Investigating Detectors for High-Frequency Gravitational Waves based on SRF Cavities

From the MAGO cavity prototype

To a high-frequency detector exceeding the quantum limit

Lars Fischer, Hamburg 29.08.2024

Master Thesis Colloquium

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

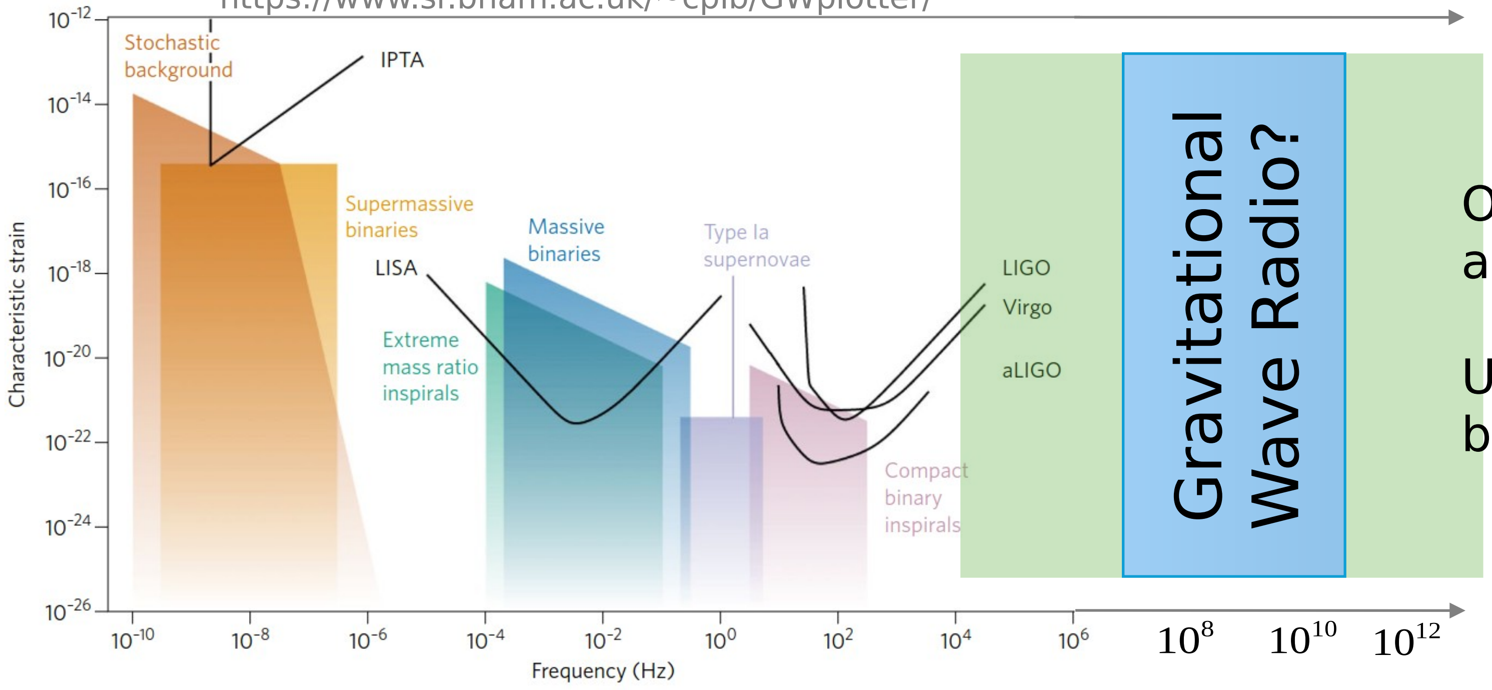
Outline

- Gravitational wave coupling mechanisms
- Sensitivity estimation of the MAGO detector
- A detector concept for high-frequency gravitational waves
- Quantum enhanced cavity searches
- Gravitational wave sources of new physics

Gravitational Waves

- Linearized general relativity: flat space + perturbation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Gravity must be spin-2: Radiation generates quadrupole moment

https://www.sr.bham.ac.uk/~cplb/GWplotter/

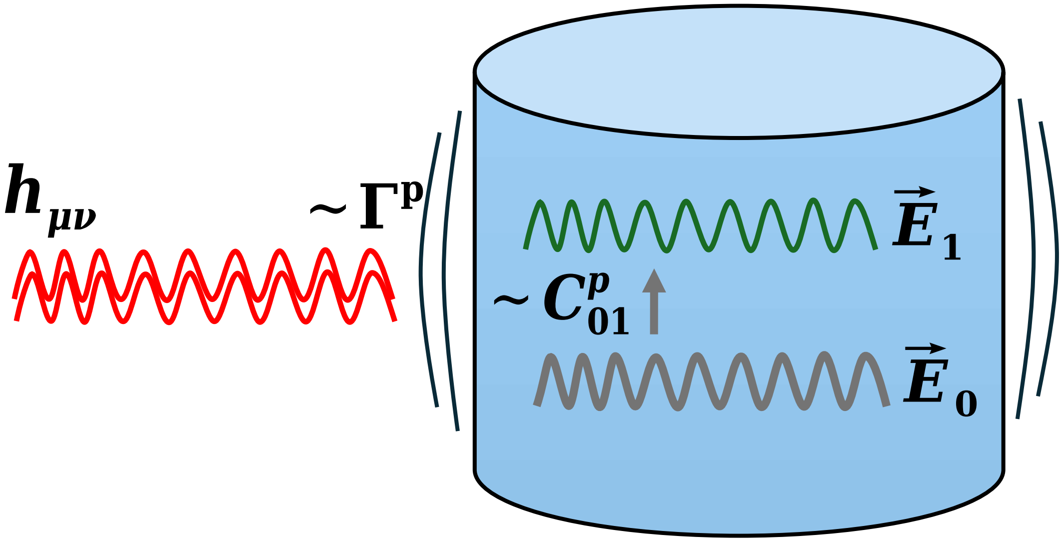


Only sources of new physics above kHz

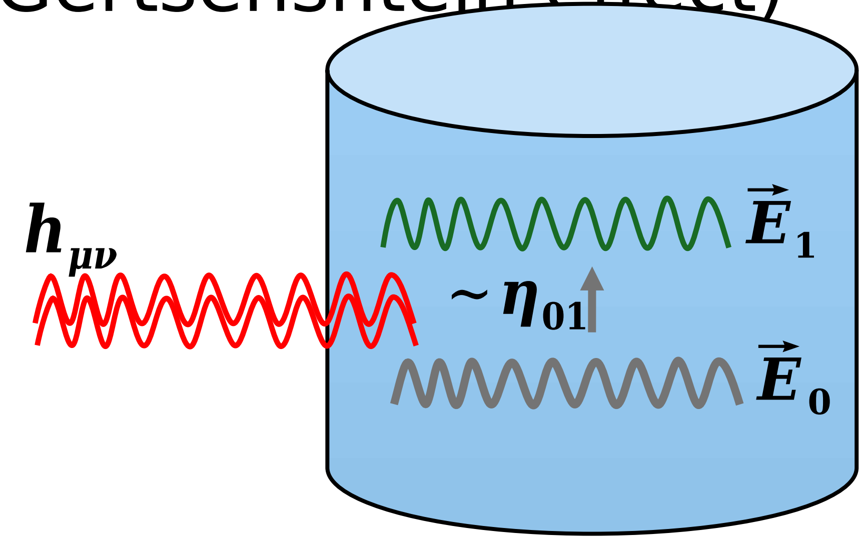
Unique experiments to probe beyond standard model physics

Gravitational Wave Coupling with EM Cavities

- Deforms the **mechanical** structure of the detector



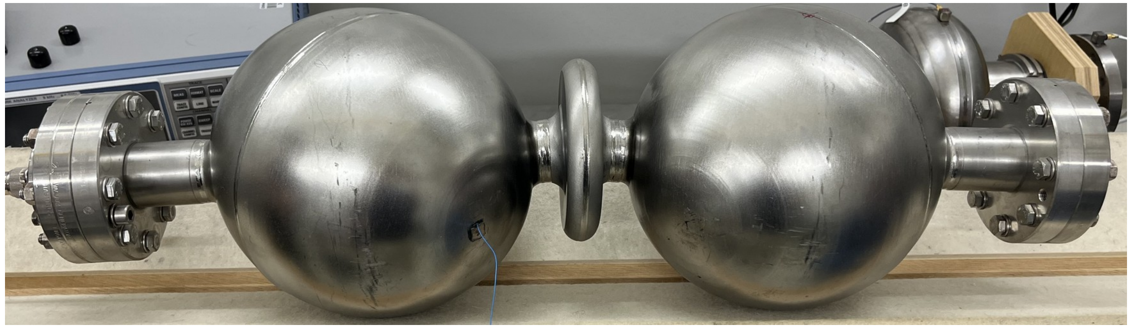
- Induces a current for an **electromagnetic** background field (Gertsenshtein effect)

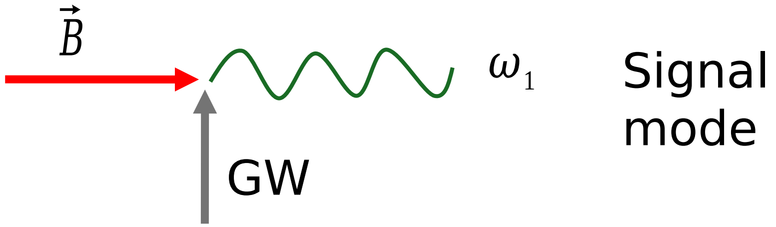


Gravitational Wave Cavity Experiments

- Superconducting cavities in a **heterodyne** set-up
- EM eigenmode loaded with EM energy
- MAGO prototype

- Normal conducting Cavities in **static magnetic** fields
- Empty EM mode and strong external B-field



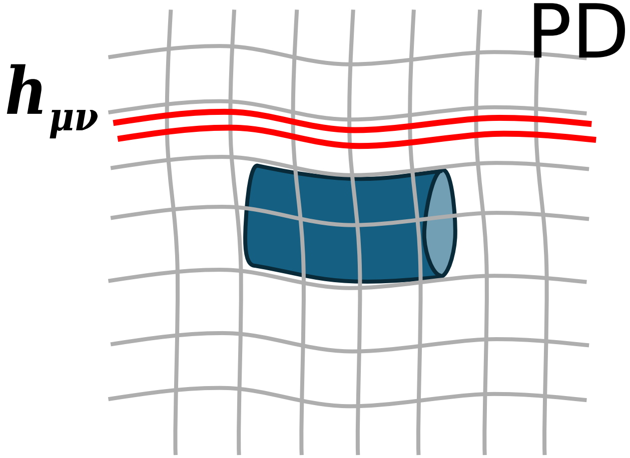


Coordinate Systems for Gravitational Waves

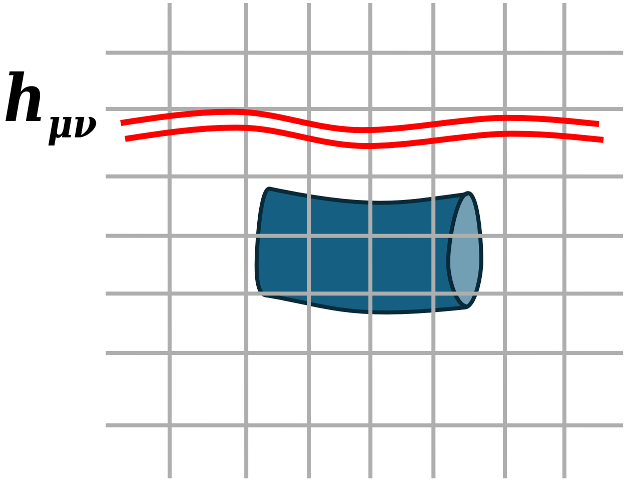
- **Transverse-traceless** (TT) frame GW take simple form
- **Proper detector** (PD) frame suited to describe laboratory

