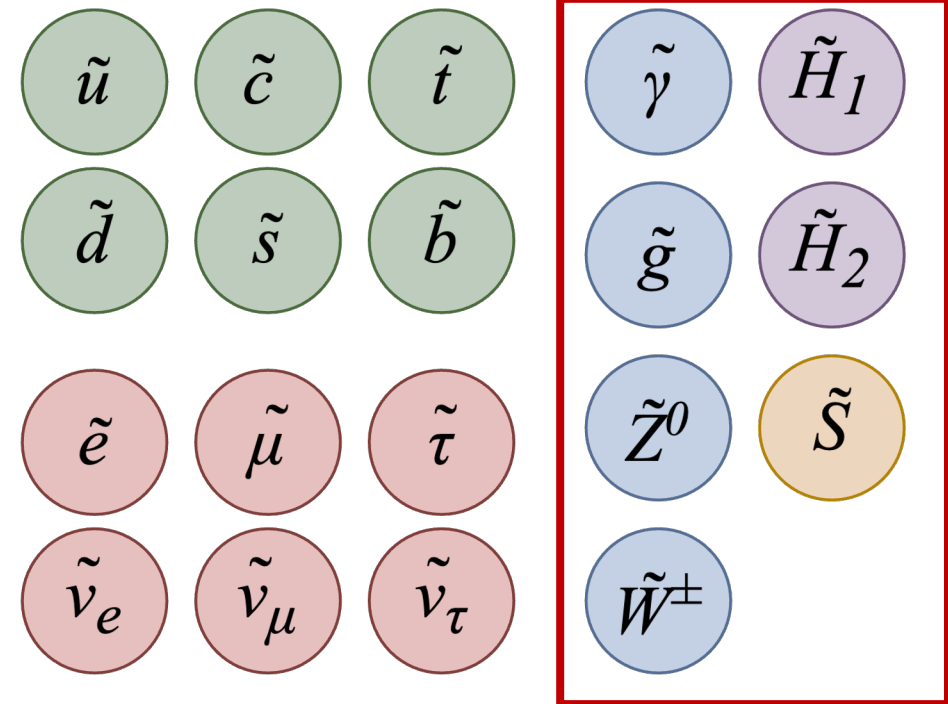
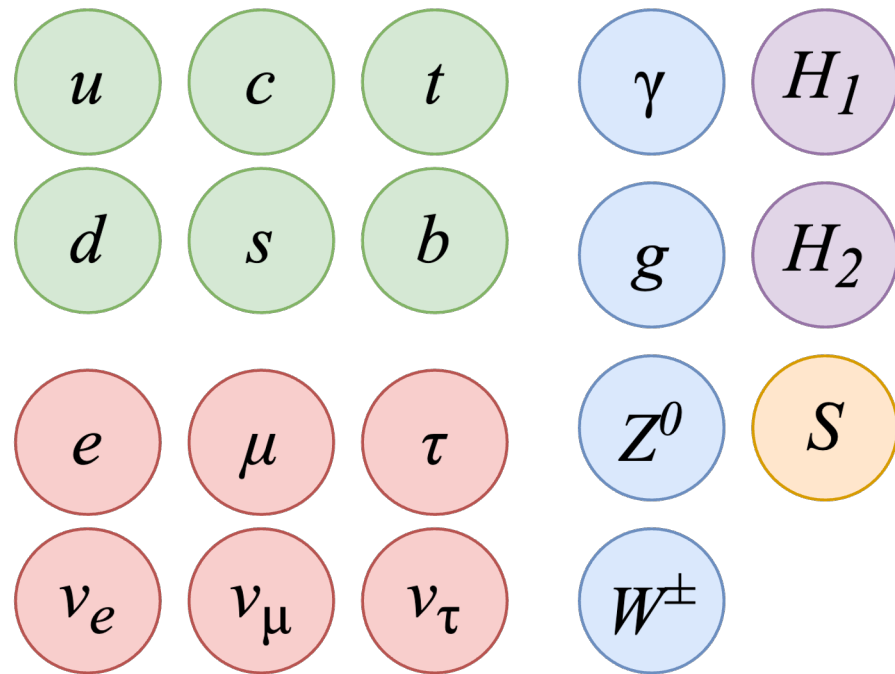


● Quarks 
 ● Leptons 
 ● Gauge Bosons 
 ● Higgs Fields 
 ● Singlet

● Squarks 
 ● Sleptons 
 ● Gauginos 
 ● Higgsino Fields 
 ● Singlino

# The NMSSM

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# Chargino Sector

Charged winos and higgsinos mix to form **charginos**  $\tilde{\chi}_{1,2}^{\pm}$

$$\text{Chargino mass matrix: } M_C = \begin{pmatrix} M_2 & m_w \sqrt{2} \sin \beta \\ m_w \sqrt{2} \cos \beta & \mu \end{pmatrix}$$

- $M_2$ : **Wino** mass parameter
- $\mu$ : **Higgsino** mass parameter
- $\tan \beta = \frac{v_2}{v_1}$ : Ratio of Higgs vevs

Mass values via diagonalisation with mixing angles  $\phi_{L,R}$

# Neutralino Sector

$\{\tilde{B}^0, \tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S}\}$  mix to form **neutralinos**  $\tilde{\chi}_{1,2,3,4,5}^0$

Neutralino mass matrix:  $M_N = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W & 0 \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W & 0 \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu & -\lambda v_2 \\ m_Z s_\beta s_W & m_Z s_\beta c_W & -\mu & 0 & -\lambda v_1 \\ 0 & 0 & -\lambda v_2 & -\lambda v_1 & 2\kappa v_s \end{pmatrix}$

- $M_1$ : **Bino** mass parameter
- $\lambda, \kappa$ : **Singlet** couplings
- $v_s$ : Singlet vev

- $c_W = \sqrt{1 - s_W^2} = \frac{m_W}{m_Z}$

- Lightest neutralino as **dark matter candidate**
- **Singlino-like** lightest neutralino, **higgsino-like** light charginos and neutralinos
- **Input points** in NMSSM parameter space: SLHA files from  
*“Consistent excesses in the LHC electroweak SUSY searches:  
GUT-based singlino/higgsino interpretation in the NMSSM”*  
[Bagnaschi et al., arXiv:2512.16783]

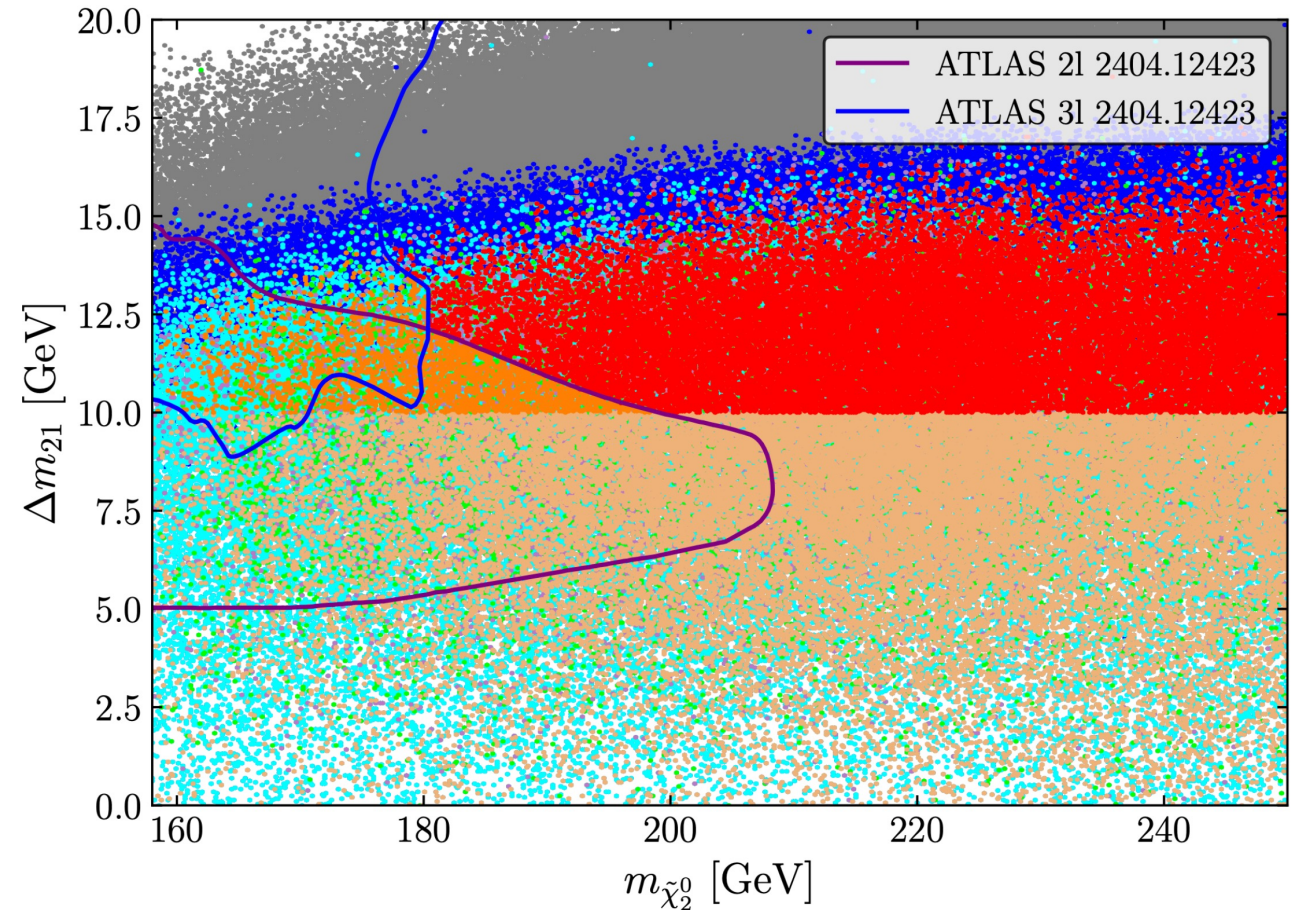
# Data Points

Excess:

