Domain Walls in the N2HDM

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The Next to 2 Higgs Doublet Model - N2HDM

Two Higgs doublets and one singlet:

$$\Phi_1 = \begin{pmatrix} \Phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix} \;\; ; \;\; \Phi_2 = \begin{pmatrix} \Phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix} \;\; ; \;\; \Phi_S = v_s + \rho_S$$

Symmetry structure:

$$\mathbb{Z}_2 : \; \Phi_1 \to \Phi_1 \; ; \; \Phi_2 \to -\Phi_2$$

$$\mathbb{Z}_2' : \Phi_1 \to \Phi_1, \; \Phi_2 \to \Phi_2, \; \Phi_S \to -\Phi_S$$

The Next to 2 Higgs Doublet Model - N2HDM

Potential of the N2HDM:

$$
\begin{aligned}
V_{N2HDM} =& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + h.c\right] \\
&+ \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 \\
&+ \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left[\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + h.c\right] \\
&+ \frac{1}{2}m_S^2\Phi_S^2 + \frac{\lambda_6}{8}\Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger\Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger\Phi_2)\Phi_S^2
\end{aligned}
$$

Domain Walls (DWs)

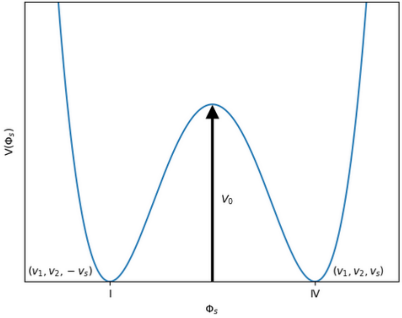


Figure: Simplified potential in terms of the singlet of the N2HDM.

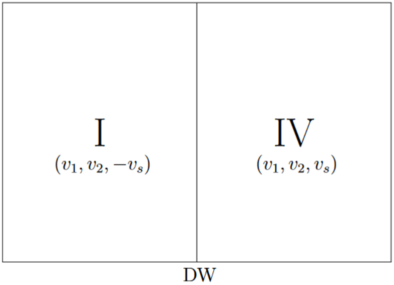


Figure: Sketch of different vacua seperated by a DW.

Domain Walls (DWs)

- Thickness of the DW: δ
- Tension of the DW:

$$\sigma = \int_{-\infty}^{\infty} dx E_{den}(x) \approx \delta V_0$$

- Tension force:

$$\rho_T \propto \frac{\sigma}{R} \propto \frac{\sigma}{t}$$

- Total energy of DWs without friction force (vacuum):

$$\rho_{DW}(t) = A\frac{\sigma}{t}$$

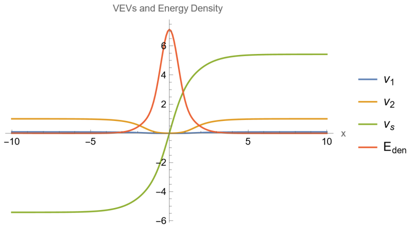


Figure: DW configuration between regions with (v_1, v_2, v_s) left and $(v_1, v_2, -v_s)$ right.

Domain Walls (DWs)

- Inside DW the VEVs of the doublets become nearly zero $\rightarrow$ restoration of EWSB
- interesting physics inside of DW.

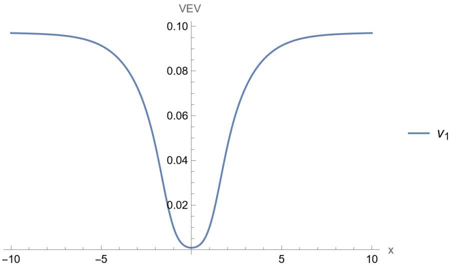


Figure: DW configuration for the doublet Φ_1 between regions with (v_1, v_2, v_s) left and $(v_1, v_2, -v_s)$ right.

The Domain Wall Problem

Total energy density of the universe without DWs:

$$\rho_c(t) = \frac{3M_{PL}^2}{4t^2}$$

$\rightarrow$ the energy density of the DWs will eventually dominate the universe.
Domination Time:

$$t_{dom} = \frac{3M_{PL}^2}{4A\sigma}$$

Possible solution: creating a bias in the potential.