EM fields in TT

EM fields in PD



$$h^{PD} \sim \omega_g^2 L_{cav}^2 h^{TT}$$



For the direct conversion choose the PD frame

Mechanical Coupling Signal Strength

- Mechanical coupling on resonance

$$E_{sig}^{mech} \sim Q_1 L_{cav}^{-1} q_p E_0$$

- Mechanical displacement of cavity walls by force

$$q_p \sim f_p / M_{cav} \begin{cases} Q_p / (\omega \omega_p) & \text{for } \omega \simeq \omega_p \\ \omega^{-2} & \text{for } \omega \gg \omega_p \end{cases}$$

- For tidal force of a GW

$$E_{sig}^{mech} \sim Q_1 \omega_g^2 h^{TT} E_0 \left\{ Q_p / (\omega ¿¿ g \omega_p) ¿ \omega_g^{-2} \right. \quad \begin{matrix} \text{for } \omega_g \simeq \omega_p \\ \text{for } \omega_g \gg \omega_p \end{matrix}$$

Electromagnetic Coupling Signal Strength

- GW induced effective current

$$j_{eff}(x) \sim h^{PD} E_0 L_{cav}^{-1} \sim \omega_g^2 \omega_0^{-1} h^{TT} E_0$$

- Signal strength of direct conversion is

$$E_{sig}^{EM} \sim Q_1 E_0 \omega_g^2 L_{cav}^2 h^{TT}$$

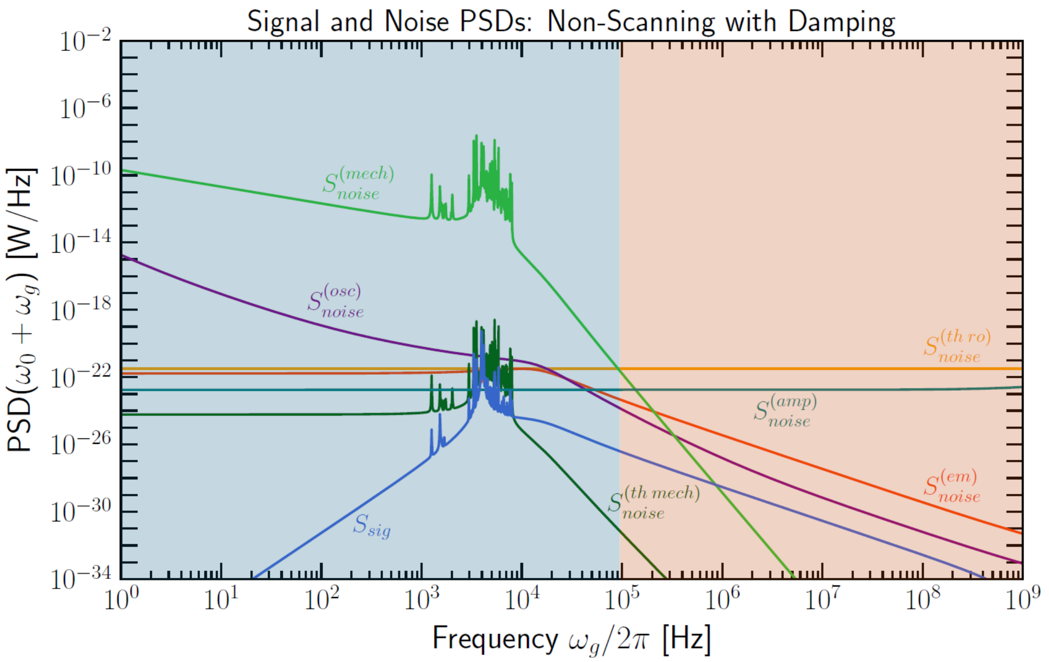
- Ratio of direct and indirect signal strength

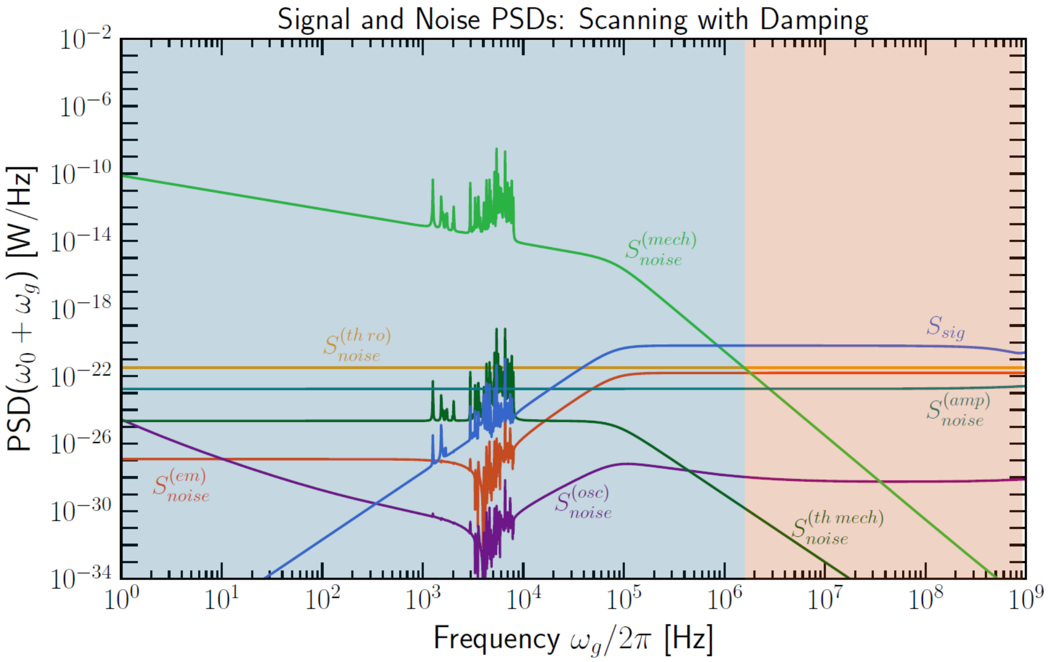
$$E_{sig}^{mech} / E_{sig}^{EM} \sim \begin{cases} Q_p c_s^{-2} & \text{for} \quad \omega_g \simeq \omega_p \\ \omega_0^2 / \omega_g^2 & \text{for} \quad \omega_g \gg \omega_p \end{cases}$$

- Indirect conversion enhanced on resonance ()

Noise Sources Power Spectral Densities (PSD)

- Including field back-action *Löwenberg, Moortgat-Pick 2307.14379*



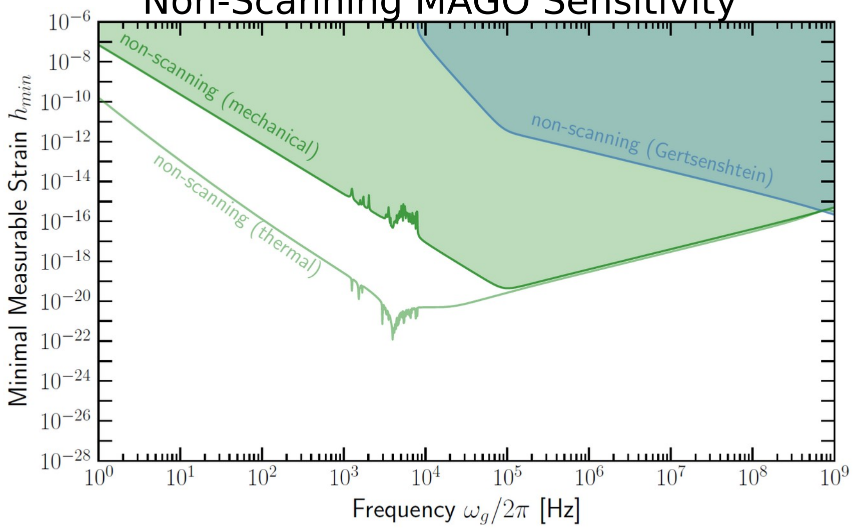


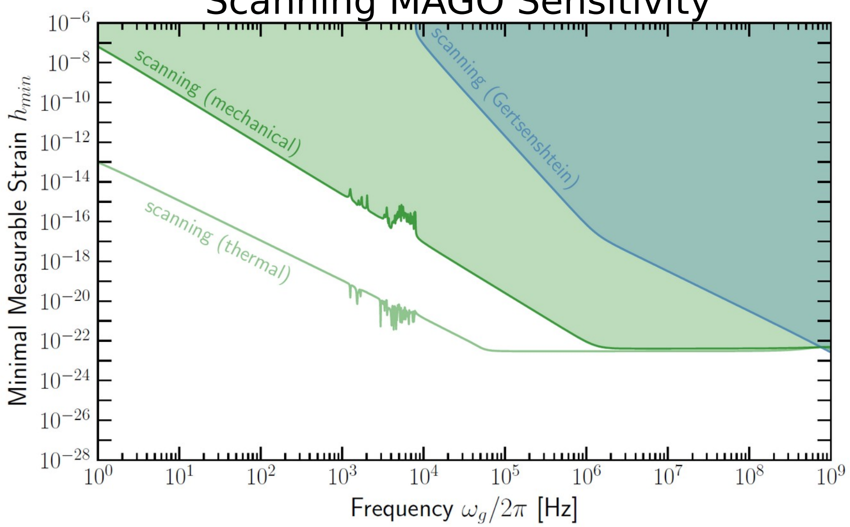
Mechanical Noise Limited Thermal Noise Limited

MAGO Detector Sensitivity

- Estimate minimal measurable strain by integrating PSDs in signal-to-noise ratio

$$h_{min} \simeq \left(\frac{t_{int}}{2\pi} \int d\omega \left(\frac{S^h_{sig}(\omega)}{S_{noise}(\omega)} \right)^2 \right)^{-1/4}$$





Overcoupling the signal mode, i.e. and

Critical coupling the signal mode, i.e. and

SRF Detectors for High-Frequencies

A regime of quantum limited noise

Direct Conversion Static B vs. Heterodyne

- Recall the signal strength for a heterodyne cavity

$$E_{sig}^{EM} \sim Q_1 E_0 \omega_g^2 L_{cav}^2 h^{TT}$$

- For direct conversion in a static B-field

$$E_{sig}^{B} \sim Q_n^B \omega_g^3 L_{cav}^3 h^{TT} B_0$$

- The signal strength ratio is

$$\frac{E_{sig}^{EM}}{E_{sig}^{B}} \sim \frac{Q_n}{Q_n^B} \frac{\omega_0}{\omega_g} \frac{E_0}{B_0} \sim \mathcal{O}(100)$$

- Static B-field dominate for

Static B-field cavities:

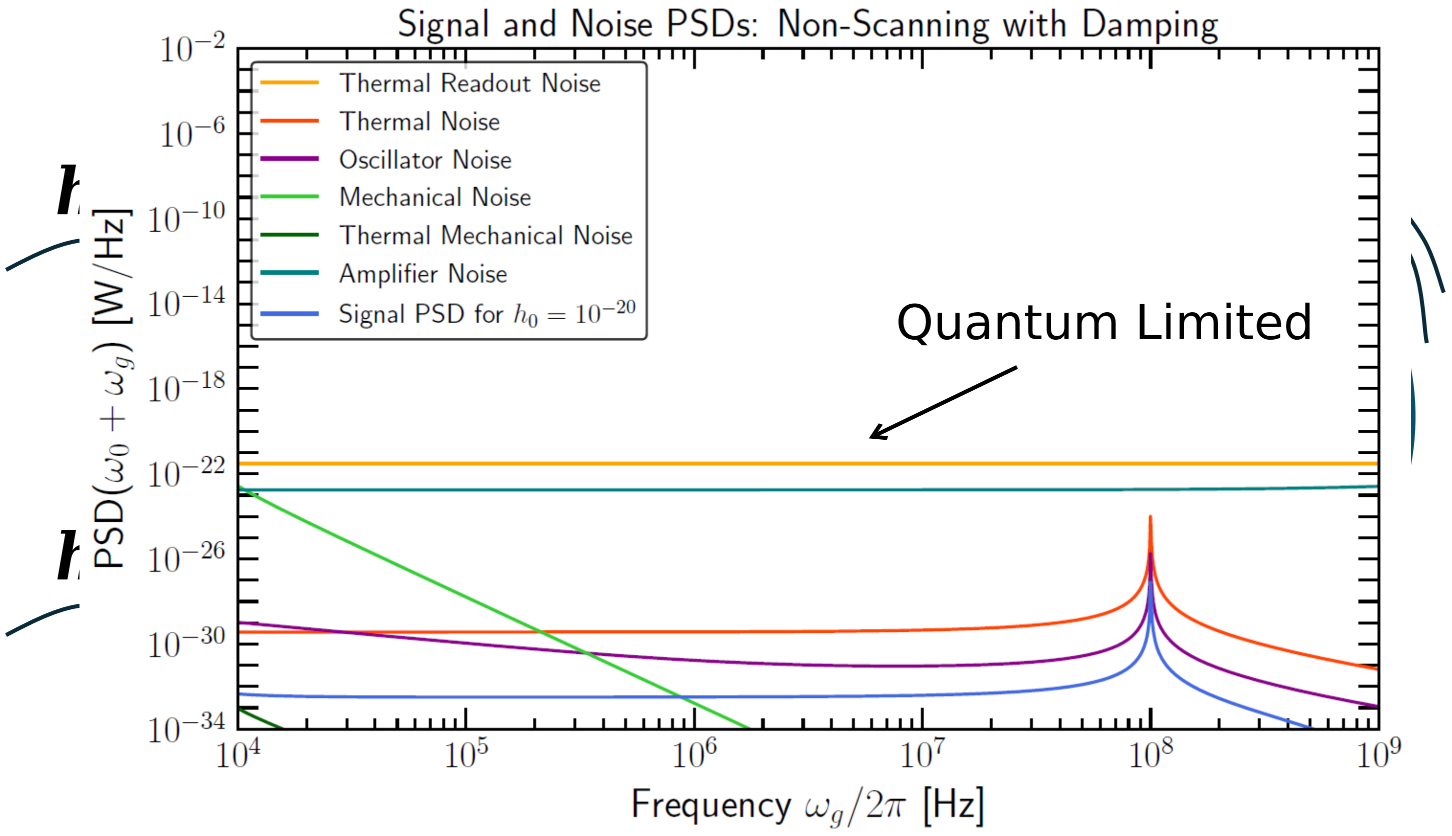
,

Heterodyne SRF cavities:

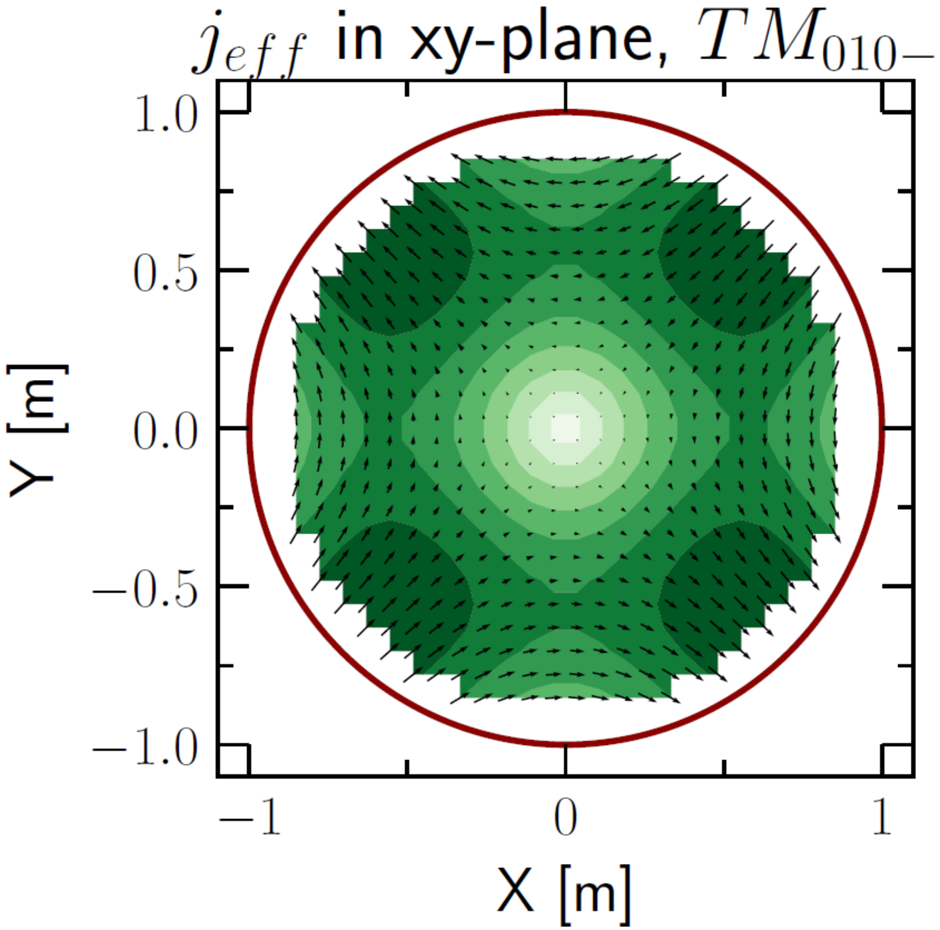
,

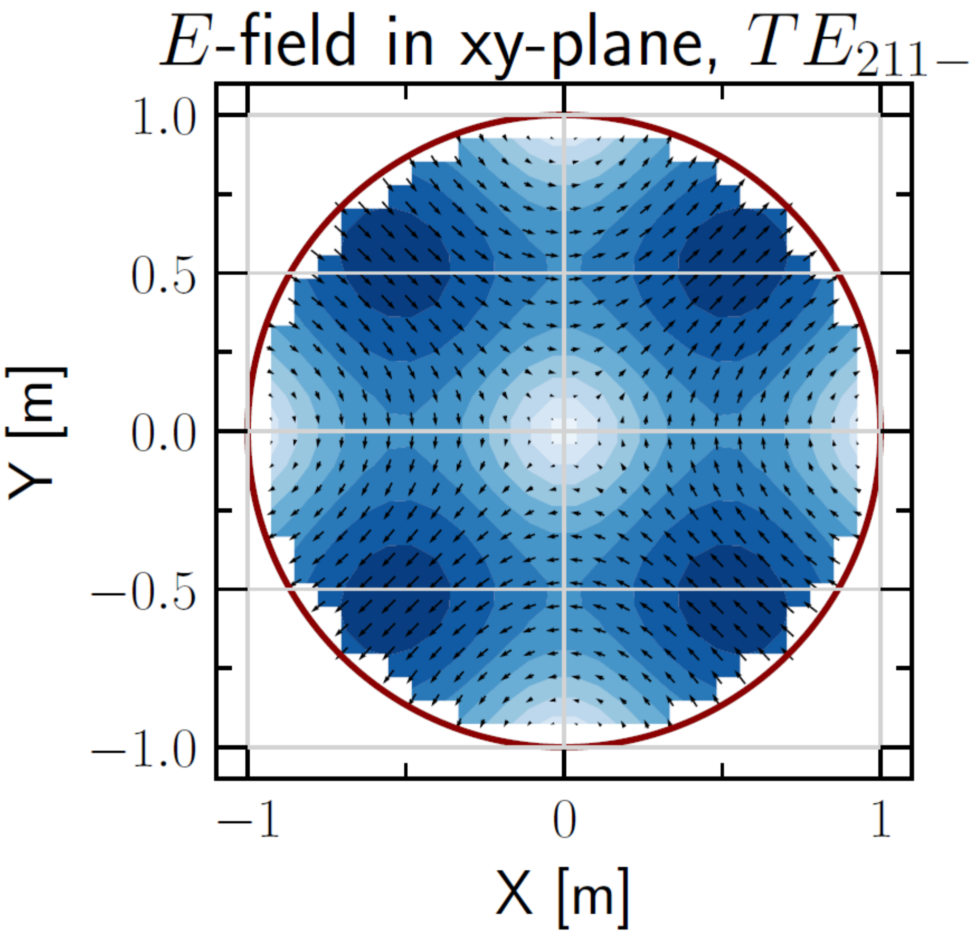
A Detector at the Quantum Limit

- A detector with GHz:



Detector for a HFGW Heterodyne Search





Peak Sensitivity

- Estimate sensitivity with signal-to-noise ratio

where is the signal bandwidth

- Heterodyne SRF cavity experiments

- Static B-field cavity experiments

Detector Signal

$$h_{00} = -\omega_g^2/c^2 x^k x^l h_{kl}^{TT} \cdot \mathcal{H}_0(\omega_g z/c),$$
$$h_{0i} = -\omega_g^2/c^2 \left[h_{ki}^{TT} x^k z - \delta_{iz} h_{kl}^{TT} x^k x^l\right] \cdot \mathcal{H}_1(\omega_g$$
$$h_{ij} = \omega_g^2/c^2 \left[\left(\delta_{iz} h_{kj}^{TT} + \delta_{jz} h_{ki}^{TT}\right) x^k z - h_{ij}^{TT} z^2$$