- Channel:  $p\bar{p} \rightarrow \tilde{\chi}_{2,3}^0 \tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 Z^* \tilde{\chi}_1^0 W^{\pm*}$
- $\Delta m_{21} = m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} \leq 20 \text{ GeV}$
- $m_{\tilde{\chi}_2^0} \approx m_{\tilde{\chi}_1^\pm} \gtrsim 200 \text{ GeV}$

Input points:

- Consistent with multiple **experimental constraints**
- $M_2 = 2M_1$  ( $\approx$  **GUT-relation**)



[Bagnaschi et al., arXiv:2512.16783]

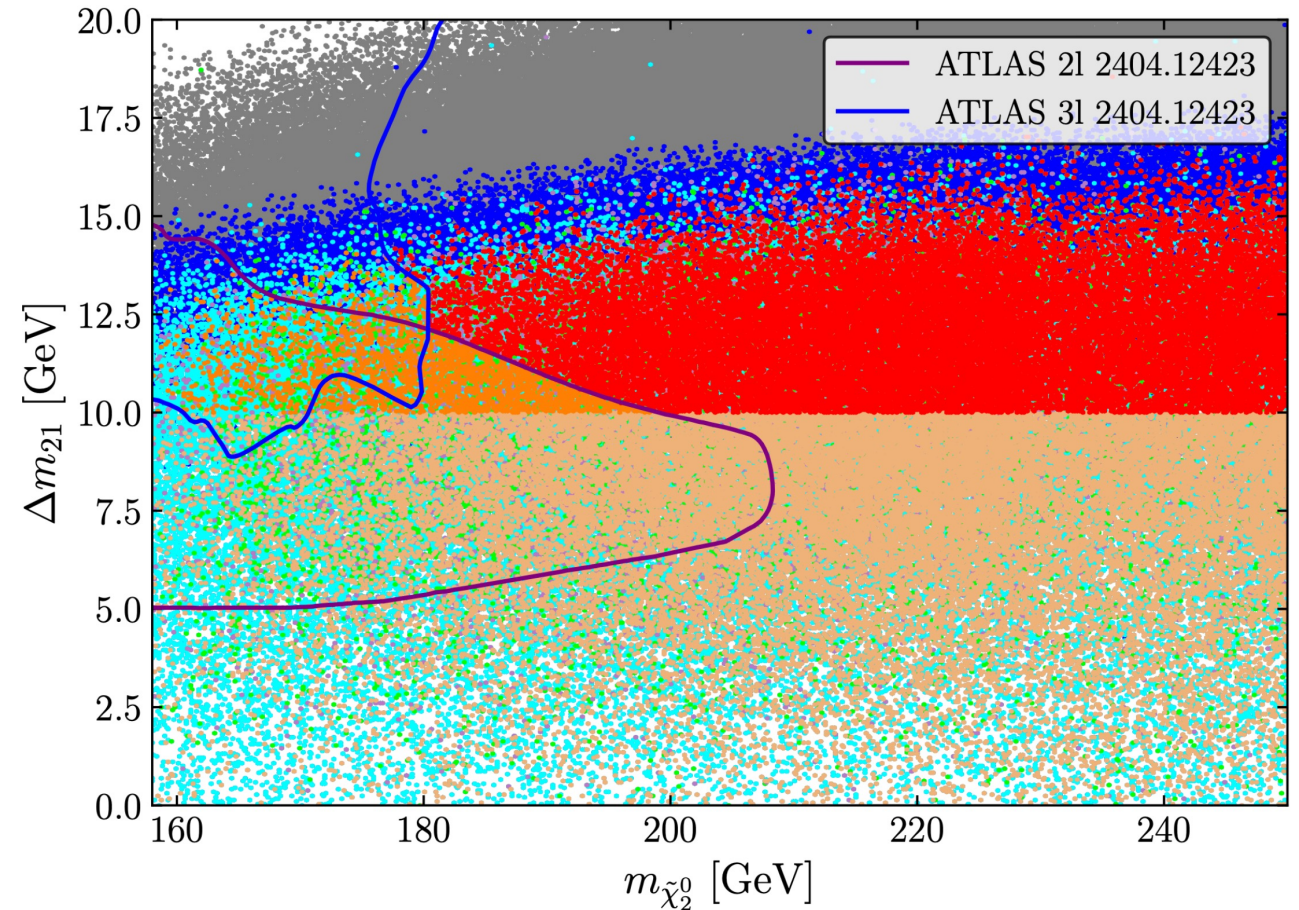
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→ here as simplification



[Bagnaschi et al., arXiv:2512.16783]

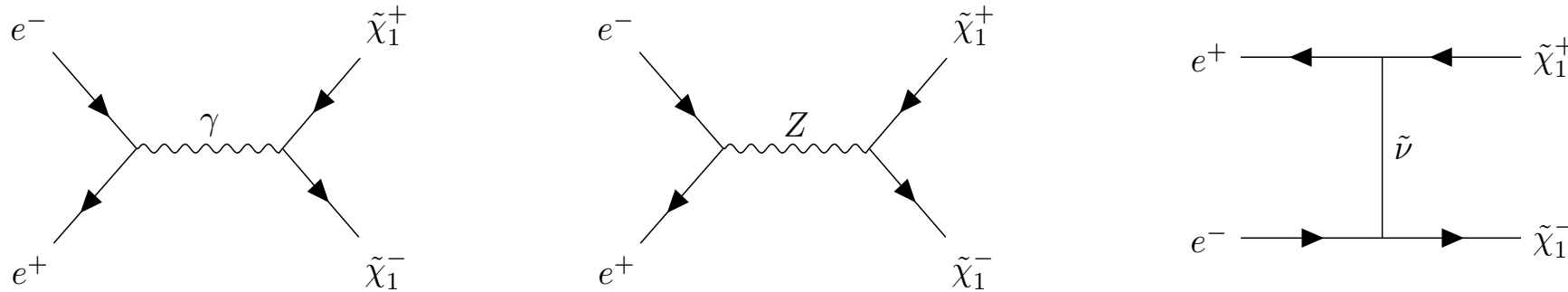
# Steps of the Analysis

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1. **Input**: benchmark points in NMSSM parameter space
2. **Hypothetical measurements** at a future  $e^+e^-$  collider
3. **Scan** of parameter space to reconstruct parameters
4. Evaluation of **DM relic density** for every parameter point

# Collider Observables

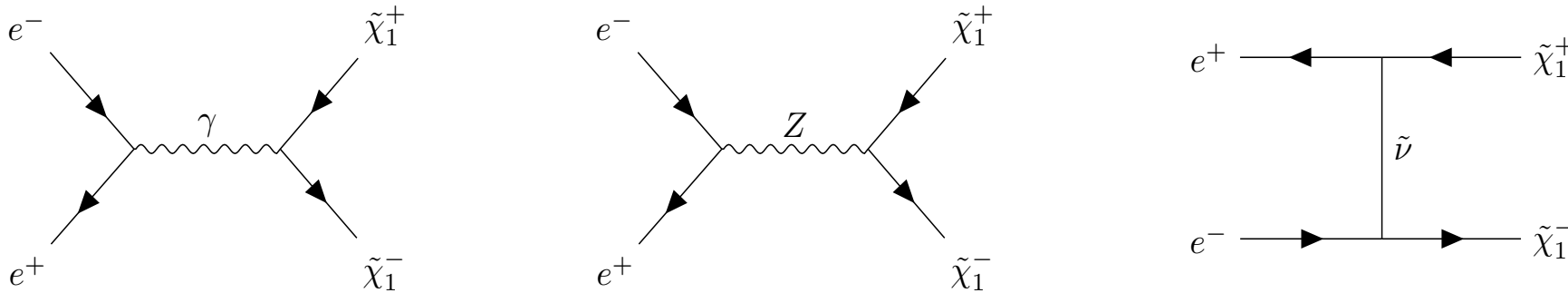
Hypothetical measurements at future  $e^+e^-$  collider (ILC), Luminosity  $L = 500 \text{ fb}^{-1}$ :



- Chargino pair production **cross section  $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$**  (4 beam configurations)
  - centre-of-mass energy  $\sqrt{s} \in \{500 \text{ GeV}, 550 \text{ GeV}\}$
  - beam polarisations  $(P_{e^-}, P_{e^+}) \in \{(-0.8, +0.6), (+0.8, -0.6)\}$

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- Lightest **chargino mass**
- Lightest three **neutralino masses**