DWs with $\mathbb{Z}_2$-symmetry breaking

- $\mathbb{Z}_2$-symmetry softly broken by m_{12}
- expectation: there will be domains with each of the four minima.
  $\rightarrow$ Two types of DWs
- Consider DW between area II and IV
  $\rightarrow$ DW configuration is equivalent to DW between I and IV

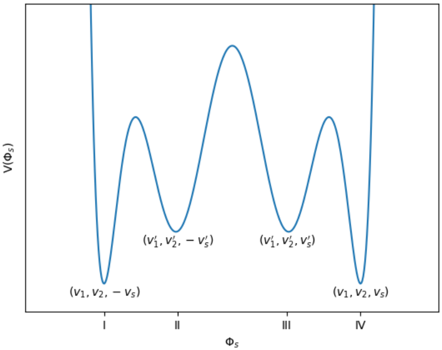


Figure: Sketch of a potential with both the physical as well as the modified minima after $\mathbb{Z}_2$-symmetry breaking.

DWs with $\mathbb{Z}_2$-symmetry breaking

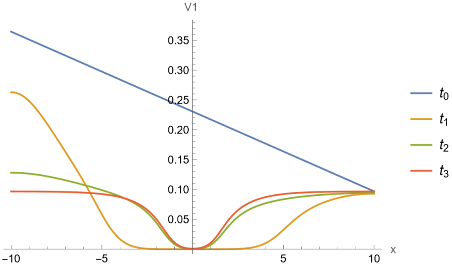


Figure: DW solution between areas II and IV using gradient flow method with different time steps.

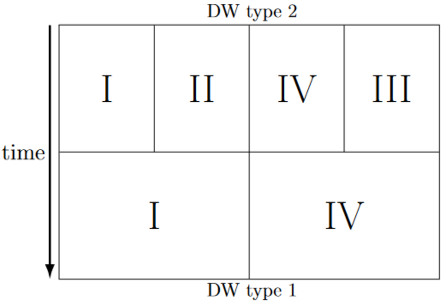
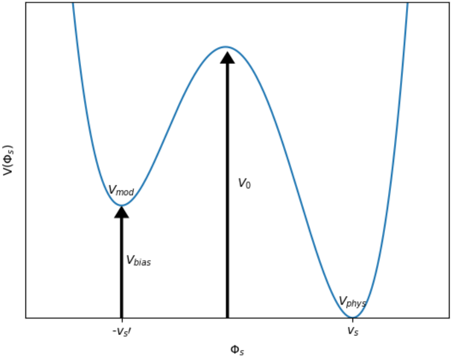


Figure: Sketch of different domains. After some time the physical vacua expand into the modified minima.

Breaking of the $\mathbb{Z}_2'$-symmetry

$$V_{N2HDM,broken} = V_{N2HDM} + \epsilon_{s3}\Phi_S^3 + \epsilon_{s1}\Phi_S + \left(\epsilon_{11}\Phi_1^\dagger\Phi_1 + \epsilon_{22}\Phi_2^\dagger\Phi_2 + \epsilon_{12}\Phi_1^\dagger\Phi_2 + \epsilon_{21}\Phi_2^\dagger\Phi_1\right)\Phi_S.$$



$$V_{bias} := V(v_s) - V(v_s') \neq 0,$$

Figure: sketch of the potential after $\mathbb{Z}_2'$-symmetry breaking.

Annihilation of the DWs

- Energy difference leads to a volume pressure force ρ_V acting on the DW.
- True minimum grows; false minimum shrinks.
- When the volume pressure force ρ_V becomes bigger than the tension force ρ_T, the DWs become unstable and collapse.

$$\rho_V = \rho_T \Rightarrow \; t_{ann} = C_{ann} \frac{A\sigma}{V_{bias}}.$$

$V_{N2HDM,broken} = V_{N2HDM} + \epsilon_{s3}\Phi_S^3$

Goal: Find which values of ϵ_{s3} allow the formation of DWs.

1. Condition for formation of DWs: $\frac{V_{bias}}{V_0} < 0.795 \quad \Rightarrow \epsilon_{s3} \geq -1.6$
2. Physical minimum must be the true minimum $\quad \Rightarrow \epsilon_{s3} < 0$