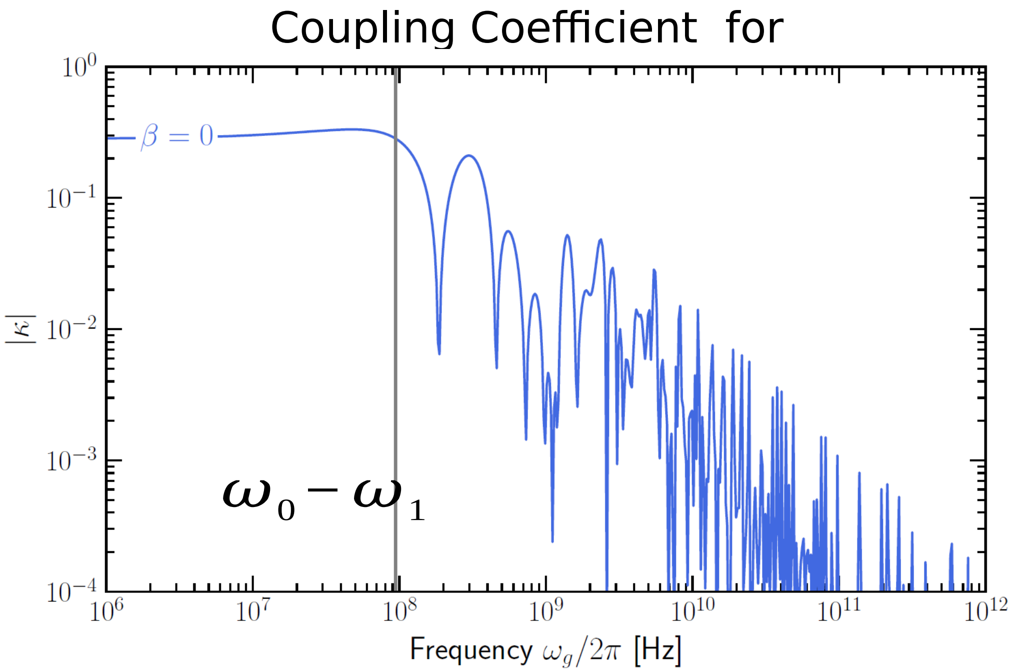
- Coupling coefficient

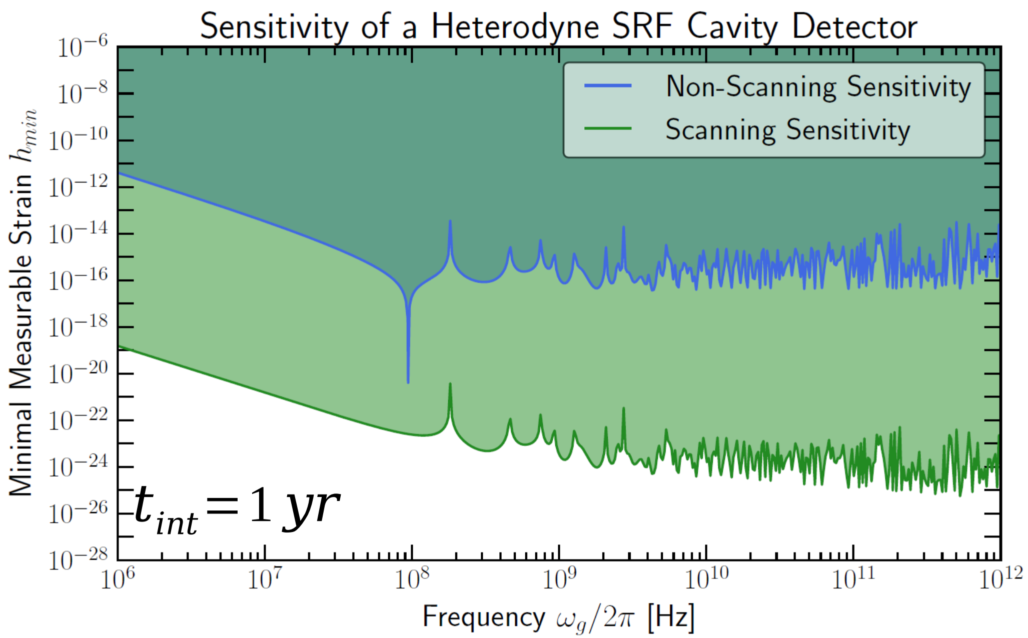
$$\kappa_{0n} = \frac{\omega_n^{-1}}{\sqrt{U_n U_0}} \int_{V_{cav}} d^3x \, \vec{E}_n^* \cdot \vec{j}_{eff}$$

- Signal power for periodic sources

Limited broadband sensitivity!

Overcoupling the signal mode for Non-scanning, i.e.

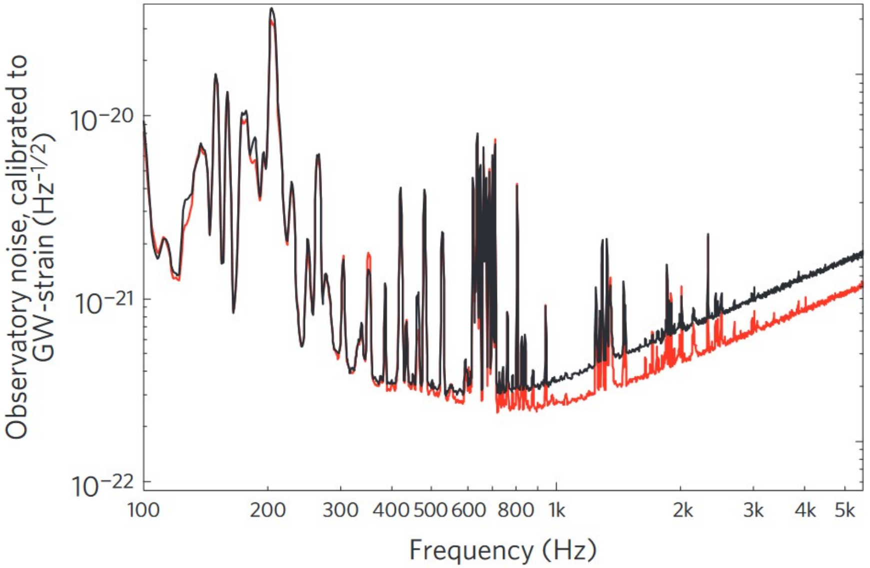




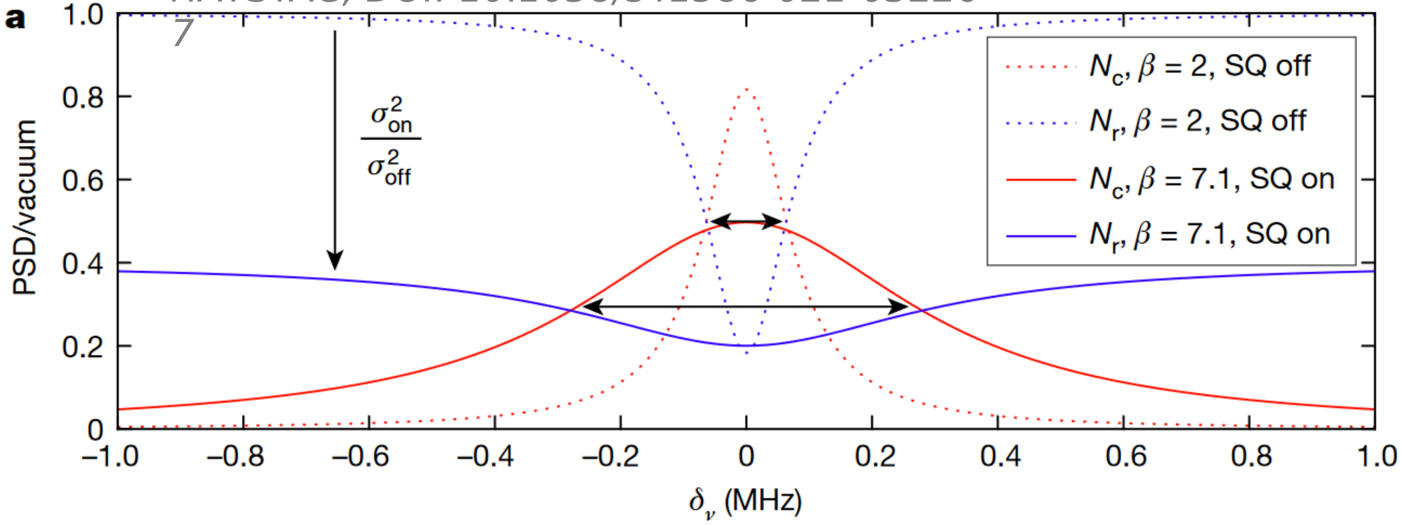
Quantum Enhanced Sea

- Squeezing increased sensitivity in GW interferometers
- Direct axion searches speed up scan rate with squeezed vacuum
- **Can squeezed states in a heterodyne cavity enhance the search for G**

GEO600, DOI: 10.1038/NPHYS2083



HAYSTAC, DOI: 10.1038/s41586-021-03226-7



Squeezed States of Light

- Until here: Classical field theory

$\vec{E}(t,x) = E_0 \; \hat{q} \; \sin kx$ $\hat{p}$ /laxwell in 1D)

$¿\,\mathscr{E}_0(\hat{a}+\hat{a}^\dagger)\sin kx$ $¿-i\mathscr{B}_0(\hat{a}-\hat{a}^\dagger)\cos kx$

Define quadrature operators: and

, for

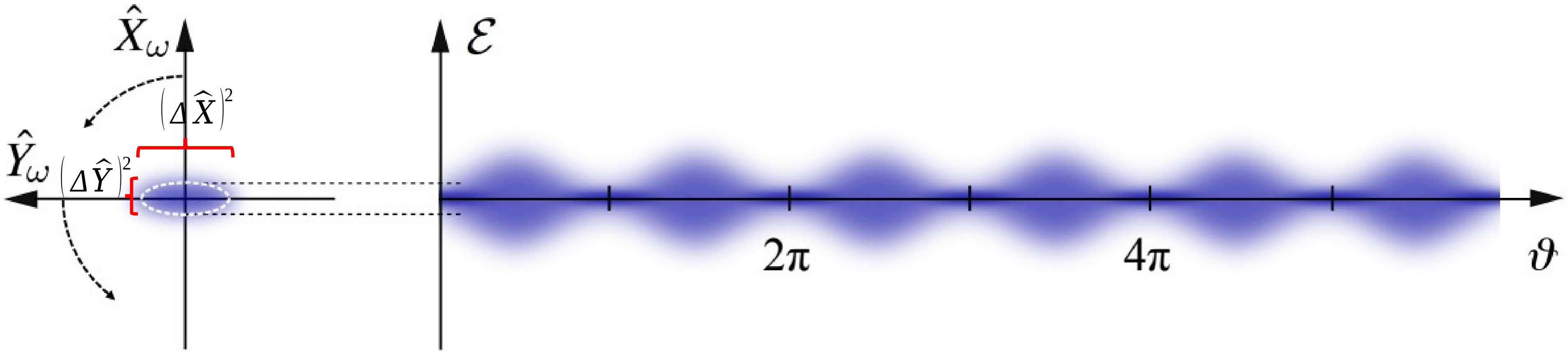
From follows the **uncertainty relation**

Squeezed light for or

Squeezed States of Light

R. Schnabel, arXiv:1611.03986

- Squeezed vacuum state



Phases space

Electric field oscillation, real space

A Cavity Model within Input-Output Theory

Based on: Malnou et al., PhsyRevX 9, 021023 (2019)

Clerk et al., arXiv:0810.4729

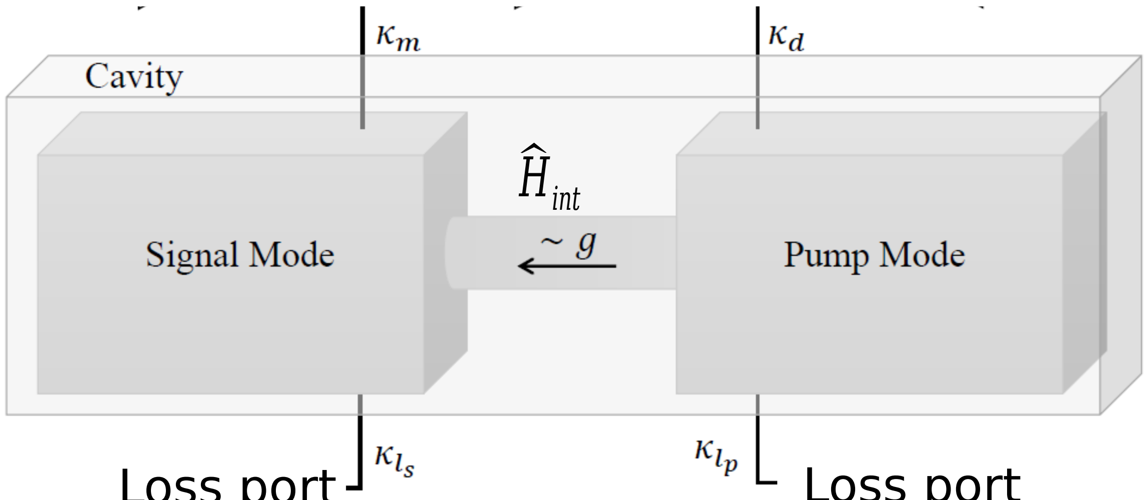
- Interaction of incoming bath modes with the cavity

$$\hat{a}_{out,x}(t)=\hat{a}_{in,x}(t)+\sqrt{\kappa_x}\,\hat{A}_{0,1}$$

- Model in it's own rotating frame

$$\hat{H}_{cav}=\hat{H}_1+\hat{H}_0+\hat{H}_{int}$$

$$\hat{H}_{int}=\hbar g\left(\hat{A}_0^\dagger\hat{A}_1+\hat{A}_0\hat{A}_1^\dagger\right)$$



