Progress report: BFB conditions in 2hdSMASH

2HDM Working Group

Michael Matlis

Hamburg, 13.08.2020

Overview

1 The νDFSZ Axion model dubbed 2hdSMASH

2 BFB conditions: Requirement

3 BFB conditions: Sylvester's criterion

4 Necessary BFB conditions: 2HDM & σ

5 Necessary BFB conditions: Higgs-Portal

6 Sufficient BFB conditions: Higgs-Portal

7 Implications & Outlook for BFB conditions in 2hdSMASH

The νDFSZ Axion model dubbed 2hdSMASH

- 2hdSMASH: 2HD-SM-Axion-Seesaw-Higgs Portal Inflation
- Based on νDFSZ Axion model:
  2HDM Type II + Complex Scalar Singlet σ + ν_R under a global **$U(1)_{PQ}$-Symmetry**
- Scalar field transformation under $U(1)_{PQ}$:

$$\sigma \to \sigma \cdot \exp(i\alpha) \ , \ H_u \to H_u \cdot \exp(iX_u\alpha) \ , \ H_d \to H_d \cdot \exp(-iX_d\alpha) \tag{1}$$

- PQ-charges: $X_u + X_d = 2X_\sigma$

The νDFSZ Axion model dubbed 2hdSMASH

> PQ-invariant DFSZ scalar potential:

$$
\begin{aligned}
V_4(H_u, H_d, \sigma) = {} & \frac{1}{2}\lambda_1 \left|H_u\right|^4 + \frac{1}{2}\lambda_2 \left|H_d\right|^4 + \lambda_3 \left|H_u\right|^2 \left|H_d\right|^2 + \lambda_4 \left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right) \\
& + \frac{1}{2}\lambda_\sigma \left|\sigma\right|^4 + \lambda_{1\sigma} \left|H_u\right|^2 \left|\sigma\right|^2 + \lambda_{2\sigma} \left|H_d\right|^2 \left|\sigma\right|^2 - \epsilon \left(H_u^\dagger H_d \sigma^2 + H_d^\dagger H_u \sigma^{*2}\right)
\end{aligned}
\tag{2}
$$

> Parametrization:

$$
|H_u| = \frac{r \sin(\beta) \sin(\alpha)}{\sqrt{2}} \quad , \quad |H_d| = \frac{r \cos(\beta) \sin(\alpha)}{\sqrt{2}} \quad , \quad |\sigma| = \frac{r \cos(\alpha)}{\sqrt{2}} \tag{3}
$$

Bounded-from-Below conditions: Requirement

- BFB-Requirement: $V_4(H_u, H_d, \sigma) > 0$

- Let $x \equiv \cos^2(\beta)$ and $y \equiv \sin^2(\alpha)$, then $V_4(x, y) > 0$:

$$
\begin{aligned}
& y^2 \overbrace{\left(\lambda_1 (1-x)^2 + \lambda_2 x^2 + 2\lambda_{34}(1-x)x\right)}^{\equiv a} \\
& + 2y(1-y) \underbrace{\left(\lambda_{1s}(1-x) + \lambda_{2s} x - 2\epsilon\sqrt{(1-x)x}\right)}_{\equiv b} + \underbrace{\lambda_s}_{\equiv c} (1-y)^2 > 0
\end{aligned}
\tag{4}
$$

- Let $x =$ const., $y \equiv Y_1$ and $1 - y \equiv Y_2$:

$$
V_4(Y_1, Y_2)\big|_{x=\text{const.}} = a \cdot Y_1^2 + 2b \cdot Y_1 Y_2 + c \cdot Y_2^2 = \begin{pmatrix} Y_1 & Y_2 \end{pmatrix} \begin{pmatrix} c & b \\ b & a \end{pmatrix} \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}
\tag{5}
$$

Bounded-from-Below conditions: Sylvester's criterion

- **Sylvester's criterion**
  A symmetric matrix A is positive definite iff all principal minors are positive. If A is positive definite all special minors are positive.