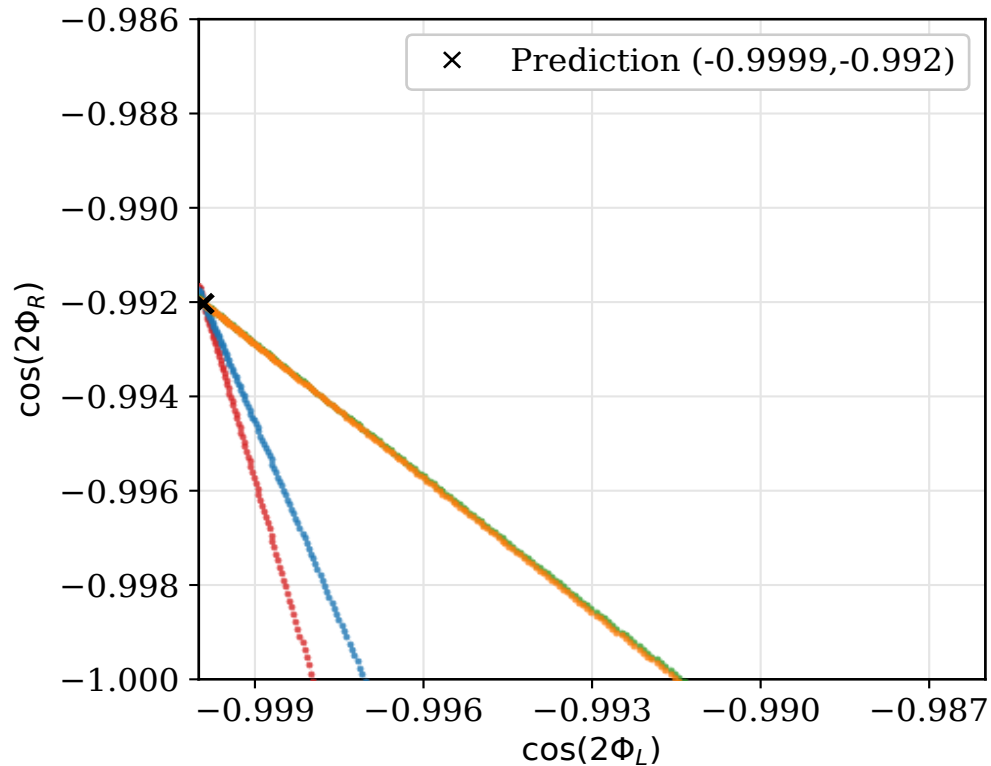
**Uncertainties:**  $\frac{\Delta P_e}{P_e} = \frac{\Delta m_{\tilde{\chi}}}{m_{\tilde{\chi}}} = 0.5\%$  (masses/polarisation\*),  $\sigma_{stat} = \frac{\sqrt{\sigma L}}{L}$  (cross section)

# Ellipse Bands in Mixing Angle Space

Cross sections  $\sigma(\cos 2\phi_L, \cos 2\phi_R)$  bilinear in mixing angles

Measurement: sets of  $(\cos 2\phi_L, \cos 2\phi_R) \rightarrow$  ellipse for every configuration

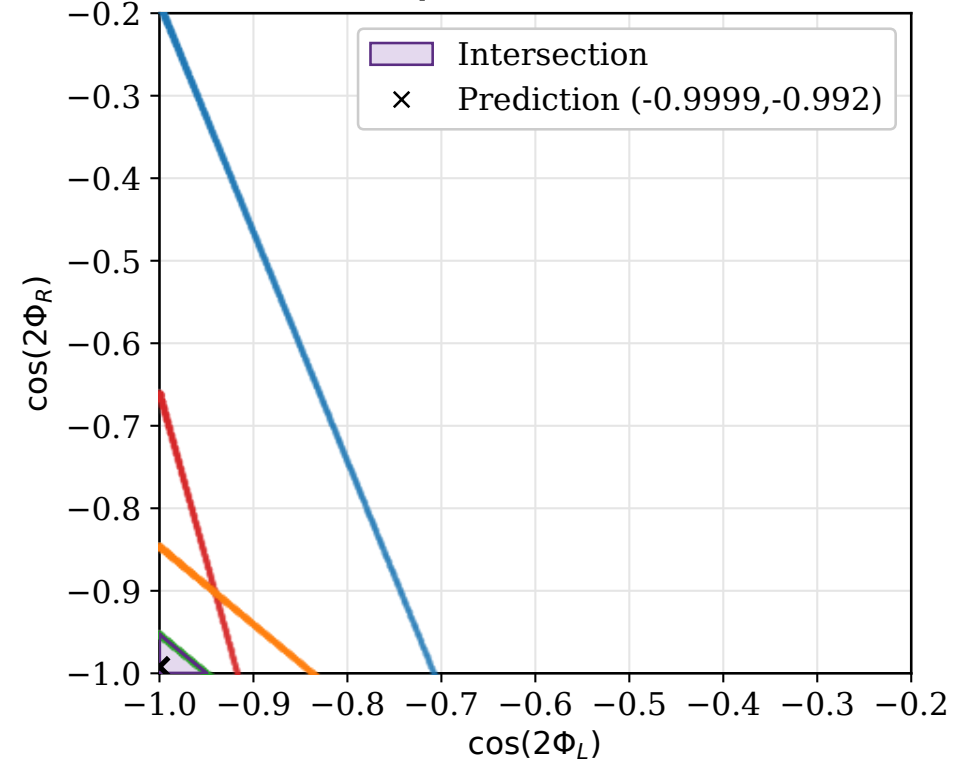
### Ellipses



Uncertainties



### Ellipse bands

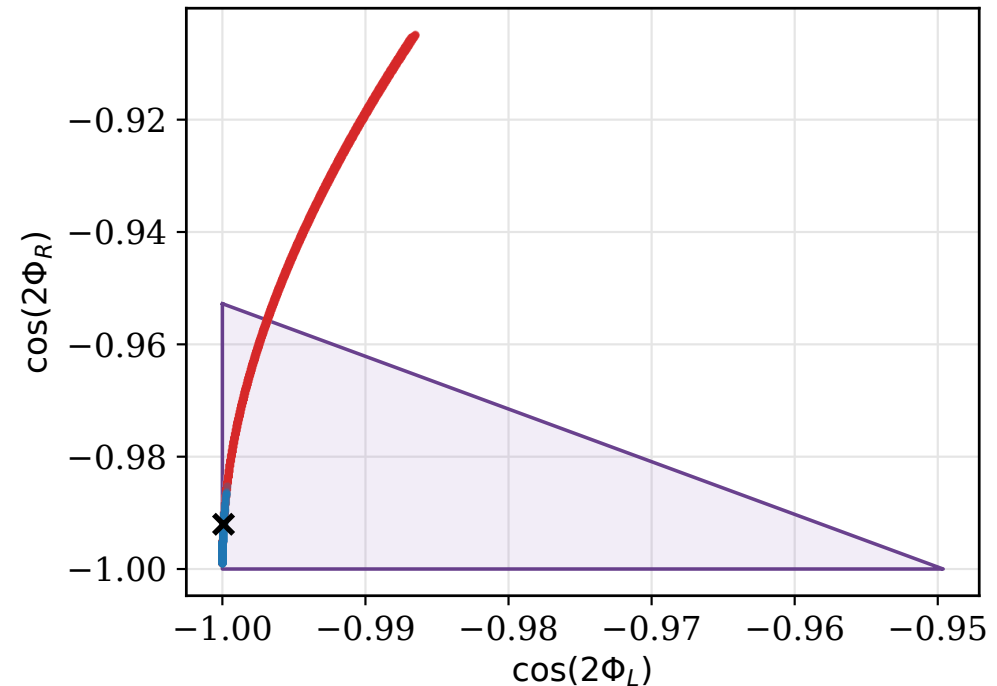


# Parameter Scan

Sets of random points in parameter space

Scan:

1. Chargino mass condition
2. Neutralino mass conditions
3. Cross section condition



- Allowed section
- Chargino mass conditions
- Theory prediction (-0.9999, -0.992)
- Chargino + neutralino mass conditions

# Results: $\mu$

Higgsino mass parameter

Relative error:

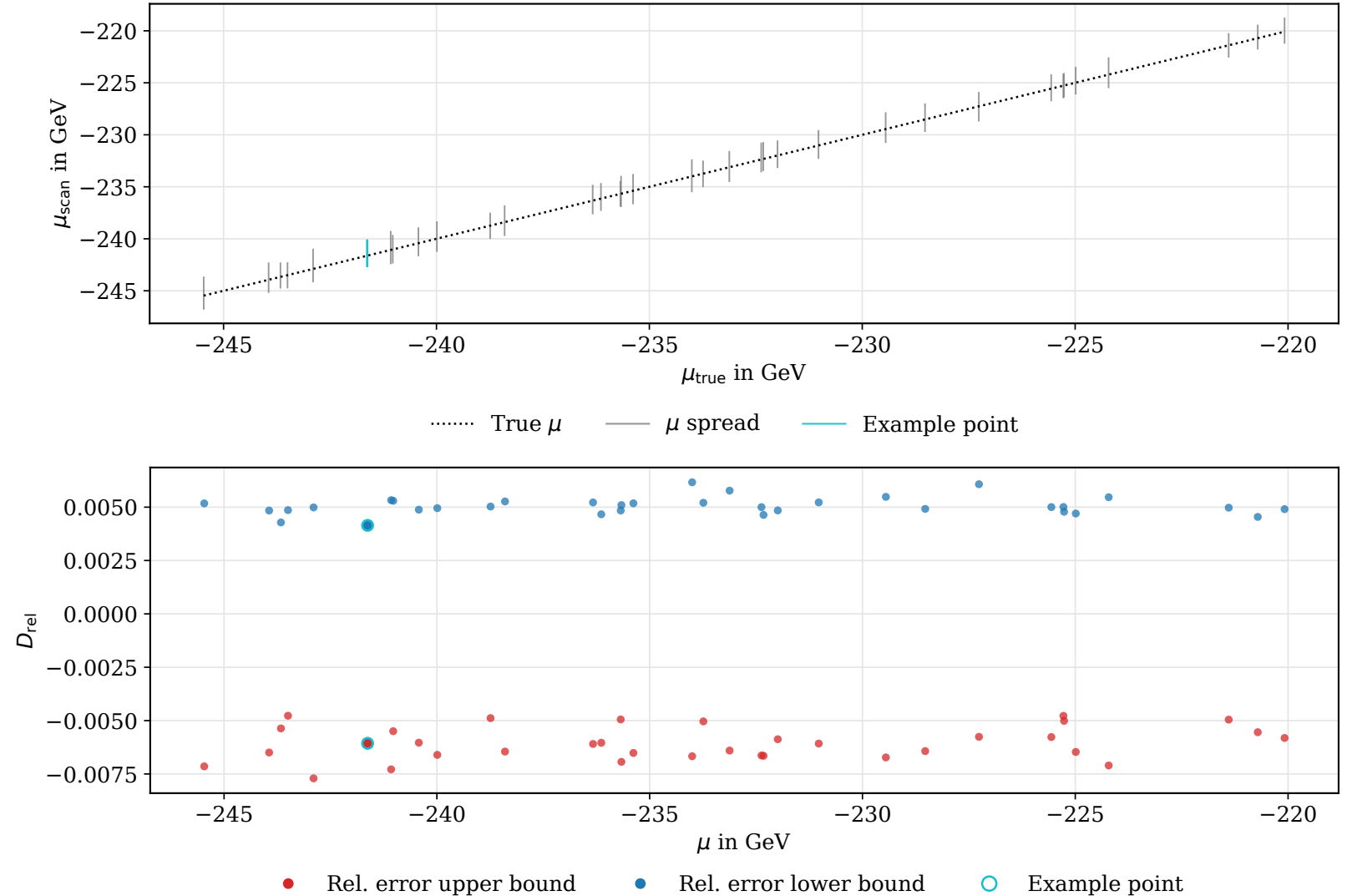
$$D_{rel} = \frac{\mu_{scan} - \mu_{true}}{\mu_{true}}$$