- Interaction of bath an cavity in Heisenberg picture
- Assuming

$$\dot{\hat{A}}_0=-ig\hat{A}_1-\frac{\kappa_d+\kappa_l}{2}\hat{A}_0+\sqrt{\kappa_d}\,\hat{a}_{in,d}+\sqrt{\kappa_{l_p}}\,\hat{a}_{in,l}$$

$$\dot{\hat{A}}_1=-ig\hat{A}_0-\frac{\kappa_m+\kappa_l}{2}\hat{A}_1+\sqrt{\kappa_m}\,\hat{a}_{in,m}+\sqrt{\kappa_{l_s}}\,\hat{a}_{in,l}$$

Solve for in Fourier space

A Cavity Model within Input-Output Theory

- Interaction of all incoming with the cavity yields the outgoing of the signal mode

$$\hat{a}_{out,s}(\omega)=\sum_n \chi^0_{ns}(\omega)\hat{a}_{in,n}(\omega)+\sum_j \chi^1_{js}(\omega)\hat{a}_{in,j}(\omega)$$

- Where the (2x2) sucsceptibility matrices are

$$\chi^0_{ns}(\omega)=\frac{ig\sqrt{\kappa_n\kappa_s}}{Z_1(\omega)Z_0(\omega)+g^2},\quad \chi^1_{js}(\omega)=\frac{-\sqrt{\kappa_j\kappa_s}Z_0(\omega)+\delta_{js}\left(Z_1(\omega)Z_0(\omega)+g^2\right)}{Z_1(\omega)Z_0(\omega)+g^2}$$

Mode „impedances“:
$$Z_1(\omega):=i\omega+(\kappa_l+\kappa_m)/2$$
$$Z_0(\omega):=i\omega+(\kappa_l+\kappa_d)/2$$

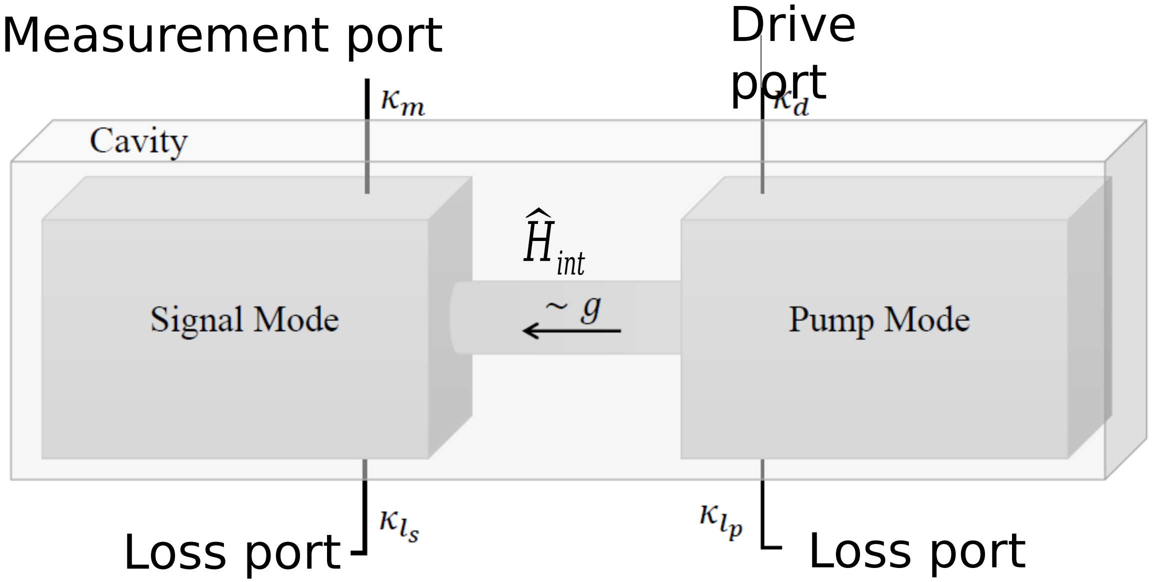
A Cavity Model in Input-Output Theory

- Follow the quadrature through individual elements

$$\vec{x}_{in,0}=\begin{pmatrix}\hat{X}_{in,d}\\ \hat{X}_{in,l}\end{pmatrix} \qquad \vec{x}_{in,1}=\begin{pmatrix}\hat{X}_{in,m}\\ \hat{X}_{in,l}\end{pmatrix}$$

- Projector for

$$\begin{pmatrix}\vec{x}\\ \vec{y}\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}I_2 & I_2\\ -iI_2 & iI_2\end{pmatrix}\begin{pmatrix}\vec{a}\\ \vec{a}^\dagger\end{pmatrix}$$



- Find susceptibility matrices for output quadratures $\vec{x}_{out,1}=\Xi_{X,0}\vec{x}_{in,0}+\Xi_{X,1}\vec{x}_{in,1}$
- Cascading field quadratures:

Squeezing (SQ) → **Cavity** → **Amplification (AMP)**

A Cavity Model in Input-Output Theory

- From input to output quadratures

$$\vec{x}_{out,1} = \boldsymbol{A}_X \left(\boldsymbol{\chi}^0(\omega)\, \boldsymbol{S}_X^0\, \vec{x}_{in,0} + \boldsymbol{\chi}^1(\omega)\, \boldsymbol{S}_X^1\, \vec{x}_{in,1} \right) = \boldsymbol{\Xi}_{X,0}\, \vec{x}_{in,0} + \boldsymbol{\Xi}_{X,\mathbf{1}}\, \vec{x}_{in,1}$$

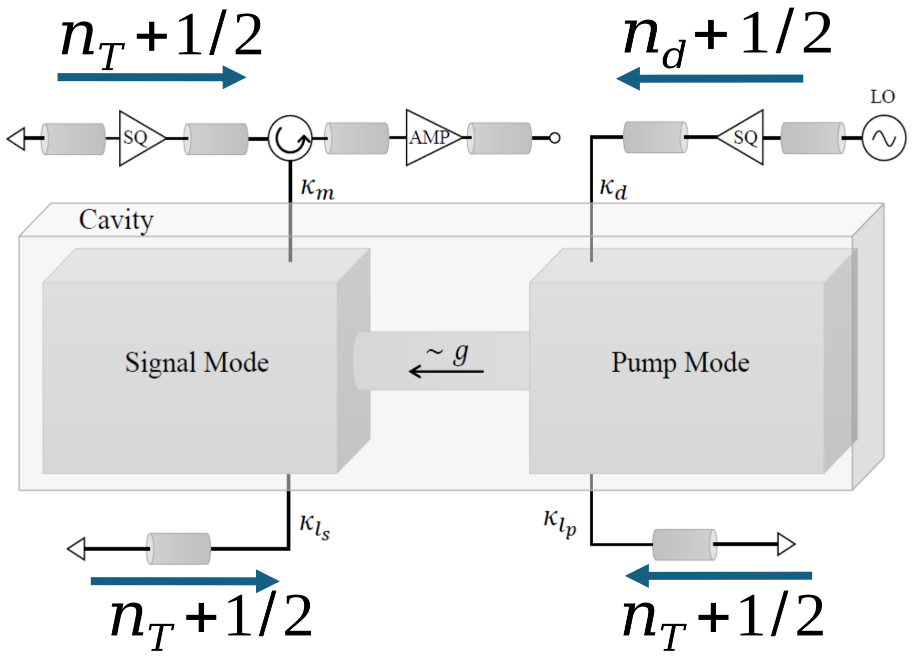
AMP signal output SQ pump input SQ signal input

- Squeezing (SQ) the signal mode

- Amplify (AMP) signal output

- Output spectral density $\boldsymbol{\Sigma}_{out,X,1} = \left\langle \left(\vec{x}_{out,1} \right)^* \left(\vec{x}_{out,1} \right)^T \right\rangle / 2\pi$

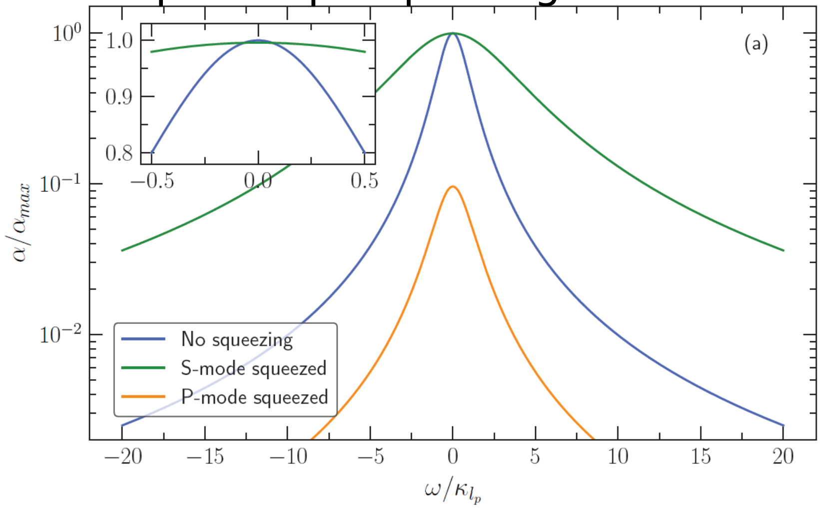
Improved Signal Visibili



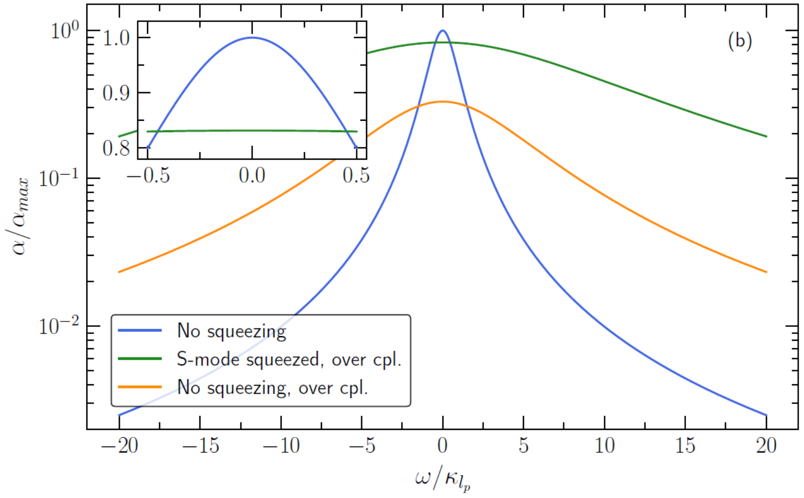
- First element in corresponds to measurement output port
- “Signal visibility” is SNR of

$$\alpha(\omega) \approx \frac{g^2 n_d \kappa_d \kappa_m / G_p}{(n_T+1/2)(\gamma(\omega)\kappa_l \kappa_m + g^2 \kappa_l \kappa_m + \beta(\omega)/G_s)}$$