- Let $A = \begin{pmatrix} c & b \\ b & a \end{pmatrix}$, then A is positive definite if:

$$\det(a) > 0 \ , \ \det(c) > 0 \ , \ \det \begin{pmatrix} c & b \\ b & a \end{pmatrix} > 0 \tag{6}$$

$$\Leftrightarrow \quad a > 0 \ , \ c > 0 \ , \ b > -\sqrt{a \cdot c} \tag{7}$$

Necessary Bounded-from-Below conditions: 2HDM & σ

> Necessary 2HDM BFB conditions:

$$a > 0 \quad \Rightarrow \quad \lambda_1(1-x)^2 + \lambda_2 x^2 + 2\lambda_{34}(1-x)x = \begin{pmatrix} x & 1-x \end{pmatrix} \begin{pmatrix} \lambda_2 & \lambda_{34} \\ \lambda_{34} & \lambda_1 \end{pmatrix} \begin{pmatrix} x \\ 1-x \end{pmatrix} > 0 \tag{8}$$

> Sylvester's criterion:

$$\lambda_{1,2} > 0 \;\; , \quad \lambda_{34} > -\sqrt{\lambda_1 \lambda_2} \tag{9}$$

> Necessary σ BFB condition:

$$c > 0 \quad \Rightarrow \quad \lambda_s > 0 \tag{10}$$

Necessary Bounded-from-Below conditions: Higgs-Portal

- Necessary Higgs-Portal BFB conditions:

$$b > -\sqrt{a \cdot c} \tag{11}$$

$$\Leftrightarrow \quad \lambda_{1s}(1-x) + \lambda_{2s}x - 2\sqrt{(1-x)x\epsilon} > -\sqrt{\lambda_s\left(\lambda_2 x^2 + \lambda_1(1-x)^2 + 2\lambda_{34}x(1-x)\right)}$$

- ϵ-term spoils linear independence

- Set $\epsilon = 0$:

$$b^2 - a \cdot c\big|_{\epsilon=0} = \begin{pmatrix} x & 1-x \end{pmatrix} \begin{pmatrix} \lambda_{2s}^2 - \lambda_2\lambda_s & \lambda_{1s}\lambda_{2s} - \lambda_{34}\lambda_s \\ \lambda_{1s}\lambda_{2s} - \lambda_{34}\lambda_s & \lambda_{1s}^2 - \lambda_1\lambda_s \end{pmatrix} \begin{pmatrix} x \\ 1-x \end{pmatrix} \tag{12}$$

- Sylvester's criterion:

$$\lambda_{1s,2s} + \sqrt{\lambda_{1,2}\lambda_s} > 0 \quad , \quad \lambda_s\lambda_{34} - \lambda_{1s}\lambda_{2s} + \sqrt{\left(\lambda_1\lambda_s - \lambda_{1s}^2\right)\left(\lambda_2\lambda_s - \lambda_{2s}^2\right)} > 0 \tag{13}$$

Sufficient Bounded-from-Below conditions: Higgs-Portal

- Sufficient Higgs-Portal BFB conditions:

$$b^2\big|_{\epsilon\neq 0} > a \cdot c \tag{14}$$

- Since $a, c > 0$ we get $b > 0$:

$$b(x, 1-x) = \begin{pmatrix} x & 1-x \end{pmatrix} \begin{pmatrix} \lambda_{2s} & \epsilon \\ \epsilon & \lambda_{1s} \end{pmatrix} \begin{pmatrix} x \\ 1-x \end{pmatrix} > 0 \tag{15}$$

- Sylvester's criterion:

$$\lambda_{1s} > 0 \tag{16}$$

$$\lambda_{2s} > 0 \tag{17}$$

$$\lambda_{1s}\lambda_{2s} - |\epsilon|^2 > 0 \tag{18}$$

Implications & Outlook for BFB conditions in 2hdSMASH

> Determine parameter space for viable inflation in…
- σ-direction $\left(\lambda_s \backsim 10^{-10}\right)$
- 2HDM-direction $\left(\lambda_{1,2,\text{mixed}} \backsim 10^{-10}\right)$
- Higgs-Portal direction $\left(\lambda_{\text{Higgs Portal}} \backsim 10^{-10}\right)$

> Determine particle masses for $[1503.02953v2]$ …
- SM + Axion (extreme decoupling $\backsim$ original DFSZ)
- Low energy(SM $+$ Axion) + High energy($H + H_\pm + A_0$)$|_{\backsim 100\text{ TeV}}$
- weak scale (SM + Axion + Heavy Higgs)

> work in progress …

Thank you!

Contact

DESY. Deutsches
Elektronen-Synchrotron

www.desy.de

Michael Matlis
Theory Group
michael.maxim.matlis@desy.de
+49–40–8998–4236