Upper bound:  $\sim 0.50\%$

Lower bound:  $\sim 0.65\%$

→ uniform accuracy

Here:  $\mu < 0$



# Results: $M_1$

Bino mass parameter

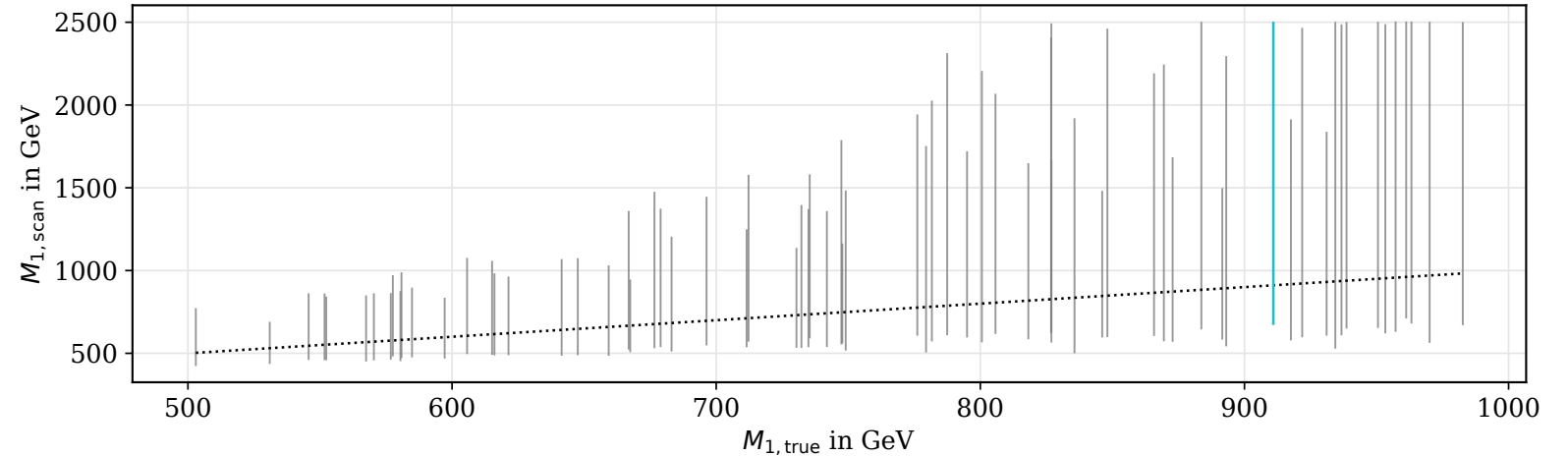
Relative error:

$$D_{rel} = \frac{M_{1,scan} - M_{1,true}}{M_{1,true}}$$

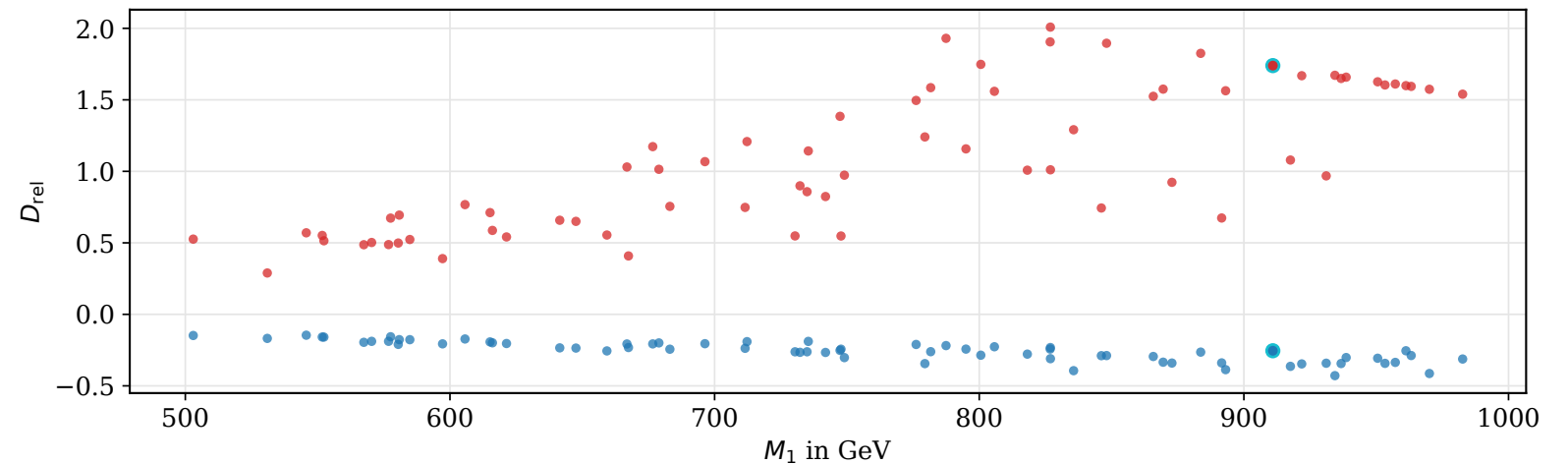
Upper bound: 30 – 200%

Lower bound: 15 – 45%

→ accuracy decreases for larger  $M_1$



..... True  $M_1$     —  $M_1$  spread    — Example point



• Rel. error upper bound    • Rel. error lower bound    ○ Example point

# Results: $\lambda$

Singlet coupling

Relative error:

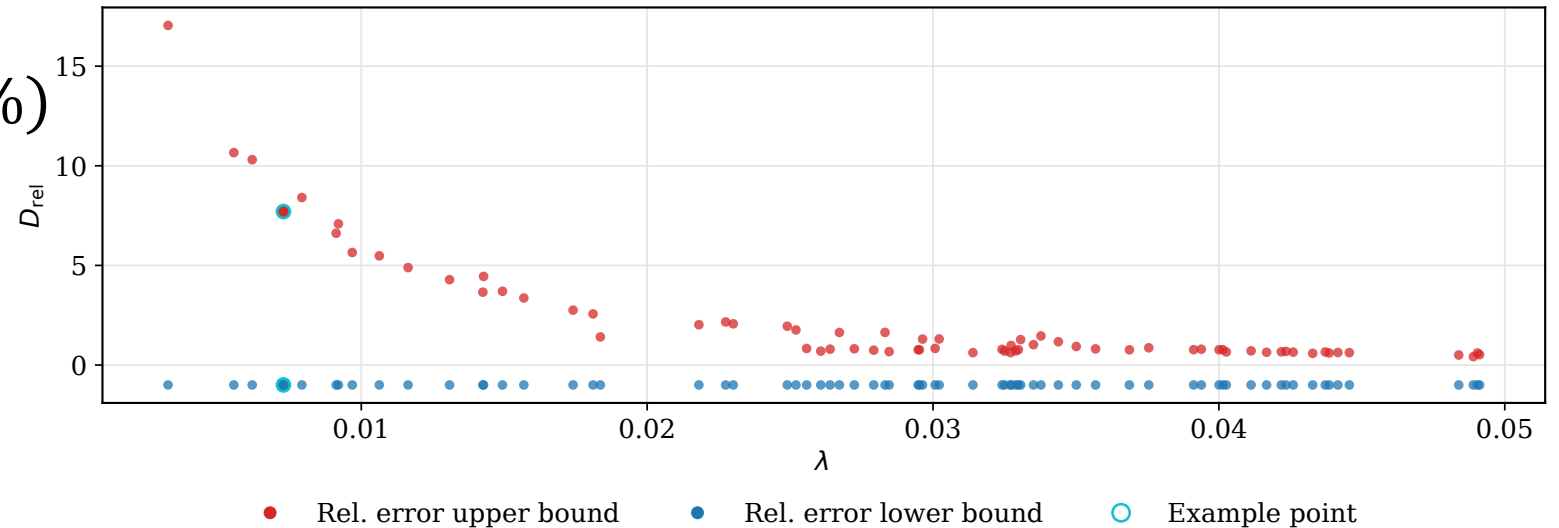
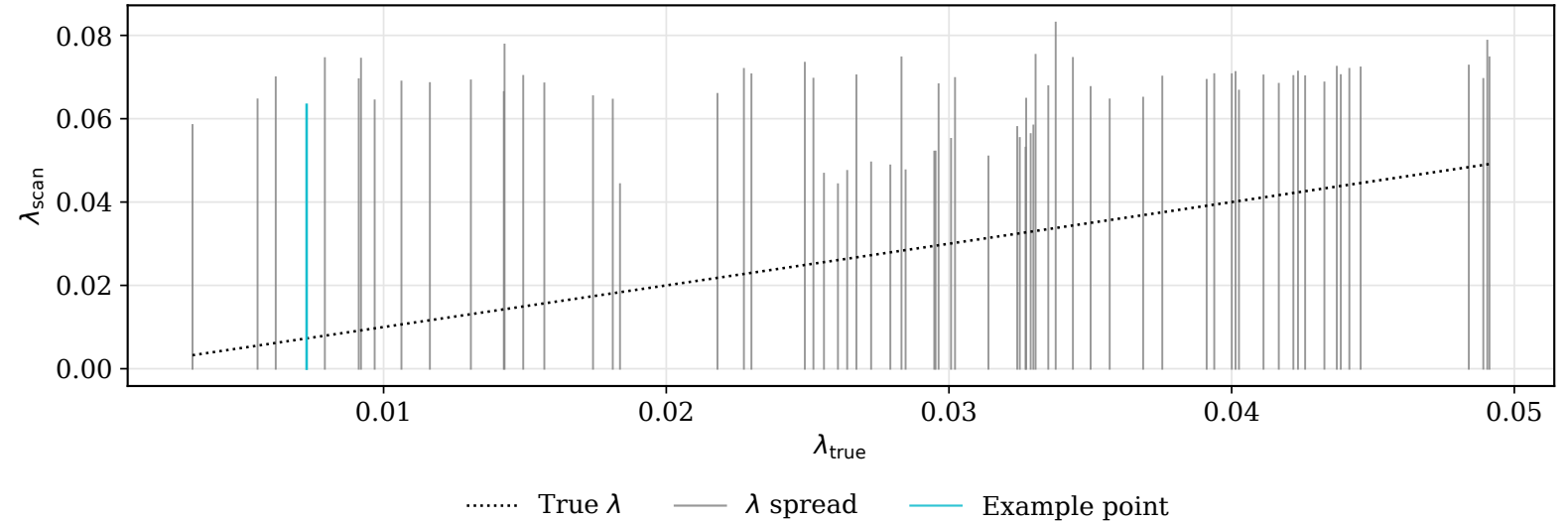
$$D_{rel} = \frac{\lambda_{scan} - \lambda_{true}}{\lambda_{true}}$$

Upper bound: 40 – 1700%

Lower bound: not found (~100%)

Similar results for  $\kappa$

→ determined by ratio  $\kappa/\lambda$ ,



# Results: $\kappa/\lambda$

Coupling ratio  $\kappa/\lambda > 0$

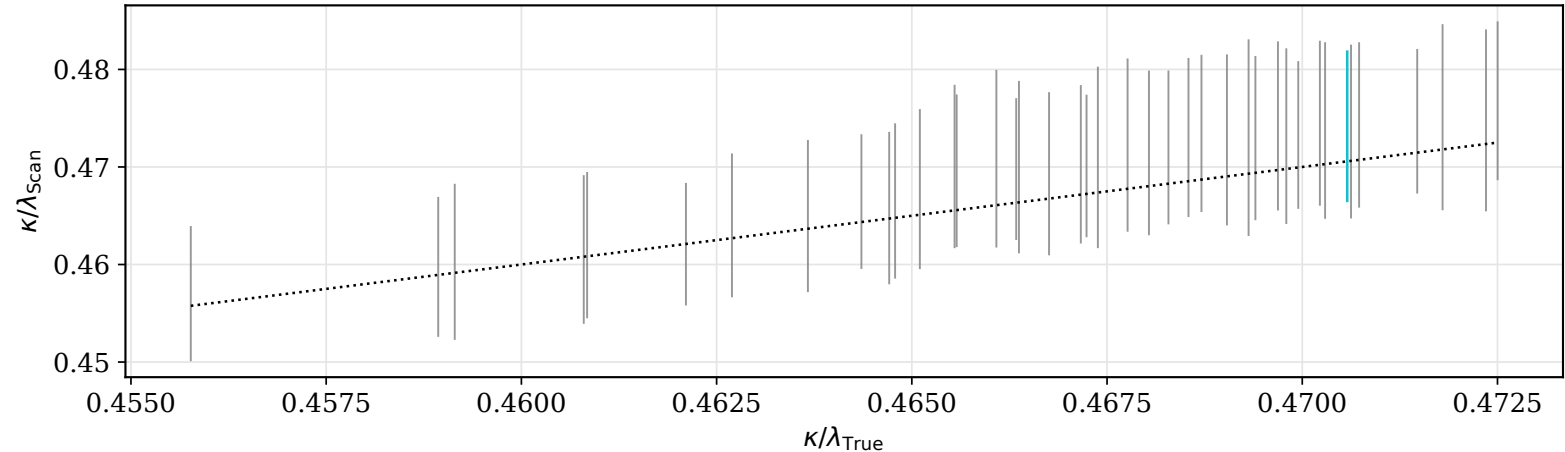
Relative error :

$$D_{rel} = \frac{\kappa/\lambda_{scan} - \kappa/\lambda_{true}}{\kappa/\lambda_{true}}$$