Squeeze pump or signal mode

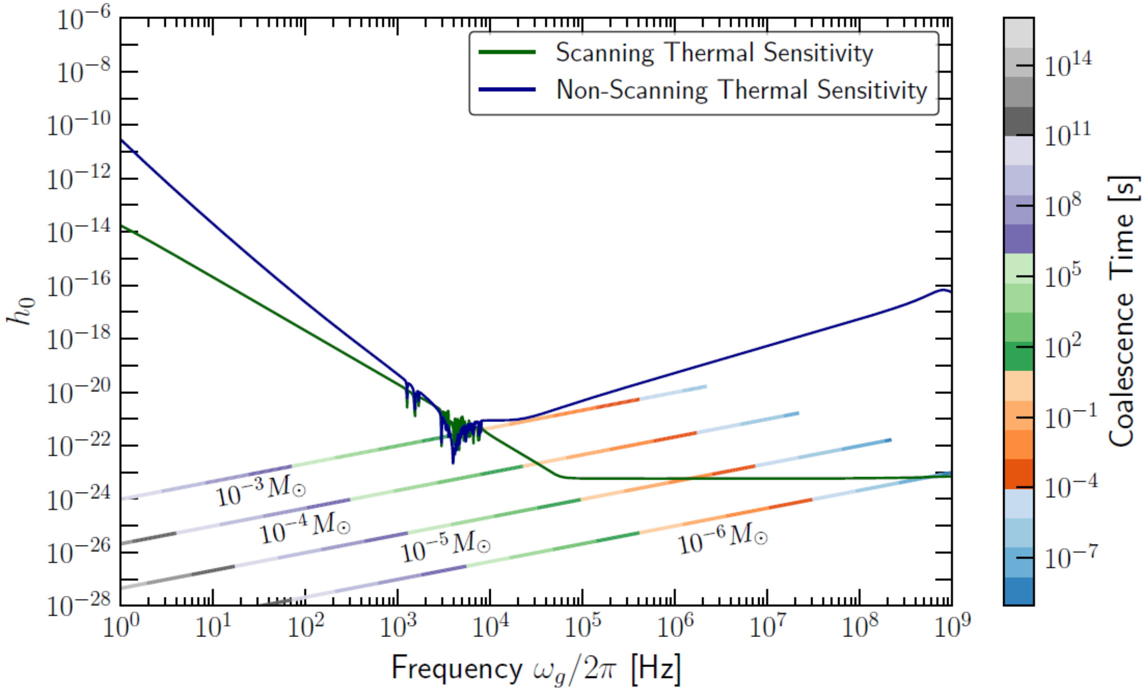


Squeeze signal mode and overcouple

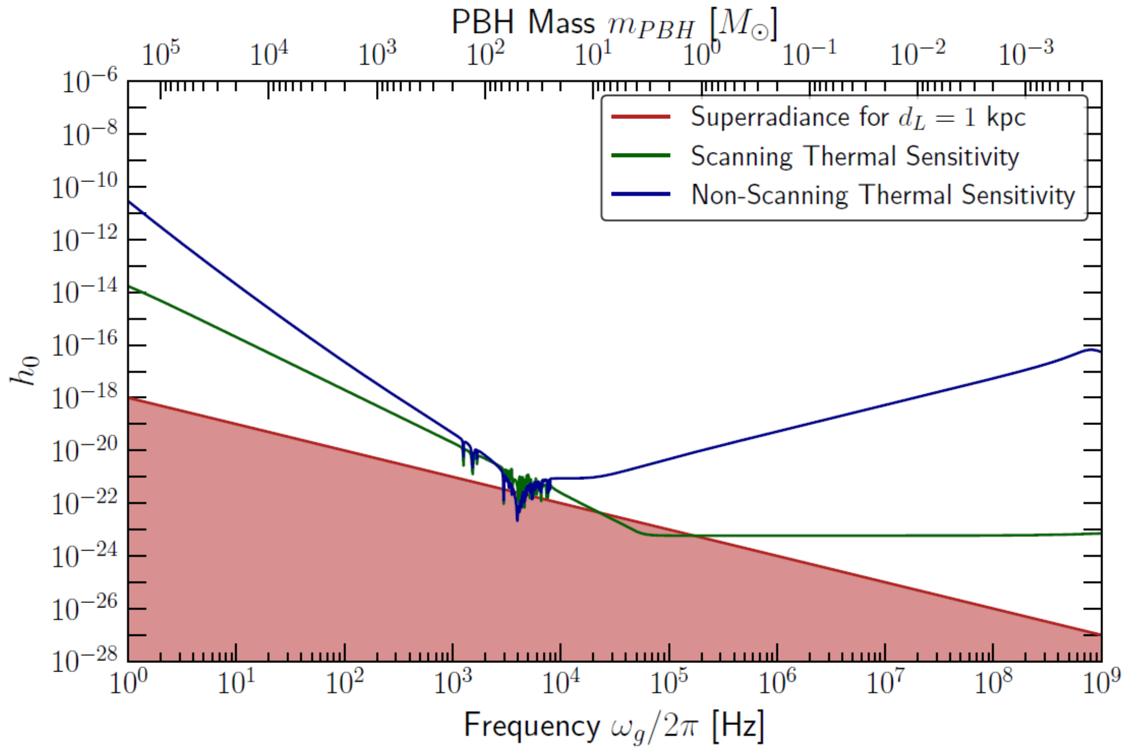


BSM Gravitational Wave Sources for MAGO

Equal mass primordial black hole binary mergers



Black hole superradiance from bosonic clouds



Conclusion

- Heterodyne Cavity experiments have a unique broadband sensitivity for
- Potential region, where direct and indirect conversion can be combined at
- In this spectral range a detector is likely operating at the quantum limit
- Inspection of a detector only using the direct conversion of the Gertsenshtein effect shows limited broadband sensitivity
- Construction of a quantized heterodyne cavity model within an input-output theory showed potential improvements with squeezed states

Ideas and Thoughts

- Expand the signal analysis to more realistic signal forms, like chirp or stochastic signals
- Quantify the detector sensitivity with less biased quantities, e.g. „effective noise strain“ or characteristic strain
- Optimizing detector geometries and tuning mechanisms for high-frequency GWs
- A detailed investigation of detector correlation and signal analysis to improve the sensitivity
- Experimentally test the opportunities of squeezing to enhance the broadband sensitivity of a quantum limited detector

Back up

The Proper Detector Frame

$$h_{00} = -\sum_{n=0}^{\infty} \frac{2(n+3)}{(n+3)!} x^k x^l x^{k_1} \ldots x^{k_n} (\partial_{k_1} \ldots \partial_{k_n} R_{0k0l})(g),$$ where $$R_{0i0j} = -\frac{1}{2}\ddot{h}_{ij}^{TT},$$

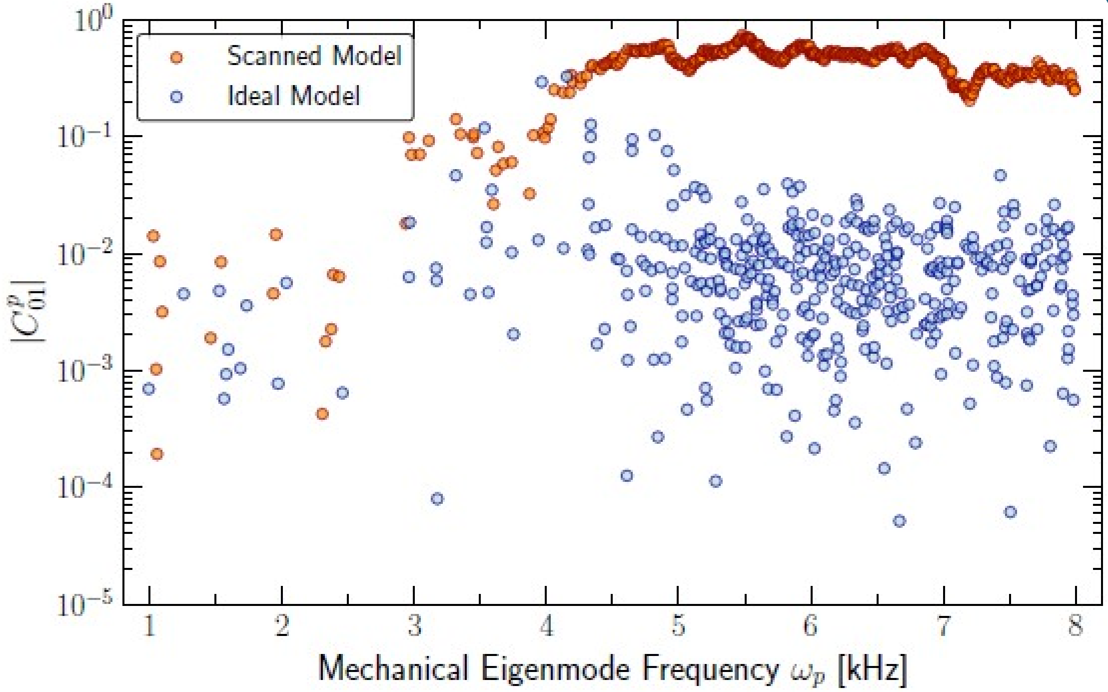
$$h_{0i} = -\sum_{n=0}^{\infty} \frac{2(n+2)}{(n+3)!} x^k x^l x^{k_1} \ldots x^{k_n} (\partial_{k_1} \ldots \partial_{k_n} R_{0kil})(g),$$ where $$R_{0kil} = \frac{1}{2}(\partial_l \dot{h}_{ki}^{TT} - \partial_i \dot{h}_{kl}^{TT}),$$

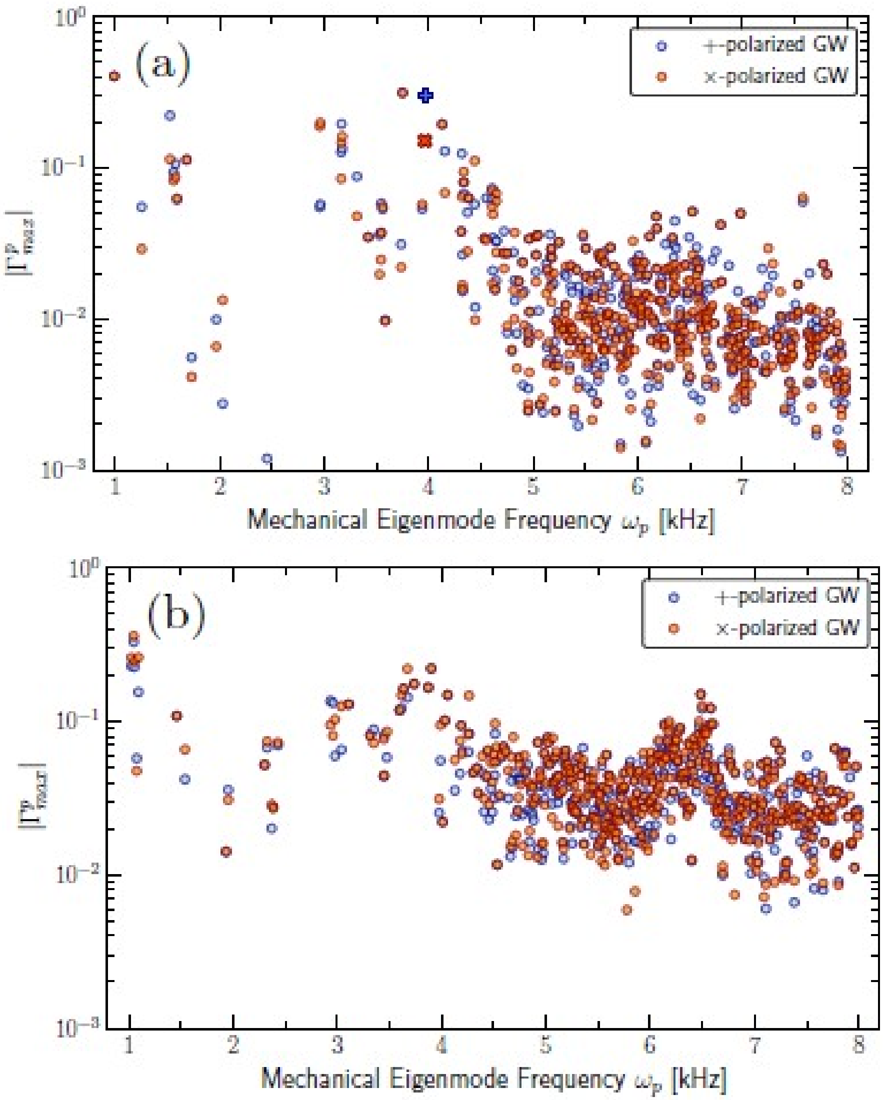
$$h_{ij} = -\sum_{n=0}^{\infty} \frac{2(n+1)}{(n+3)!} x^k x^l x^{k_1} \ldots x^{k_n} (\partial_{k_1} \ldots \partial_{k_n} R_{ikjl})(g).$$ where $$R_{ikjl} = \frac{1}{2}(\partial_k \partial_j h_{il}^{TT} - \partial_i \partial_j h_{lk}^{TT} - \partial_l \partial_k h_{ij}^{TT} + \partial_l \partial_i h_{jk}^{TT}),$$

and $$h_{ab}^{TT}(\vec{k}, t) = \begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix}_{ab} \times e^{i(\omega_g t - k_z z)}.$$