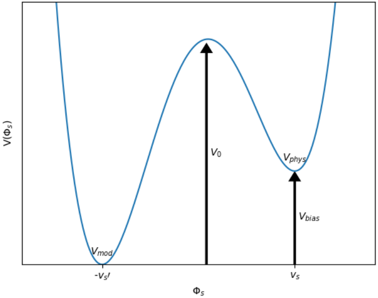
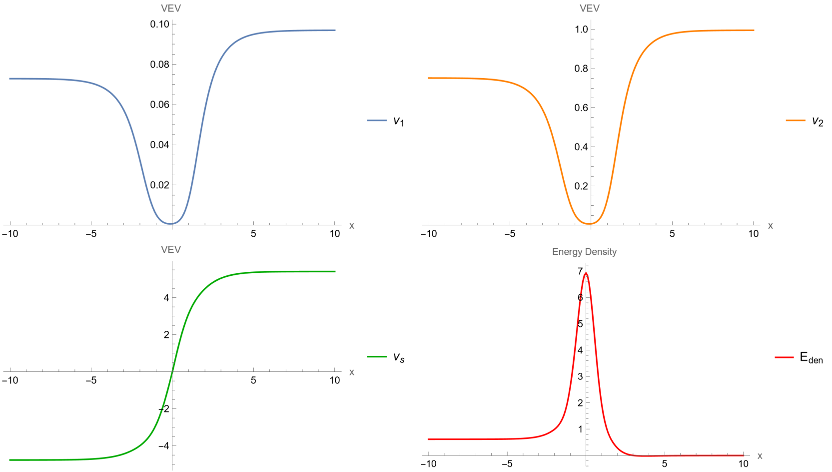


Figure: Sketch of the potential after $\mathbb{Z}_2'$-symmetry breaking, with $\epsilon_{s3} > 0$. With these minima, the non physical minimum would expand into the physical minimum.

DW Solution for $V_{N2HDM,broken} = V_{N2HDM} - 0.3\Phi_S^3$



Solving the DW problem

- $t_{ann} = \mathcal{O}(10^{-26})\ s$
  for $\epsilon_{s3} = \mathcal{O}(-1)$
- Domination time of the DW in the non broken case:
  $$t_{dom} \approx 38459\ s$$
- For which values of ϵ_{s3} is the DW problem no longer solved?

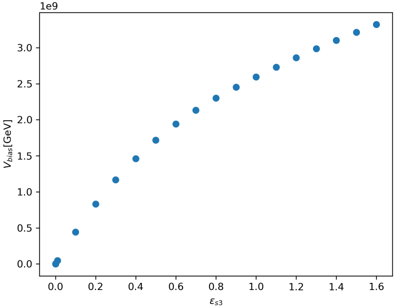


Figure: V_{bias} in terms of different ϵ_{s3}.

$$t_{ann} \propto \frac{1}{V_{bias}}$$

Solving the DW problem

- When $t_{ann} > t_{dom}$ the DW is no longer solved
- $t_{ann} = \mathcal{O}(10^{-20})$ s for $\epsilon_{s3} = \mathcal{O}(-10^{-6})$
- $t_{ann} \propto \frac{1}{\epsilon_{s3}}$ for small ϵ_{s3} $\rightarrow$ we expect problems at $\epsilon_{s3} \approx \mathcal{O}(-10^{-30})$

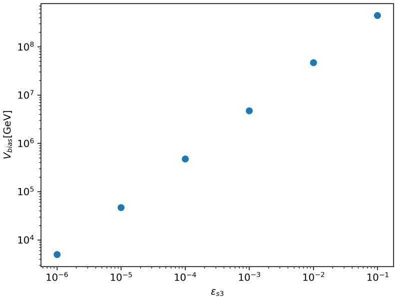


Figure: V_{bias} in terms of different ϵ_{s3}.

Summary

- We calculated the DW solution using gradient flow method
  $\rightarrow$ EWSB restoration inside DW
- having a term that softly breaks the $\mathbb{Z}_2$-symmetry leads to the same DW configuration as not having one.
- DW problem: after $t_{dom} = \mathcal{O}(10^4\ s)$ the energy density of DW dominates the universe.
- Solution to the DW problem: creating a bias in the potential by breaking the $\mathbb{Z}_2'$-symmetry
  $\rightarrow$ for $V_{N2HDM,broken} = V_{N2HDM} + \epsilon_{s3} \cdot \Phi_S^3$ with $-1.6 \geq \epsilon_{s3} < -10^{-6}$

Backup - $V_{N2HDM,broken} = V_{N2HDM} + \epsilon_{11}\Phi_1^\dagger\Phi_1\Phi_S$

- allowed values so that DWs can form: $-10000 \leq \epsilon_{11} < -10^{-2}$
- $t_{ann} = \mathcal{O}(10^{-20})$ for $\epsilon_{s1} = \mathcal{O}(-10^{-2})$
- $t_{ann} = \mathcal{O}(10^{-24})$ for $\epsilon_{11} = \mathcal{O}(-10^{5})$
- Domination time of the DW in the non broken case:

$$t_{dom} \approx 38459\ s$$

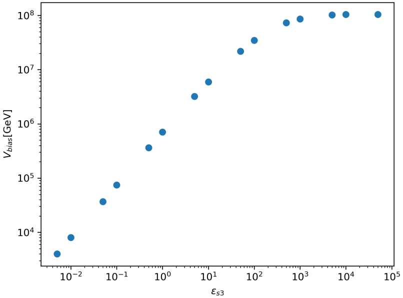


Figure: V_{bias} in terms of different ϵ_{11}.