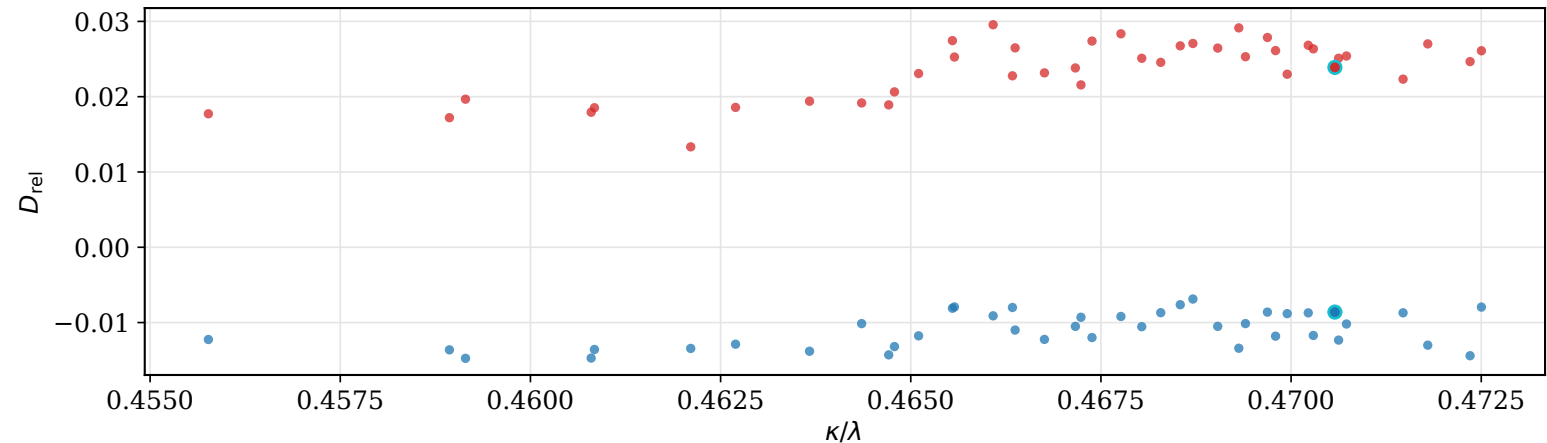
Upper bound:  $\sim 2.5\%$

Lower bound:  $\sim 1.0\%$

→ only little variation, precise reconstruction



..... True  $\kappa/\lambda$     —  $\kappa/\lambda$  spread    — Example point



• Rel. error upper bound    • Rel. error lower bound    ○ Example point

# Results: DM relic density $\Omega h^2$

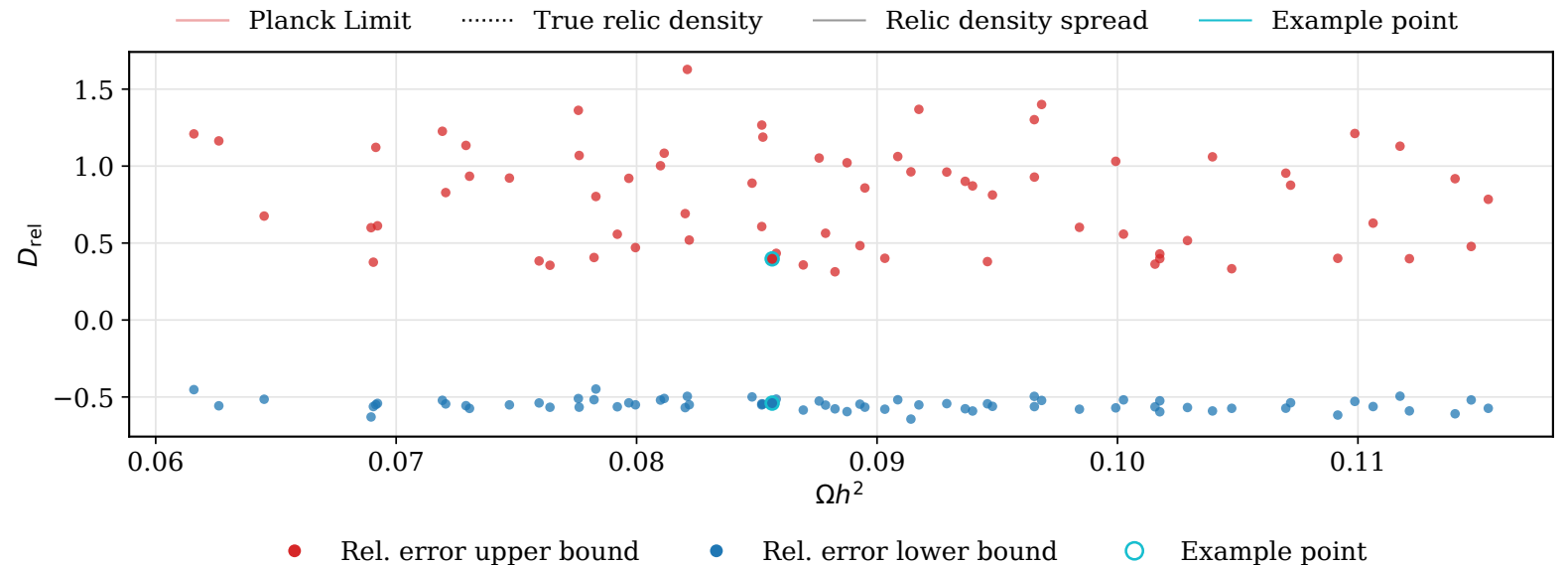
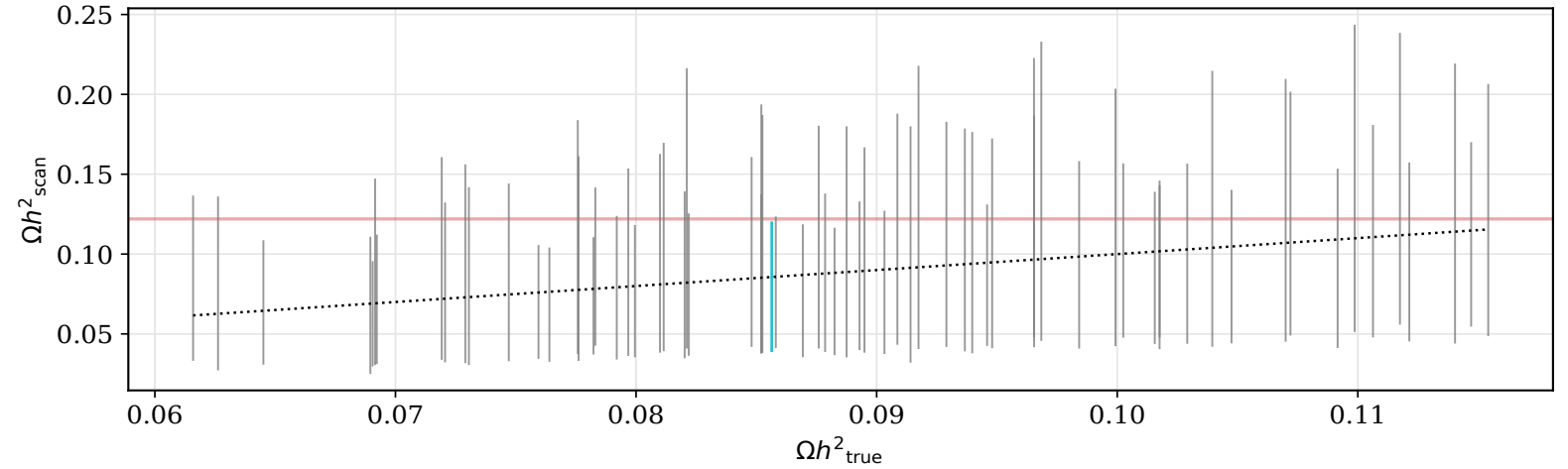
Inferred relic density

Relative error:

$$D_{rel} = \frac{\Omega h^2_{scan} - \Omega h^2_{true}}{\Omega h^2_{true}}$$

Upper bound: 30 – 160%

Lower bound: ~55%



# Conclusion & Outlook

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Reconstruction of NMSSM parameters and DM relic density for singlino/higgsino dominated scenario with  $e^+e^-$  measurements.

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Vastly varying reconstruction accuracy for parameters  
( $\mu$ :  $\sim 0.5\%$ ,  $\kappa$ : up to 1750%)

→ determined by parameter dependencies

Large errors in inferred relic density: 30 – 160%

→ high sensitivity on changes in higgsino/singlino parameters

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Further analyses:

- Improve precision through **additional observables**
- Include  $M_2$  as an independent parameter

# BACKUP

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# Results: $\mu$

Higgsino mass parameter

Relative error:

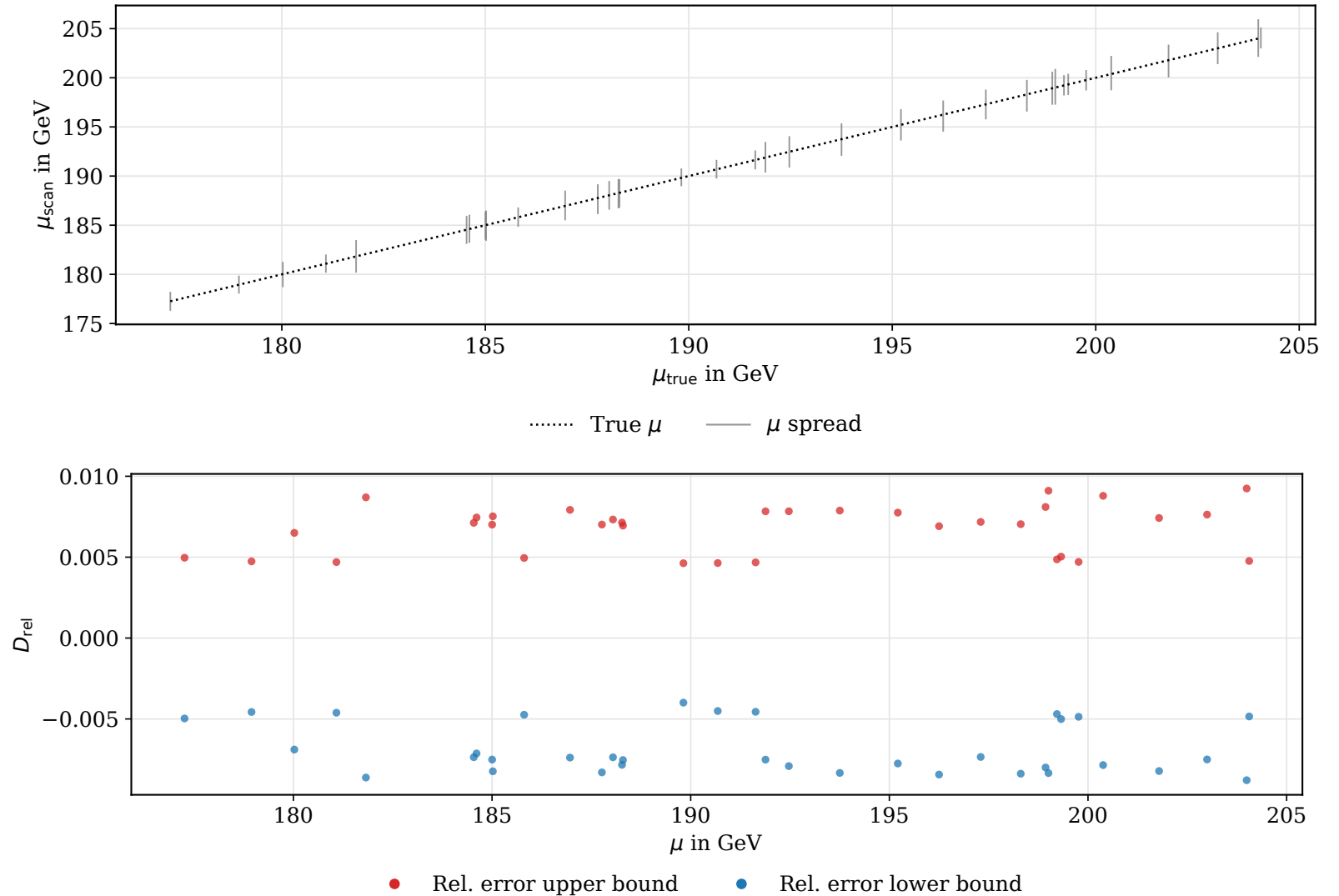
$$D_{rel} = \frac{\mu_{scan} - \mu_{true}}{\mu_{true}}$$