Numerical Evaluation of the Coupling

- Mechanical-EM and GW-mechanic

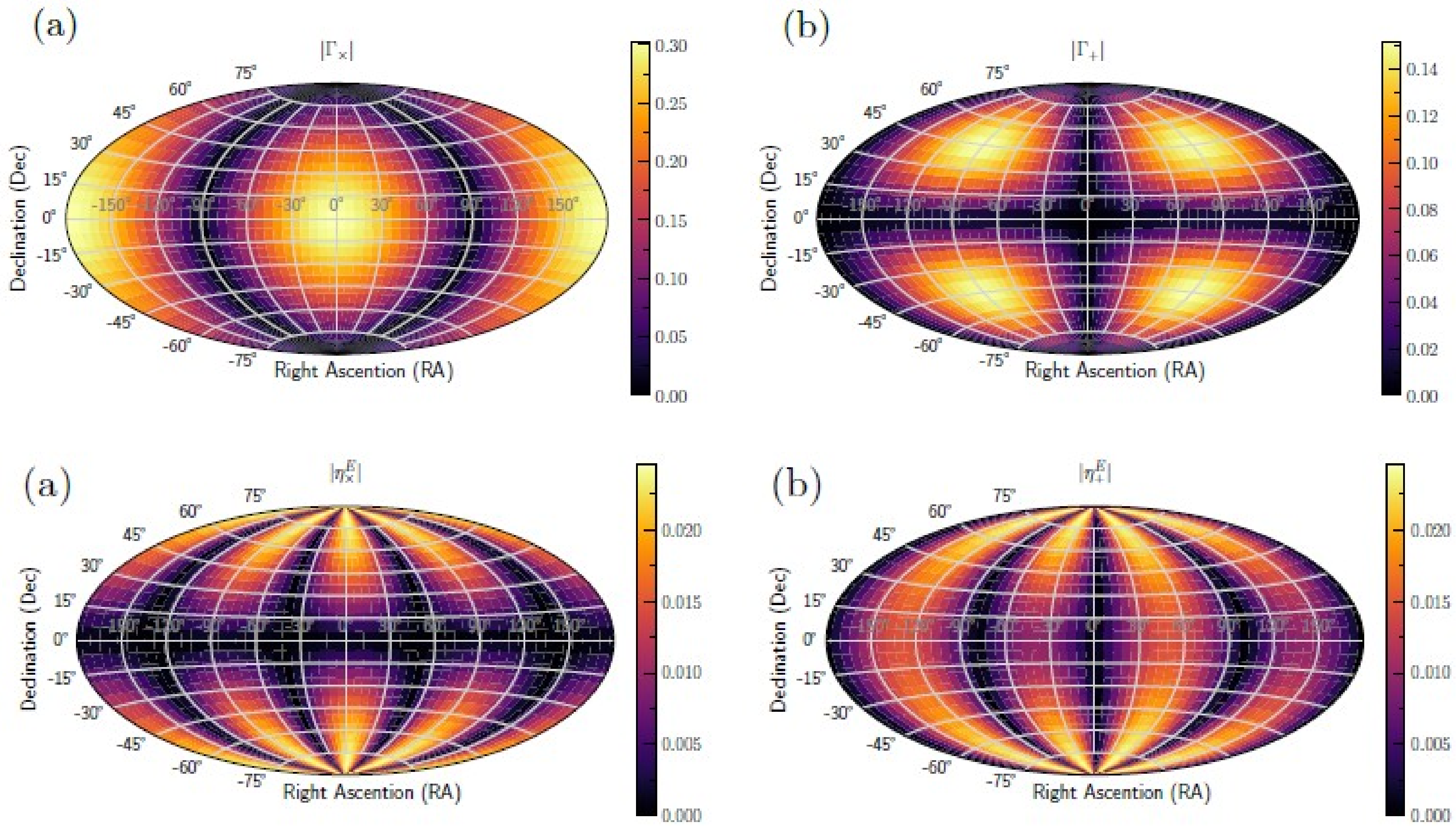




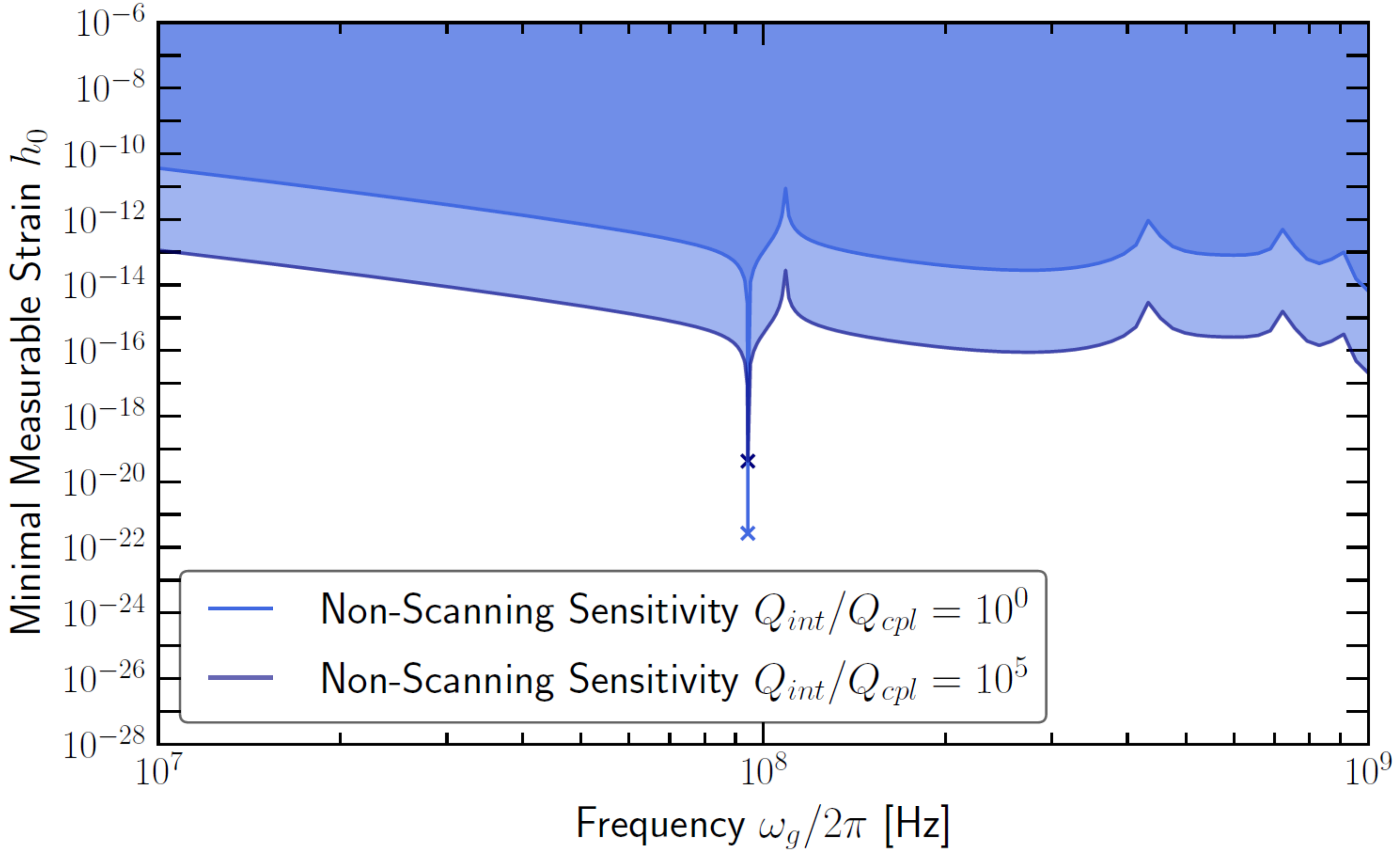
Gravitational Wave Coupling all Directions

GW-mechanical

GW-E-field



Overcoupling Heterodyne Cavity



Input-Output Theory

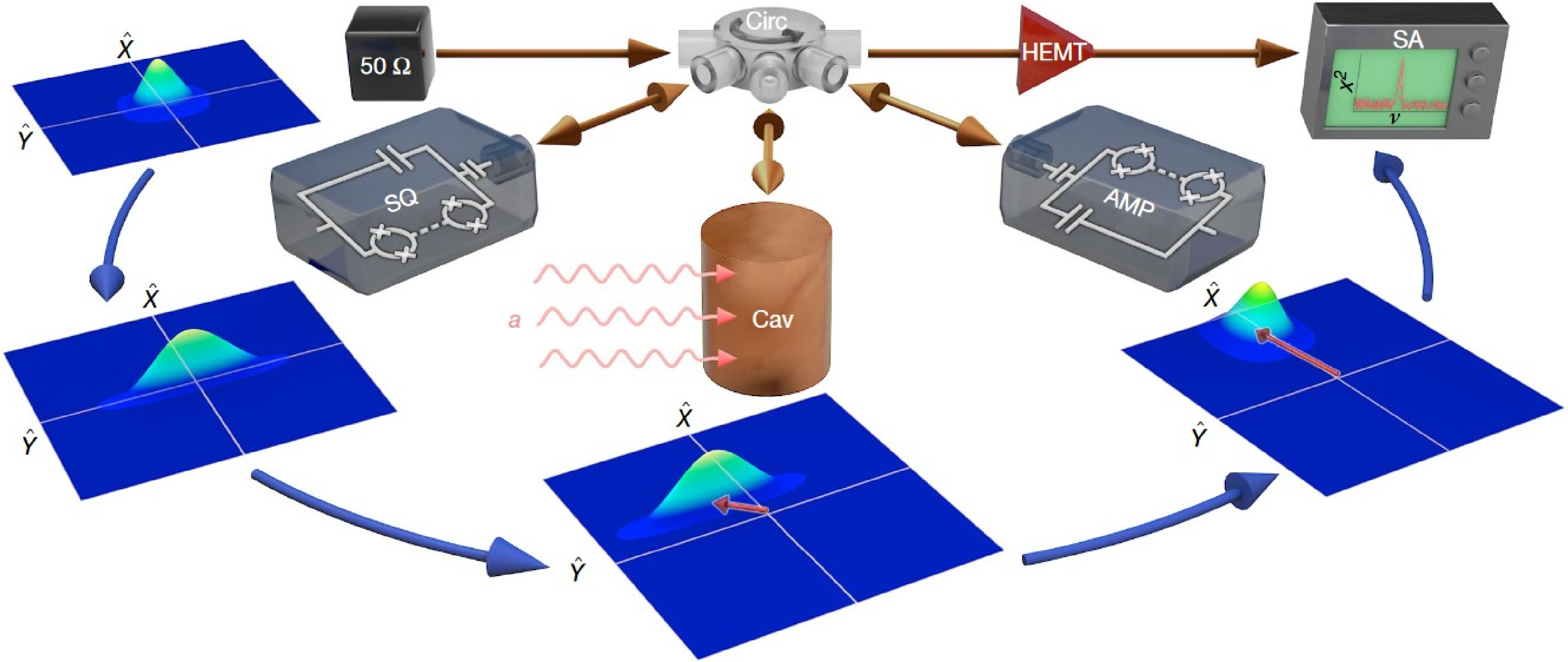
$$\hat{a}_{\rm in}(t) = (2\pi\rho)^{-1} \sum_q \exp\left[-i\omega_q(t-t_0)\right] \cdot \hat{a}_q(t_0)$$

$$\frac{d\hat{A}}{dt} = \frac{i}{\hbar}[\hat{H}_{\rm cav}, \hat{A}] - \frac{\kappa}{2}\hat{A} - \sqrt{\kappa}\hat{a}_{\rm in}(t),$$