$$t_{ann} \propto \frac{1}{V_{bias}}$$

Backup - Parameter Point

PP_1 :Variables for the fixed PP	
Mass of the SM Higgs :	$m_h = 125.09$ GeV
Mass of the Heavy Higgs :	$m_H = 713.24$ GeV
Mass of the singlet :	$m_S = 95.68$ GeV
Mass of the CP-odd Higgs :	$m_A = 811.20$ GeV
Mass of the charged Higgs :	$m_{H^\pm} = 677.38$ GeV
Mixing angle β :	$tan(\beta) = 10.26$
Standard model VEV :	v = 246.0 GeV
VEV of the singlet :	vs = 1333.47 GeV
Mixing angles α_i	$\alpha_1 = 1.57$; $\alpha_2 = 1.22$; $\alpha_3 = 1.49$
$\mathbb{Z}_2$-symmetry breaking mass:	$m_{12} = 221.12$ GeV

Table: The PP_1 used for the calculation of the DW solution.

Backup - Potential inside and outside of the DW

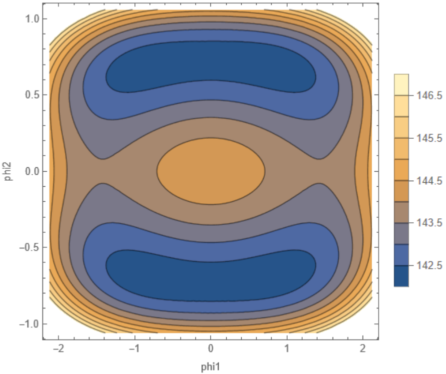


Figure: Plot of the potential outside of the DW. There are degenerate minima.

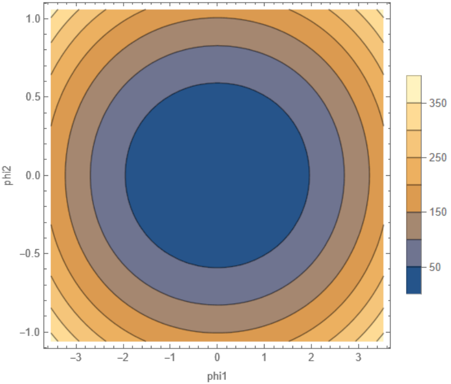


Figure: Plot of the Potential inside of the DW. The only minimum at $v_1 = v_2 = 0$, therefore there is no EWSB.

Backup - Formation of DWs with broken $\mathbb{Z}_2'$-symmetry

- probabilities p_+ and p_- that the scalar field takes a positive or negative VEV in a specific region of space
- Minimal value at which a large cluster of the non physical vacuum appears: $p_c = 0.311$

- formation of DWs suppressed for large bias

$$\frac{p_-}{p_+} \approx exp\left(-\frac{V_{bias}}{V_0}\right).$$

$$\Rightarrow \frac{V_{bias}}{V_0} < ln\left(\frac{1-p_c}{p_c}\right) = 0.795 .$$

Backup - Mass Matrix

- Minimization conditions:

$$\left.\frac{\partial V}{\partial \Phi_1}\right|_{\substack{\Phi_1=v_1\\ \phi_2=v_2\\ \Phi_S=v_S}} = \left.\frac{\partial V}{\partial \Phi_2}\right|_{\substack{\Phi_1=v_1\\ \phi_2=v_2\\ \Phi_S=v_S}} = \left.\frac{\partial V}{\partial \Phi_S}\right|_{\substack{\Phi_1=v_1\\ \phi_2=v_2\\ \Phi_S=v_S}} = 0,$$

- Mass matrix:

$$M_\rho^2 = \begin{pmatrix} \lambda_1 c_\beta^2 v^2 + t_\beta m_{12}^2 & \lambda_{345} c_\beta s_\beta v^2 - m_{12}^2 & \lambda_7 c_\beta v v_S \\ \lambda_{345} c_\beta s_\beta v^2 - m_{12}^2 & \lambda_2 c_\beta^2 v^2 + m_{12}^2/t_\beta & \lambda_8 s_\beta v v_S \\ \lambda_7 c_\beta v v_S & \lambda_8 s_\beta v v_S & \lambda_6 v_S^2 \end{pmatrix}.$$

- Rotation matrix:

$$R_{N2HDM} = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -(c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & -(c_{\alpha_1} s_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}.$$