Upper bound: 0.45 – 0.95%

Lower bound: 0.40 – 0.90%

→ uniform accuracy

Here:  $\mu > 0$



# Results: $\kappa$

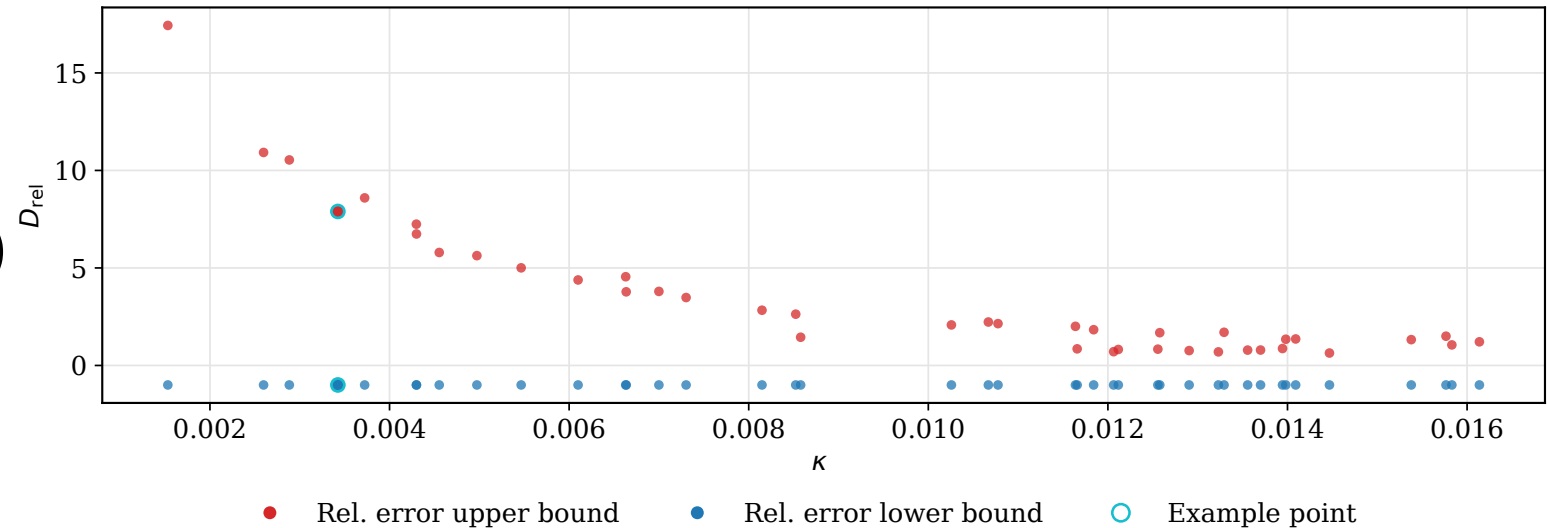
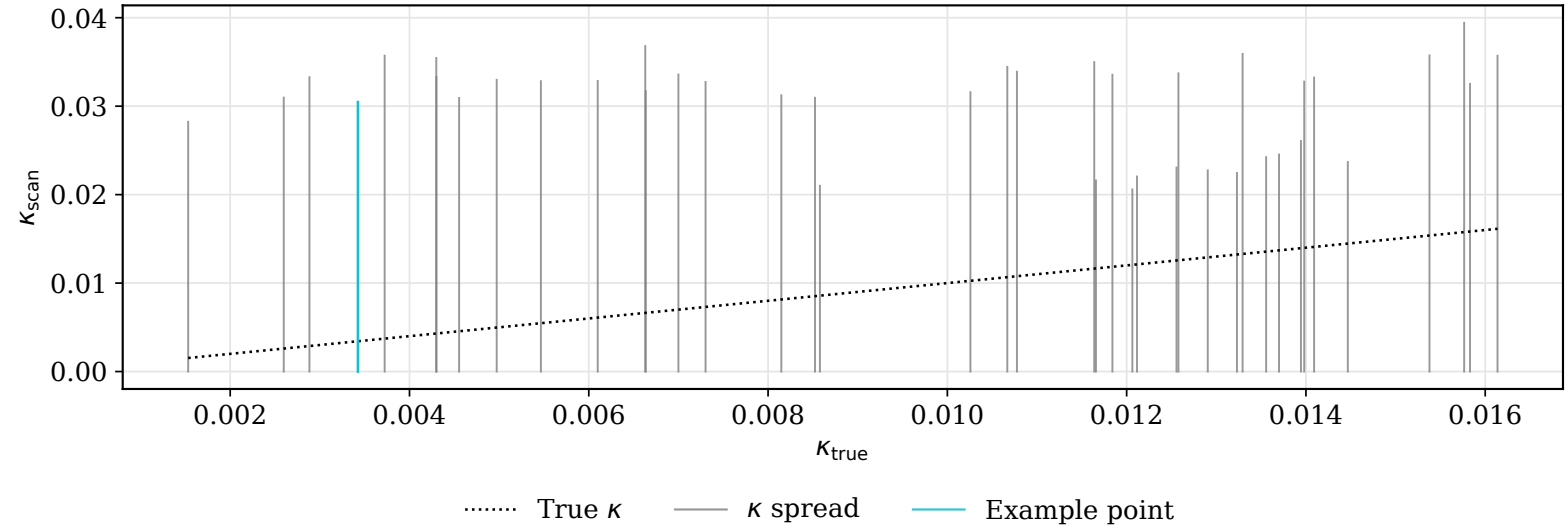
Coupling parameter  $\kappa > 0$

Relative error :

$$D_{rel} = \frac{\kappa_{scan} - \kappa_{true}}{\kappa_{true}}$$

Upper bound: 65 – 1750%

Lower bound: not found (~100%)



# Results: $\kappa$

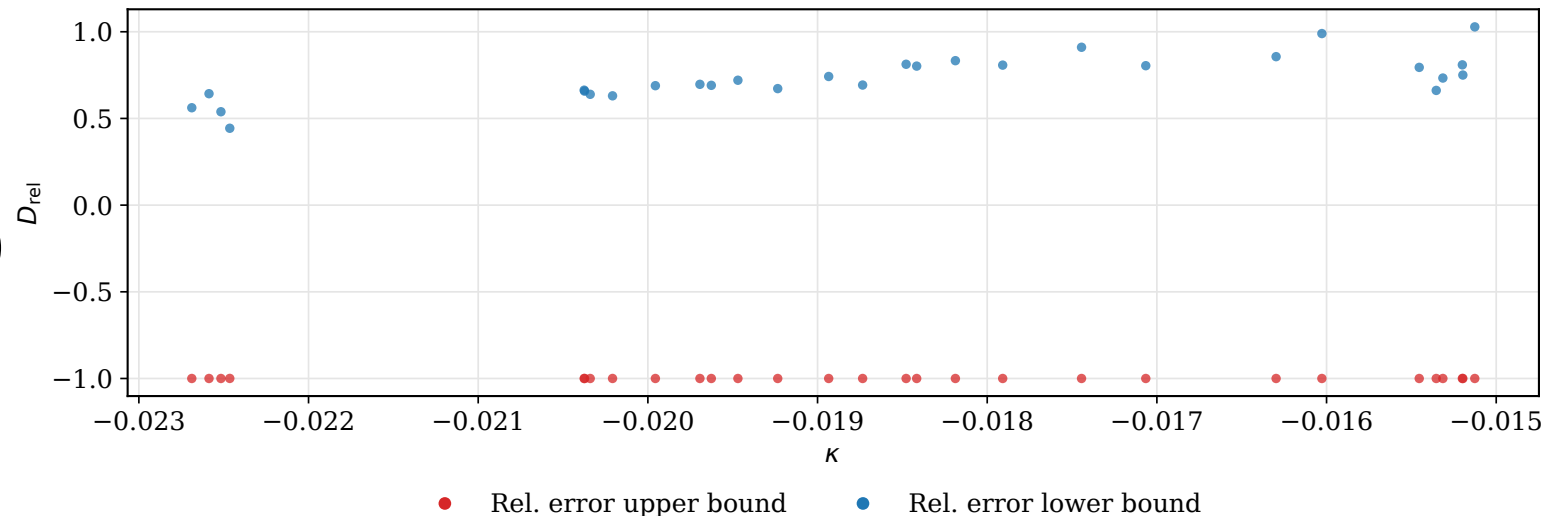
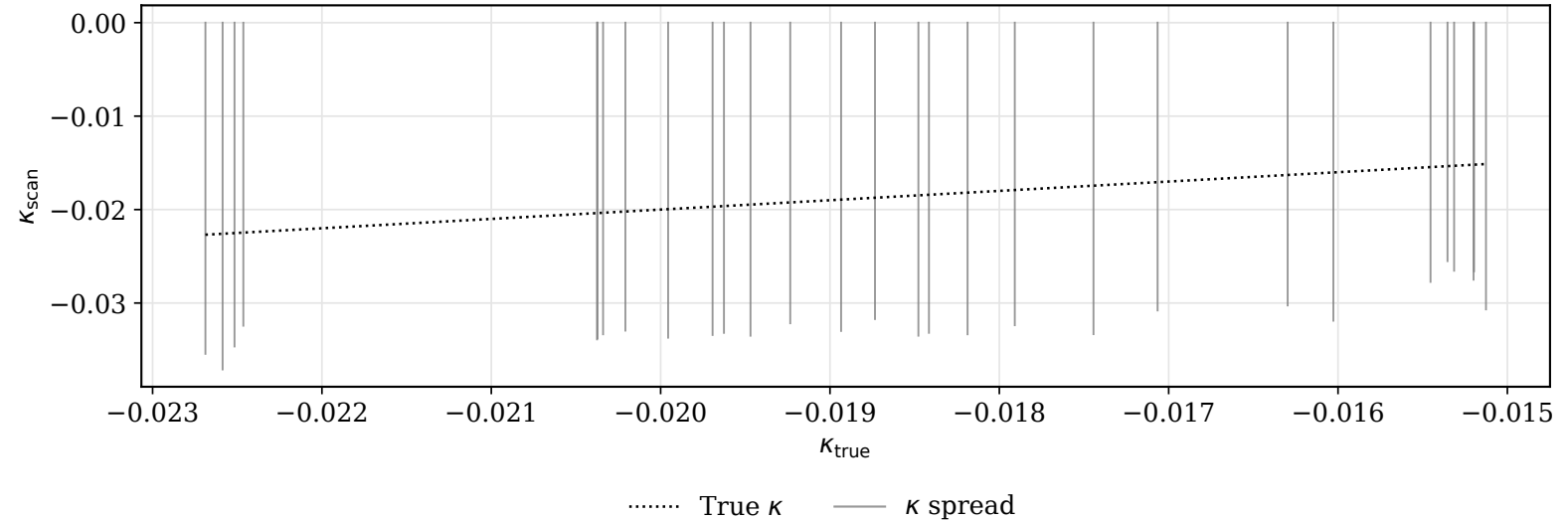
Coupling parameter  $\kappa < 0$

Relative error :

$$D_{rel} = \frac{\kappa_{scan} - \kappa_{true}}{\kappa_{true}}$$

Upper bound: 45 – 100%

Lower bound: not found (~100%)



# Higgsino/Singlino-like Neutralinos

The three lightest neutralino masses are governed by higgsino/singlino block of  $M_N$ :

$$M_{N,3 \times 3} = \begin{pmatrix} 0 & -\mu & -\lambda v_2 \\ -\mu & 0 & -\lambda v_1 \\ -\lambda v_2 & -\lambda v_1 & \frac{2\mu\kappa}{\lambda} \end{pmatrix}$$

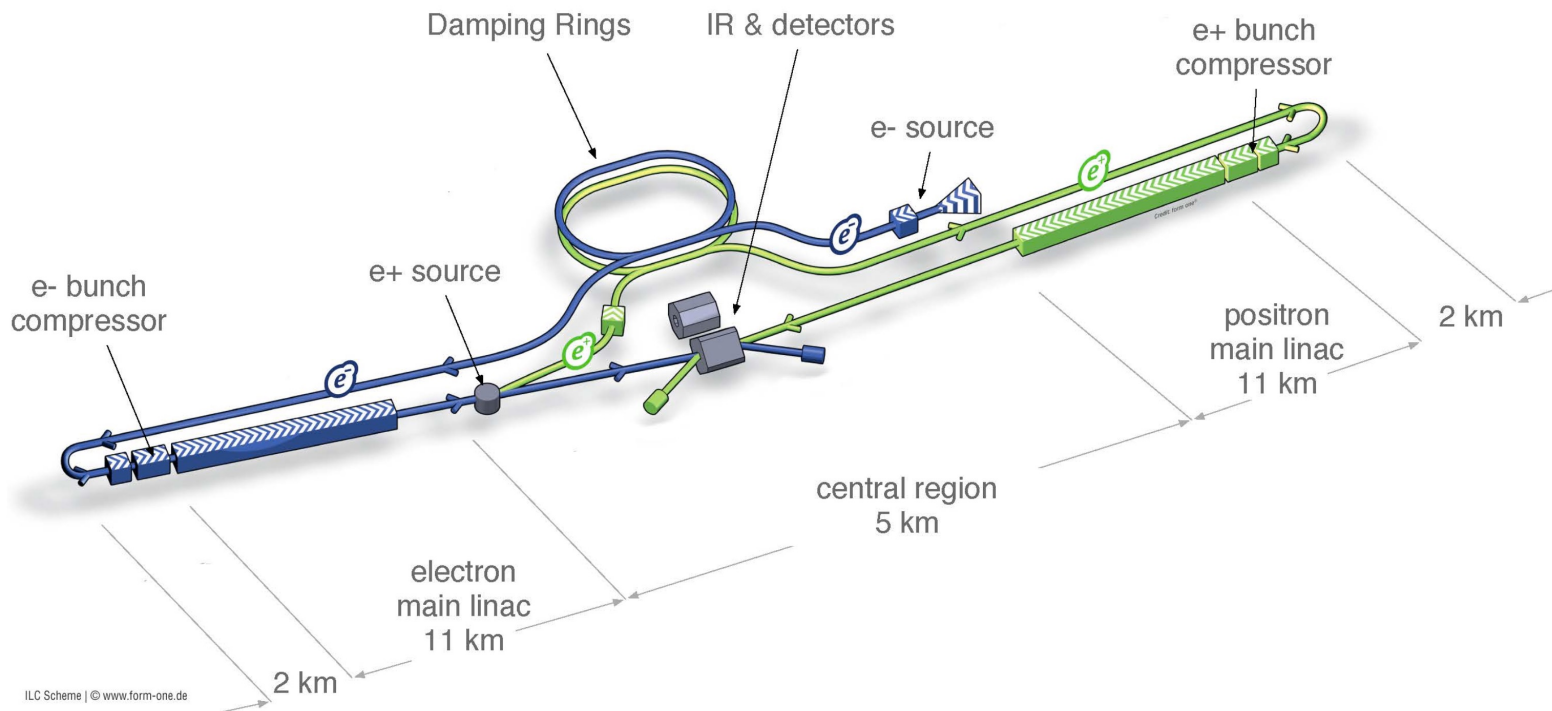
$$\frac{M_{N,3 \times 3}}{\mu} = \begin{pmatrix} 0 & -1 & -\frac{\lambda v_2}{\mu} \\ -1 & 0 & -\frac{\lambda v_1}{\mu} \\ -\frac{\lambda v_2}{\mu} & -\frac{\lambda v_1}{\mu} & \frac{2\kappa}{\lambda} \end{pmatrix}$$

- Overall mass scale:  $\mu$
  - Relative structure:  $\frac{\kappa}{\lambda}$
- only weak direct sensitivity on  $\lambda$  and  $\kappa$

# International Linear Collider

Future  $e^+e^-$  collider with spin polarised beams

- Main goals: measure Higgs + top masses, BSM searches
- $\sqrt{s} = 500 \text{ GeV}$  , possible upgrades up to 1 TeV
- Polarisation of 80% for  $e^-$  beam and 30 – 60% for  $e^+$  beam



[arXiv:2004.14628]

# Results: $\kappa/\lambda$

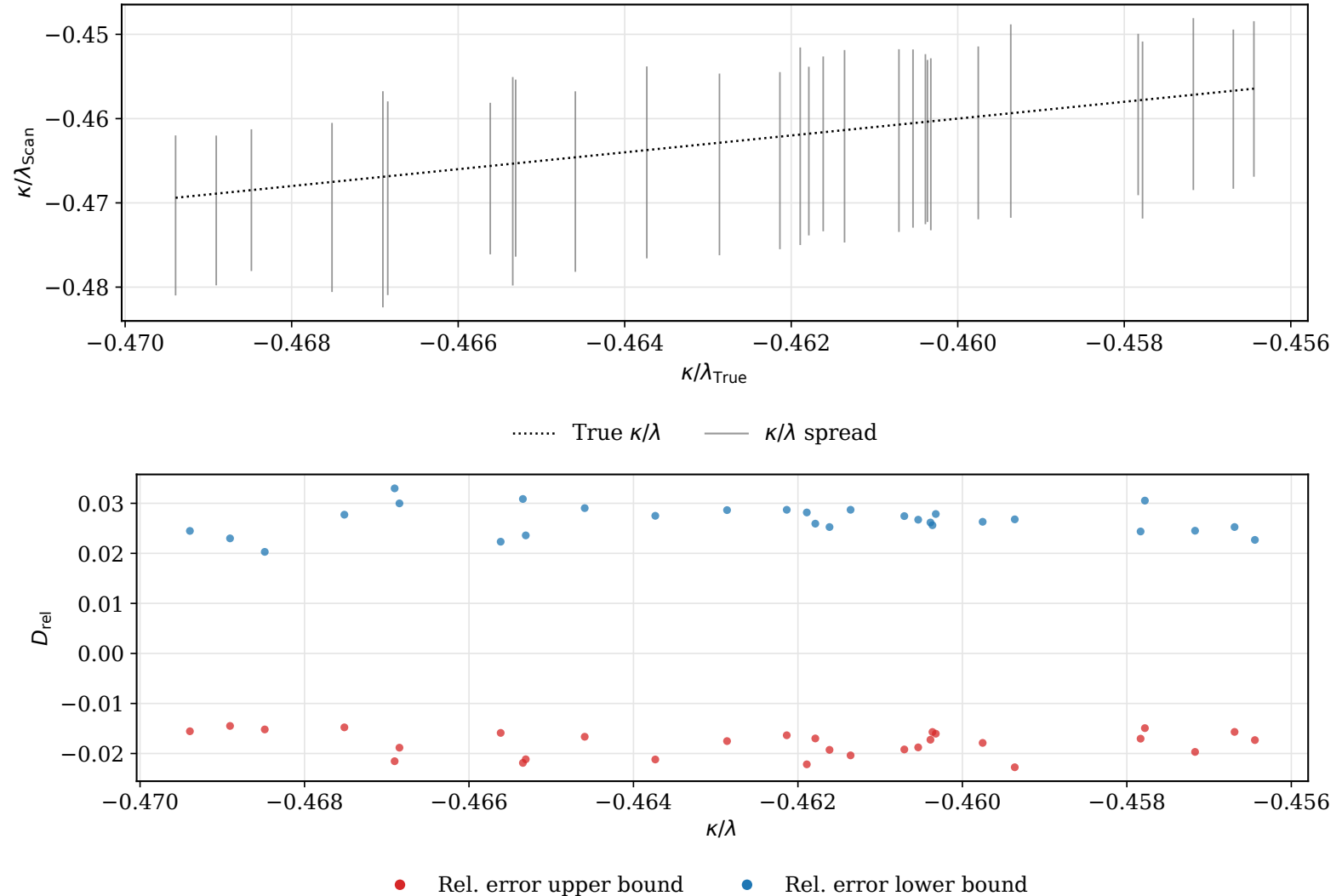
Coupling ratio  $\kappa/\lambda < 0$

Relative error :

$$D_{rel} = \frac{\kappa/\lambda_{scan} - \kappa/\lambda_{true}}{\kappa/\lambda_{true}}$$

Upper bound: 1.5 – 2.5%

Lower bound: 2 – 3.5%



# Determination of $M_1$

Chargino mass dependence on  $M_1$  for two example parameter points