Squeezing in HAYSTAC

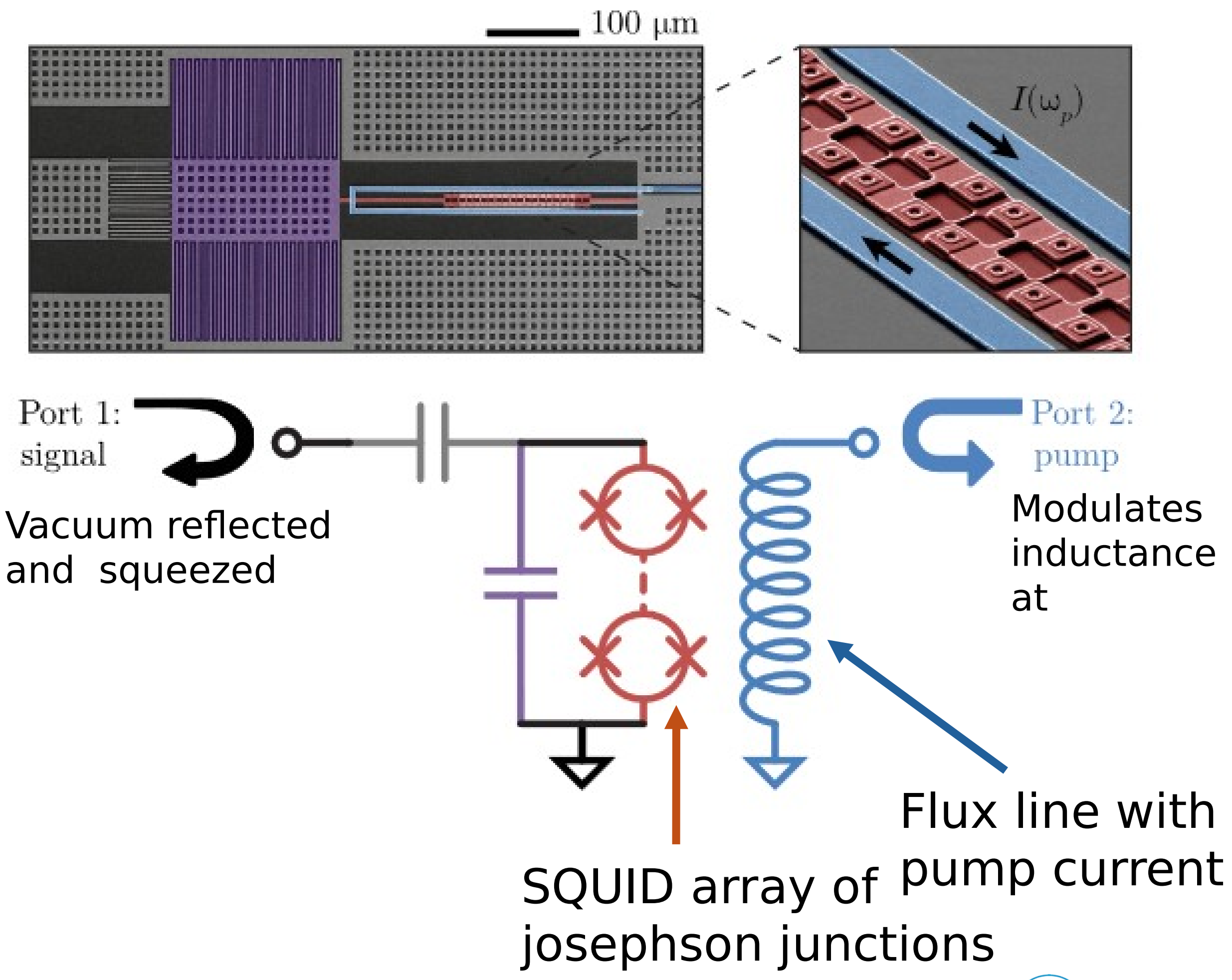
HAYSTAC, DOI: 10.1038/s41586-021-03226-7

High-electron-mobility transistor (HEMT) amplifier



Flux-pumped Josephson Parametric Amp.

- Parametric processes with **nonlinear** „optical“ devices
- crystal material/SQUID with nonlinear susceptibility
- amplified or deamplified depending on the phase
- In RF use Josephson Parametric Amplifier (JPA)
- Nonlinearities in Josephson-junction, effective inductance of an LC-circuit



Improved Models in Input-Output Theory

- Including transmission losses along all transmission lines

$$\alpha(\omega) \approx \frac{g^2 \lambda (1-\lambda+\lambda/G_p) n_d \kappa_d \kappa_m}{(n_T+1/2)\left(B(\omega)(1-\lambda)+\lambda\left[\gamma(\omega)\kappa_l \kappa_m + g^2 \kappa_l \kappa_m + (1-\lambda+\lambda/G_s)\beta(\omega)\right]\right)}$$

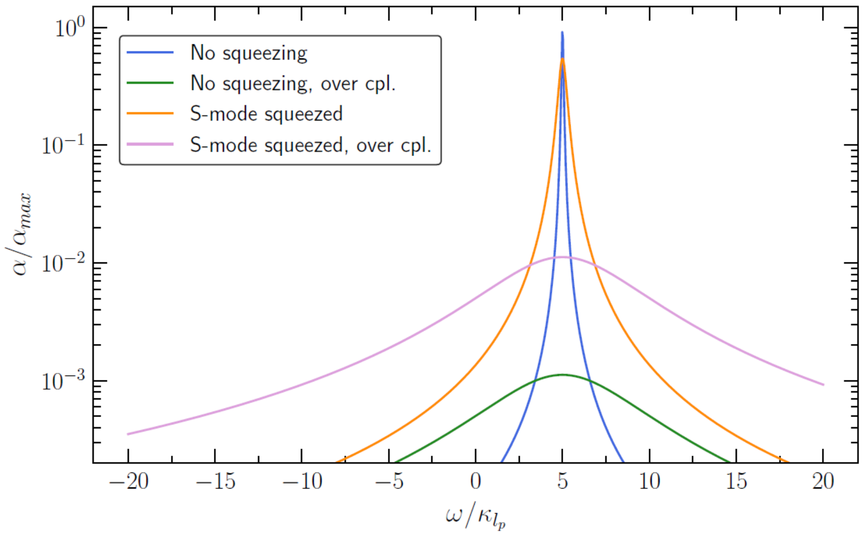
: New terms with transmission efficiency

- Add a third cavity with GW quanta to calculate for

$$\hat{H}_g = \hbar \omega_g (\hat{q}^\dagger \hat{q} + 1/2)$$

$$\hat{H}_{int} = \hbar g (\hat{q}^\dagger \hat{A}_0^\dagger \hat{A}_1 + \hat{q} \hat{A}_0 \hat{A}_1^\dagger)$$

$$\alpha(\omega_g+\omega_0) \approx \frac{q^2 n_d \kappa_d \kappa_m / G_p}{(n_T+1/2)(\kappa_l \kappa_m + q^2 \kappa_l \kappa_m + \text{¿} \beta(\omega_g+\omega_0)|^2 / G_s)}$$



A Cavity Model within Input-Output Theory

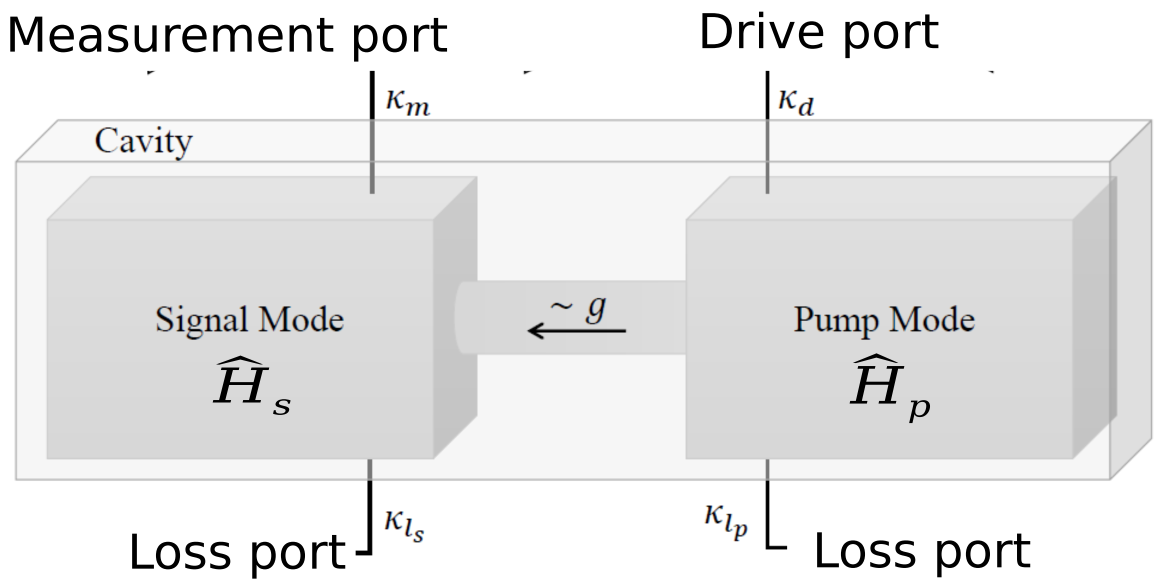
$$\hat{H}_{s,p} = \hat{H}_{cav} + \hat{H}_{bath} + \hat{H}_{int}$$

$$\hat{H}_{bath} = \sum \hbar \omega_q \hat{a}_q^\dagger \hat{a}_q$$

$$\hat{H}_{cav,s} = \hbar \omega_s \left(\hat{A}_s^\dagger \hat{A}_s + 1/2 \right)$$

$$\hat{H}_{cav,p} = \hbar \omega_p \left(\hat{A}_p^\dagger \hat{A}_p + 1/2 \right)$$

in rotating wave approx.



Markov approximtaion to simplify cavity and bath interaction

coupling of bath and cavity mode:

density of states: